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Modified model for cluster decay

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Abstract

George Gamow theorized the model of alpha-decay, which can be used to formulate the relationship between energy of alpha particle and the tunneling probability. In the present work we have employed Gamow's theory for evaluating the decay rates of cluster emitters. The $T_{1/2}$ of the cluster depends on the penetration probability. In this present work the square well model is modified by smoothening the value of the potential inside the nucleus which we will call as S-potential. The modification was made in the model by taking two points δ and ζ, where δ corresponds to the radius of the cluster nuclei and ζ corresponds to the Q value. We show that the calculated half-lives are in good match with the experimental $T_{1/2}$ within one order of magnitude. Gamow's theory has been experimentally verified using Geiger-Nuttall (G.N.) law. G.N. Law holds true for cluster decay using S-Model as can be seen by our calculations. A plot between the Q (MeV) and $T_{1/2}$(sec) of cluster is found to be linear. To our surprise the S-potential (Smoothened) gives a better match the experimental data as compared to Viola Seaborg method.

Keywords: Cluster decay, Pre-formation factor, penetration probability, Half-life, Geiger-Nuttall law.

Introduction

Cluster radioactivity is an interesting field for the nuclear physicists since early 80's. The spontaneous emission of nuclei with higher A values compared to α-particles and without neutron emission was proposed by Sandulescu et al[1]. Cluster decay was first seen by Rose and Jones[2] with Radium nucleus where carbon-14 cluster was emitted in the year 1984[2]. Similarly there was a report of emission of other clusters also[3].Even-even neutron rich nuclei were emitted in such decays and have been experimentally observed, except for ^{23}F[4]. In this present work the square well model is modified by smoothening the value of the potential inside the nucleus which we will call as S-potential or S-Model (S stands for smoothened potential) for a Gamow like theory i.e. square well potential in one dimension.

A simple formula for half-life was derived using the Schrödinger equation in one dimension. The $T_{1/2}$were calculated and was found to be in good match with the experimental $T_{1/2}$ available[4]. The trans-lead region means ^{14}C, $^{18,20}O$, $^{24,26}Ne$, $^{28,30}Mg$, ^{34}Si have the maximum cluster formation probability and minimum half-life, this shows that α like cluster are probable from trans-lead region[6-8]. The verification of Gamow's law was done by Geiger-Nuttall law and was found to be linear which shows the validity of Gamow's law of cluster radioactivity.

Model

The first theoretical approach of the alpha decay was given by George Gamow in the year 1928[9]. This was one of the strong evidence of the success of quantum mechanics. A particle partially bound within a potential well has a certain probability upon each encounter with the barrier of appearing as a free particle on the other side. Classically a particle cannot overcome a barrier but it tunnels according to quantum mechanics. In spherical symmetric potential there is a discontinuous jump of the potential which tends to be not physical because the force there becomes infinite. The detailed analysis of the spherical symmetric potential was studied previously[10]. For obtaining a better outcome, a modification was made in the model by smoothening the value of the potential inside the nucleus which we termed as S-potential or S-Model.

The modified model is sketched and shown in Figure-1.

Instead of flat line in the Gamow potential points A and C are joined by a smooth curve BD is joined by a straight line.

Point C corresponds to distance R from the origin and point D corresponds to distance ζ from the origin where $\zeta = \frac{2z_1z_2e^2}{E}$. B is a point chosen on the curve AC such that $\delta = \frac{\sqrt{2E/m_c}}{\omega}$ which corresponds to the radius of the cluster nuclei. CD corresponds to a linear drop in the maximum potential at C to cut off potential at D given by ζ corresponding to Q value. This leads to smoothening of the potential in the region AC and removes the abrupt change of potential at distance R.

The force is absent on the emitted cluster particle when it is present in the range $0 < r < \delta$ followed by a force of attraction present in the range of $\delta < r < R$.

