



## Setting up a geoid model in Cotonou

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### Abstract

*The reference considered for altimetry measurements in Benin would be accurate only if the geoid of all French West Africa was the same as the geoid of Benin. But it is important to know precisely, the geoid of the place to determine the altitude. The Republic of Benin does not yet have a national model of geoid. The definition and modeling of the geoid of Benin are imperative to facilitate the task to the actors of measurement and localization. To better determine the position of the points to be observed, a theoretical grid of 1km was applied throughout the territory of the Commune of Cotonou and led to the choice of a hundred points. Raw observational data was calculated using PrepaComp software (PrepaComp: Software for switching from Leica GSI format to OBS format) developed by IGN-France for the determination of elevation differences. The geometric geoid determined on the leveled points and the gravitational world geoid will result in two different reference planes. The gap between these two planes is called the trend. The undulation N of the geometric geoid at each of these points has been determined. So the geoid Nglc\_2017 (Rippling of the local geoid of Cotonou) determined is an accuracy of two centimeters (2cm).*

**Keywords:** Geoid, altitude, precision, trend, ondulation.

### Introduction

Since the 1990s, the measurement of the location is done by satellite positioning. Precision of the positioning by GPS method has been steadily expanding and accelerated in the 2000s with the advent of GNSS networks. The GNSS technique provides information on the vertical component that is not directly usable. The passage of this information to "an altitude above average sea level", which is usable by everyone, requires the exploitation of a model called "geoid model".

**Problem statement:** In the world, geoid models vary from one country to another. Thus, several global models and national models exist. The best-known global models are EGM96 (Earth Gravitational Model 1996) and EGM08 (Earth Gravitational Model 2008). Their Overall accuracy is estimated at the metric level for EGM96 and in the range of 15 to 20cm for EGM08. National models such as RAF08 in France can reach an accuracy of 1 to 2cm. The general leveling network of Benin, the platform of the current orthometric height measurement system in Benin, was established between 1947 and 1963 by direct leveling methods. The reference for the altitudes was defined as the mean sea level as determined from data collected by the tide gauge of the Dakar Hydrographic Service.

The reference considered for altimetry measurements in Benin would be accurate only if the geoid of all French West Africa was the same as the geoid of Benin. Which can not be possible. But it is important to know precisely, the geoid of the place to

determine the altitude. The Republic of Benin does not yet have a national model of geoid. The definition and modeling of the geoid of Benin is imperative to the actors of measurement and localization.

### Methodology

To better determine the position of the points to be observed, a theoretical grid of 1km was applied throughout the territory of the Commune of Cotonou and led to the choice of a hundred points. Raw observational data was calculated using PrepaComp software developed by IGN-France for the determination of elevation differences. These height differences determined in the first leg are compared with that of the return and then the expected tolerance in order to validate the leveling section realized. The altitude of all the points is determined by direct leveling with a Leica DNA03 level and an invar of two (02) meters. A stability check was made from the triplets, the closure of the paths did not exceed five (05) millimeters with an average of closures less than two (02) millimeters. This leveling allowed to set up forty (40) macaroons on the sections of leveling.

Figure-1 shows the configuration of the mesh made in the context of this study. For a good estimation of the signal, the method of kriging was used. Noise is obtained by subtracting the estimated signal from the effective residue<sup>1</sup>.

According to Gosselin<sup>2</sup>, point kriging is used to interpolate a point value from a dataset.



**Figure-1:** Project of theoretical grid of 1 km of the points to be determined with the GNSS and the direct leveling on the city of Cotonou<sup>3</sup>.

**Calculation of the geoid of the city of Cotonou:** The geoid is an equipotential surface of the Earth's gravitational field. This surface is very close to the shape of the Earth under the continents and coincides perfectly with the average level of the seas at rest<sup>4</sup>. In theory, the geoid determined on the leveled GPS points and the gravitational world geoid should coincide. However, in practice, they will lead to two different reference planes because of the long wavelength errors present in the global model. They are random sure the input data used for each of these methods.

In general, the deviations  $dNi$  between the geoid gravimetric and geometric decomposed into two parts:  $dNi = E(dNi) + vi$ . The deterministic term  $E(dNi)$  denotes the trend and models the long-wavelength variations of the  $dNi$  deviations. The random term  $vi$  is the residual and models the variations of the  $dni$  deviations at short wavelength.

Residues ( $vi$ ) have a zero average. We say that there is a tendency when  $E(dNi)$  is non-constant in space: we also say that the mean is non-stationary. Once the trend is detected, the tool "trend analysis Geostatistical Analyst" arcGIS software will define which order (first, second, third, ...) is this trend. For this, the tool projected the points in two directions North and East on map plane. A polynomial curve has been adapted to each projection.