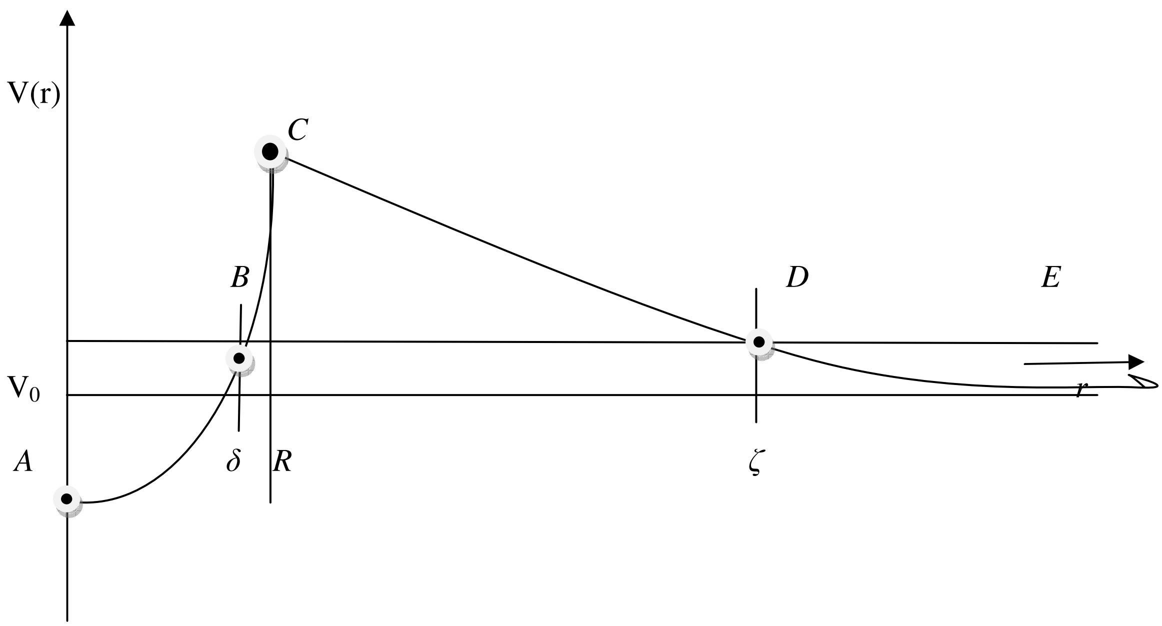

Figure-1: S-Model on the shape of V (r); the inside potential has been assumed to be of the (spherical) harmonic kind.

The coulomb repulsive force becomes active outside the barrier where $r \rightarrow R$. This approach would be closer to the actual physical process and shall give more accurate results.

The potential function is now:

$$V(r) = \frac{1}{2}m_c\omega r^2 \qquad \delta \leq r \leq R$$
$$= \frac{2z_1z_2e^2}{r} \qquad R \leq r \leq \zeta$$

The Gamow's factor "G", calculated to be:

$$G = \sqrt{\frac{2m_c}{\hbar^2}}\left[\int_\delta^R dr\sqrt{\left(\frac{1}{2}m_c\omega r^2 - E\right)}\right] + \int_R^\zeta dr\sqrt{\left(\frac{2Ze^2}{r} - E\right)} \quad (1)$$

$$G = \sqrt{\frac{2m_c}{\hbar^2}}\left[\left[\frac{R}{2}\sqrt{\frac{1}{2}m_c\omega^2R^2 - E} - \frac{\delta}{2}\sqrt{\frac{1}{2}m_c\omega^2r^2 - E} + \frac{E}{\omega\sqrt{2m_c}}ln\left(\frac{\delta m_c\omega^2+\omega\sqrt{2m_c}\sqrt{\frac{1}{2}m_c\omega^2\delta^2-E}}{Rm_c\omega^2+\omega\sqrt{2m_c}\sqrt{\frac{1}{2}m_c\omega^2R^2-E}}\right)\right] + \left[\frac{2Z_1Z_2e^2\pi^2}{h\,v_c} - \frac{8\pi e}{h}\sqrt{\frac{2\mu Z_1Z_2r_0}{2}}\right]\right] \quad (2)$$

Equation (1) shows the estimated decay rate after solving the model using Gamow's theory of alpha decay. Then the second term appears to be the correction in decay rate with respect to Gamow's result. The penetration probability is given by $P = e^{-2G}$