The order of the trend was determined by analyzing the shape of the polynomial curve that adjusts the projected data. i. Trend surface of order 1 (trend plane):  $Z = AX + BY + C$ , ii. 2nd order trend surface (quadratic surface):  $Z(X, Y) = A_{00} + A_{01} Y + A_{02} Y^2 + A_{10} X + A_{11} XY + A_{20} X^2$ , iii. Trend surface of order  $n$ :  $Z = AX + BY + \dots + q.X^n Y^n + w$ .

The first and second order trend surfaces are the most used because their coefficients are not too numerous and can be interpreted.

Residues ( $vi$ ) were then analyzed by separating them into signal ( $si$ ) and noise ( $ni$ ):  $Vi = si + ni$ . The signal represents the correlated part of the residues and the noise, uncorrelated errors.

The signal was determined by interpolating the residuals of the neighboring points by the kriging method which is a statistical interpolation method. The noise was obtained after subtracting the estimated signal from the residue.

In this section, the coefficients  $A_{00}$ ,  $A_{01}$ ,  $A_{02}$ ,  $A_{10}$ ,  $A_{11}$  and  $A_{20}$  The trend and interpolated signal values were applied to each node of the selected global gravity geoid grid to make it consistent with Benin's General Leveling Network. This geoid is called ripple of the local geoid of the city of Cotonou  $N_{glc\_2017}$ : i.  $N_{glc\_2017} = N_{GM} - \text{trend} - \text{signal}$ , ii.  $N_{GM}$ : Global gravitational geoid ripple, iii.  $N_{glc\_2017}$ : Rippling of the local geoid of Cotonou.

**Geometric geoid calculation (leveled GPS points):** After determining the geoid value on GPS points leveled by a single difference, the geoid model on the city of Cotonou is created from the altitude anomaly values for each observed GPS point. A total of fifty-four (54) leveled GPS points were determined in the city of Cotonou.

**Validation of leveled GPS points:** To validate the leveled GPS points, it is necessary to combine the GPS observations of a semi-points and the attachment of these points to Benin's

general leveling network. Thus disposing of a semi of specific points in the two reference systems, the undulation  $N$  of the geoid was determined at each of these points from the following relation:  $N = h - H$  ( $h$ : the height above the ellipsoid,  $H$ : the height above the quasi-geoid).

Validation of leveled GPS points consisted of detecting outliers by nearby point analysis theory. For corrugations, it can therefore be said that spatially distributed objects are correlated with each other according to their respective distances. Aberrant values are detected by excessive relative quantities relative to their nearest neighbors. The "Neighbourmap of the GeoXp packages" function in the statistical software R can be used to

detect these aberrant values using the dispersion diagram (Scatterplot). They appear as points far from the diagonal. The Figure-2 shows the aberrant points by the analysis of the correlation of the neighboring points. The points in blue that move away from the diagonal are considered aberrant and will not be used in the rest of the calculations. A simple click on the points aligned on the diagonal (points in red) makes it possible to display on the map the points validated. This study detected ten (10) aberrant points on all fifty-four (54) leveled GPS points. Of the remaining forty-four (44) points, thirty-four (34) points are used to adapt the global geoid to the leveled GPS points and ten (10) were used to assess the accuracy of the determined final geoid.