By substituting the value of equation (2) in the above expression

As decay constant; $\lambda = nP$ ... (3)

$$ln\,\lambda = \ln n - 2\sqrt{\frac{2m_c}{\hbar^2}}\left[\left[\frac{R}{2}\sqrt{\frac{1}{2}m_c\omega^2R^2 - E} - \frac{\delta}{2}\sqrt{\frac{1}{2}m_c\omega^2r^2 - E} + \frac{E}{\omega\sqrt{2m_c}}ln\left(\frac{\delta m_c\omega^2+\omega\sqrt{2m_c}\sqrt{\frac{1}{2}m_c\omega^2\delta^2-E}}{Rm_c\omega^2+\omega\sqrt{2m_c}\sqrt{\frac{1}{2}m_c\omega^2R^2-E}}\right)\right] + \left[\frac{2Z_1Z_2e^2\pi^2}{h\,v_c} - \frac{8\pi e}{h}\sqrt{\frac{2\mu Z_1Z_2r_0}{2}}\right]\right] \quad (4)$$

Before the cluster nuclei tunnels through barrier it exist in the parent nuclei leading to the production of cluster radioactivity[11]. We can write that the decay constant is given by the equation, $\lambda = nPP_0$, where: n is the frequency of assault. P is the penetration probability.

P_0 is known as the Pre-formation factor[12].

In general it is seen that the mass of the alpha particle is lighter than mass of the cluster formed.

$$ln\,\lambda = \ln n - 2\sqrt{\frac{2m_c}{\hbar^2}}\left[\left[\frac{R}{2}\sqrt{\frac{1}{2}m_c\omega^2R^2 - E} - \frac{\delta}{2}\sqrt{\frac{1}{2}m_c\omega^2r^2 - E} + \frac{E}{\omega\sqrt{2m_c}}ln\left(\frac{\delta m_c\omega^2+\omega\sqrt{2m_c}\sqrt{\frac{1}{2}m_c\omega^2\delta^2-E}}{Rm_c\omega^2+\omega\sqrt{2m_c}\sqrt{\frac{1}{2}m_c\omega^2R^2-E}}\right)\right] + \left[\frac{2Z_1Z_2e^2\pi^2}{h\,v_c} - \frac{8\pi e}{h}\sqrt{\frac{2\mu Z_1Z_2r_0}{2}}\right]\right] + ln\,P_0 \quad (5)$$

The above equation is the modified value of λ of emitted particles.

$T_{1/2}$ is defined as the time taken by a nucleus to reduce to half of its original values. It can be calculated by the formula:

$$log_{10}T_{1/2} = log_{10}0.693 - log_{10}\,\lambda \qquad (6)$$

By using equations (5) and (6) the decay constants and half lives of different nuclei can be calculated respectively. For a comparison we decided to calculate Log $T_{1/2}$(sec) using the Viola Seaborg formula for cluster radioactive elements[19].

Validity of Geiger - Nuttal Law for Cluster Decay: Gamow's theory can be verified using Geiger-Nuttall law. A correlation exists between the decay constant and the alpha kinetic energy such that large decay energies are usually associated with large decay constants (small half lives). A successful effort to express the correlations quantitatively was first made by Geiger and Nuttall[13]. Who found an empirical relation between the alpha particles' range in air and the decay constant. This regularity may be converted into an equivalent relation between the decay constants and alpha energy by use of the range-energy curve for the alpha particles in air. We have calculated the relation of GN law for cluster elements by modifying the law in context of cluster decay. This is equivalent Geiger Nuttall law for cluster decay.

Relation between $T_{1/2}$ and Q (MeV) $log_{10}T_{1/2} = \frac{1}{\sqrt{Q}}$

Our work is emphasized on the elements in the region *N*>126 and *Z*>82.

Table-1: Experimental Log $T_{1/2}$(sec) with the calculated Log $T_{1/2}$(sec) for S-Model and using Viola Seaborg formula[1,2].

Nuclei	Cluster	Q MeV	P_0	P Exp	P S-Model	Log $T_{1/2}$ S-Model	Log $T_{1/2}$ Viola Seaborg	Log $T_{1/2}$ (exp)	Log (T_{exp}/T_{cal})
^{222}Ra	^{14}C	31.84	5.152×10^{-5}	9.238×10^{-29}	5.05×10^{-33}	14.44	14.62	11.00	- 0.118
^{224}Ra	^{14}C	30.53	2.844×10^{-7}	2.920×10^{-31}	2.65×10^{-33}	17.79	17.43	15.77	-0.052
^{226}Ra	^{14}C	28.21	9.523×10^{-8}	2.982×10^{-36}	7.13×10^{-33}	18.06	22.83	21.25	0.070
^{226}Th	^{14}C	30.55	6.411×10^{-5}	3.823×10^{-33}	9.09×10^{-33}	14.89	20.03	15.30	0.011
^{226}Th	^{18}O	45.73	3.383×10^{-7}	6.618×10^{-31}	9.80×10^{-33}	17.10	20.78	15.30	-0.048
^{228}Th	^{20}O	44.73	5.038×10^{-11}	1.904×10^{-32}	7.65×10^{-33}	20.26	22.41	20.69	0.0147
^{230}U	^{22}Ne	61.40	7.402×10^{-11}	3.759×10^{-30}	1.11×10^{-33}	19.91	23.63	18.20	-0.038
^{230}Th	^{24}Ne	57.76	3.115×10^{-14}	3.771×10^{-33}	7.08×10^{-34}	23.34	25.63	24.61	0.023
^{230}U	^{24}Ne	61.36	1.537×10^{-10}	1.901×10^{-30}	8.34×10^{-33}	19.89	23.69	18.20	-0.038
^{232}U	^{24}Ne	62.31	2.775×10^{-14}	6.669×10^{-29}	8.01×10^{-33}	23.47	22.47	20.39	-0.061
^{234}U	^{24}Ne	58.84	2.529×10^{-15}	5.420×10^{-34}	8.04×10^{-33}	24.52	27.05	26.54	0.034
^{236}U	^{24}Ne	55.96	2.514×10^{-16}	1.307×10^{-38}	1.13×10^{-30}	23.38	31.15	32.17	0.138
^{234}U	^{26}Ne	59.47	2.869×10^{-15}	2.147×10^{-33}	1.05×10^{-33}	24.36	26.19	25.90	0.026
^{236}U	^{26}Ne	56.75	2.095×10^{-16}	8.146×10^{-38}	1.00×10^{-33}	25.53	50.76	31.47	0.090
^{232}U	^{28}Mg	74.33	2.90×10^{-17}	2.911×10^{-31}	9.47×10^{-33}	26.31	26.32	25.74	-9.51×10^{-3}
^{234}U	^{28}Mg	74.13	3.369×10^{-17}	2.471×10^{-31}	8.71×10^{-33}	26.34	26.57	25.74	-0.010
^{236}Pu	^{28}Mg	79.67	6.733×10^{-16}	6.694×10^{-27}	8.92×10^{-33}	25.08	23.38	20.00	-0.098
^{238}Pu	^{28}Mg	75.93	1.805×10^{-17}	1.165×10^{-31}	9.70×10^{-33}	26.56	27.46	26.34	-3.61×10^{-3}
^{236}U	^{30}Mg	72.48	2.617×10^{-18}	9.881×10^{-34}	3.16×10^{-33}	27.76	28.49	29.27	0.024
^{238}Pu	^{30}Mg	77.0	3.329×10^{-17}	2.143×10^{-30}	3.49×10^{-32}	25.75	26.26	24.83	-0.015
^{238}Pu	^{32}Si	91.21	8.471×10^{-19}	2.697×10^{-29}	3.86×10^{-32}	31.21	26.81	25.30	-0.091

Note: Denotes $T_{1/2}$(s) is from reference no 14, Q(MeV), P_o and P(exp) from reference no 15.

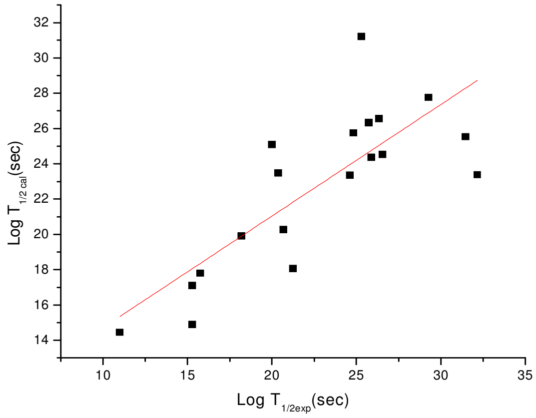

Figure-2: Experimental and calculated values of Log T1/2 in sec.

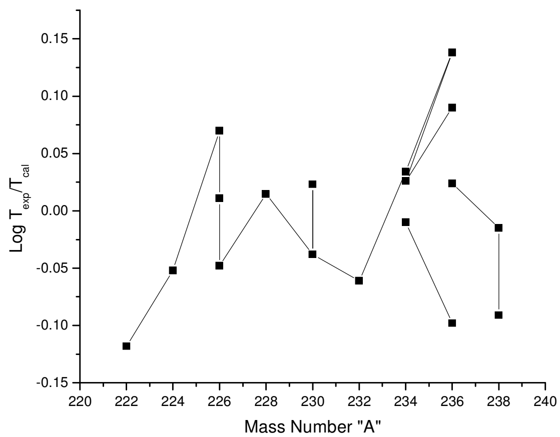

Figure-3: Relation between the A and Log (Texp /Tcal) of the cluster decay particles. The different isotopes of elements are connected by line segments. It can be observed that Log (Texp /Tcal) has a domain of ± 0.125.

Results and discussion

We have already studied the detailed analysis of the simple spherical symmetric potential[10]. We have seen that in simple spherical symmetric potential which was used by Gamow there is a potential immediately outside becomes unphysical. Therefore we smoothened these potential by taking the term δ and ζ . We modified the potential which we call the S-potential. With the help of pre-formation factor and the other details the half lives have been calculated[17,18]. Table-1 shows represents the parent nuclei and the emitted cluster nuclei, the Q values, pre-formation factor and penetration probability for experimental and for S-Model respectively. The radius of nuclei was kept at 1.2fm for our calculations in S-Model. The assault frequencies are kept in the magnitude of 10^{22} s^{-1}. The variation of v is very small. The Q-values of nuclei are taken from the experimental data[14,15]. The variation range of penetration probability (experimental) is very wide from 10^{-27} to 10^{-38}, much larger in comparison with the assault frequency[18]. The penetration probability of S-Model was calculated to be 10^{-30} to 10^{-34}. From figure 2; we see that linearity between experimental and calculated values of Log $T_{1/2}$ match to great degree. Figure 3 shows the graph between the A value and Log (T_{exp}/T_{cal}) of the cluster decay particles. The different isotopes of elements are connected by line segments. We conclude that Log (T_{exp}/T_{cal}) has a domain of ± 0.125.. Table 2 represents the energies of the different nuclei with their corresponding half-lives. A graph is plotted to show the variation of energies and the logarithmic value of half-lives in figure 4.The graph shows a linear nature which proves the validity of Geiger-Nuttall law. To our surprise the S-potential (Smoothened) gives a better match with the experimental data as compared to calculations using the Viola Seaborg formula as shown in Table-1.

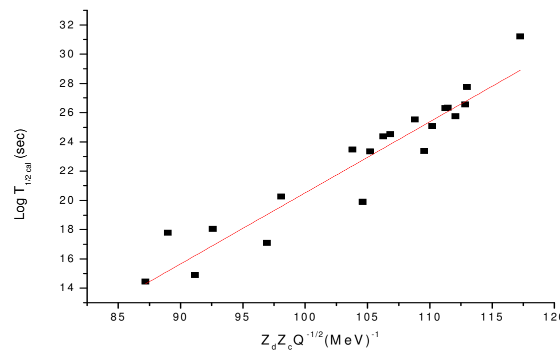

Figure-4: The comparison between the $Z_dZ_cQ^{-1/2}$ and the logarithmic value of calculated $T_{1/2}$ of cluster decay. The linear relation proves the Geiger-Nuttall law which shows the validity of Gamow's theory.

Conclusion

We have modified the potential used in Gamow's theory and applied to the problem of cluster decay which we call S-model. In this model instead of flat line in the Gamow potential we have taken points A and C which are joined by a smooth curve and further BD is joined by a straight line. Point C corresponds to distance R from the origin and point D corresponds to distance ζ from the origin where $\zeta = \frac{2z_1z_2e^2}{E}$. B is a point chosen on the curve AC such that $\delta = \frac{\sqrt{2E/m_c}}{\omega}$. CD corresponds to a linear drop in the maximum potential at C to cut off potential at D given by ζ corresponding to Q value. This leads to smoothening of the potential in the region AC and removes the abrupt change of potential at distance R. Using this modification we estimate the Gamow's factor G and consequently λ. We can conclude that the results obtained using the S-Model matches to a greater extent with the experimental data as compared to calculated $T_1/2$ using Viola-Seaborg formula.

Table-2: The tabular columns show the parent, the emitted cluster particle, Z_d ,Zc, Q-values, Z_d Z_c $Q^{-1/2}$,the logarithmic values of $T_{1/2}$ (sec) by both experiment and calculation of S-Model[1,2].

Nuclei	Cluster	Z_d	Z_c	Q MeV	$Q^{-1/2}$ MeV	$Z_d Z_c Q^{-1/2}$ $(MeV)^{-1}$	Log $T_{1/2}$ (data)sec	Log $T_{1/2}$ (cal)sec	$T_{1/2\ sec}$ (data)	$T_{1/2\ sec}$ (cal)
^{222}Ra	^{14}C	82	6	31.84	0.1772	87.18	11.00	14.44	1.0×10^{11}	2.75×10^{14}
^{224}Ra	^{14}C	82	6	30.53	0.1809	89.00	15.77	17.79	5.9×10^{15}	6.16×10^{17}
^{226}Ra	^{14}C	82	6	28.21	0.1882	92.59	21.25	18.06	1.8×10^{21}	1.14×10^{18}
^{226Th}	^{14}C	84	6	30.55	0.1809	91.17	15.30	14.89	$>2.0\times10^{15}$	7.76×10^{14}
^{226Th}	^{18}O	82	8	45.73	0.1478	96.95	15.30	17.10	$>2.0\times10^{15}$	1.25×10^{17}
^{228Th}	^{20}O	82	8	44.73	0.1495	98.07	20.69	20.26	5.0×10^{20}	1.81×10^{20}
^{230}U	^{22}Ne	82	10	61.40	0.1276	104.63	18.20	19.91	$>1.6\times10^{18}$	8.12×10^{19}
^{230}Th	^{24}Ne	82	10	57.76	0.1315	105.2	24.61	23.34	4.1×10^{24}	2.18×10^{23}
^{230}U	^{24}Ne	80	10	61.36	0.1276	104.63	18.20	19.89	$>1.6\times10^{18}$	7.76×10^{19}
^{232}U	^{24}Ne	82	10	62.31	0.1266	103.81	20.39	23.47	2.5×10^{20}	2.95×10^{23}
^{234}U	^{24}Ne	82	10	58.84	0.1303	106.84	26.54	24.52	3.5×10^{26}	3.31×10^{24}
^{236}U	^{24}Ne	82	10	55.96	0.1336	109.55	32.17	23.38	1.5×10^{32}	2.39×10^{23}
^{234}U	^{26}Ne	82	10	59.47	0.1296	106.27	25.90	24.36	8.1×10^{25}	2.29×10^{24}
^{236}U	^{26}Ne	82	10	56.75	0.1327	108.81	31.47	25.53	3.0×10^{31}	3.38×10^{25}
^{232}U	^{28}Mg	80	12	74.33	0.1159	111.26	25.74	26.31	5.5×10^{25}	2.04×10^{26}
^{234}U	^{28}Mg	80	12	74.13	0.1161	111.45	25.74	26.34	5.6×10^{25}	2.18×10^{26}
^{236}Pu	^{28}Mg	82	12	79.67	0.1120	110.20	20.00	25.08	1.0×10^{20}	1.20×10^{25}
^{238}Pu	^{28}Mg	82	12	75.93	0.1147	112.86	26.34	26.56	2.2×10^{26}	3.63×10^{26}
^{236}U	^{30}Mg	80	12	72.48	0.1174	112.99	29.27	27.76	1.9×10^{29}	5.75×10^{27}
^{238}Pu	^{30}Mg	82	12	77.0	0.1139	112.07	24.83	25.75	6.7×10^{24}	5.62×10^{25}
^{238}Pu	^{32}Si	80	14	91.21	0.1047	117.26	25.30	31.21	2.0×10^{25}	1.62×10^{31}

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