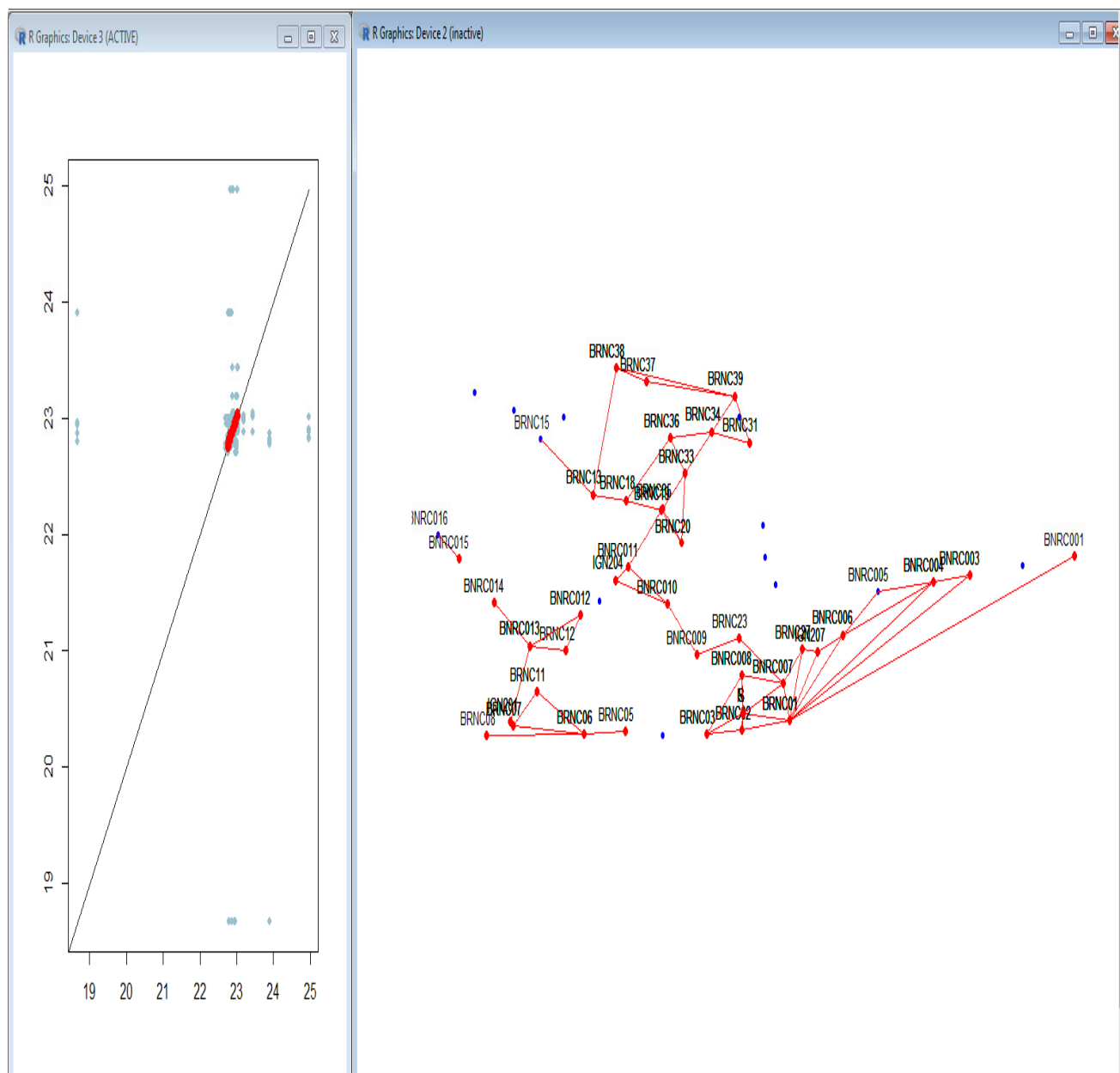


Figure-2: Dispersion diagram (Scatterplot).

## Results and discussion

**Evaluation of global models (EGM96 and EGM08) compared to RNGB (General Leveling Network of Benin):** This evaluation was made by estimating absolute and relative accuracy.

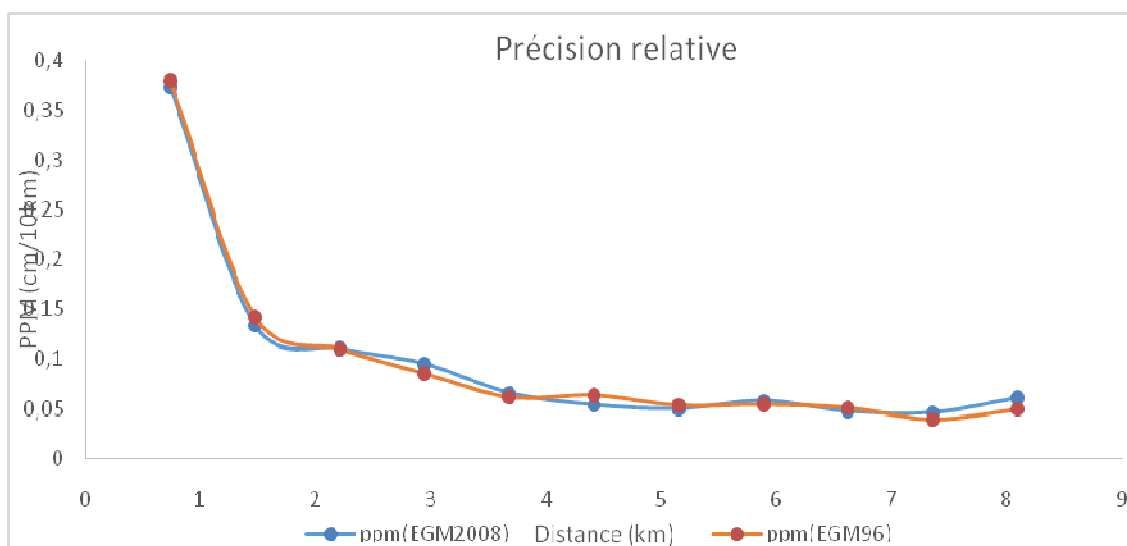
**Absolute accuracy:** The statistical analysis of the deviations obtained from the ripples of the global models (EGM96 and EGM08) with the leveled GPS points is given in Table-1.

**Relative precision:** The two solutions give approximately the same precision, but it is the model EGM08 which was retained for the continuation of the work of realization of the grid of conversion of ellipsoidal heights.

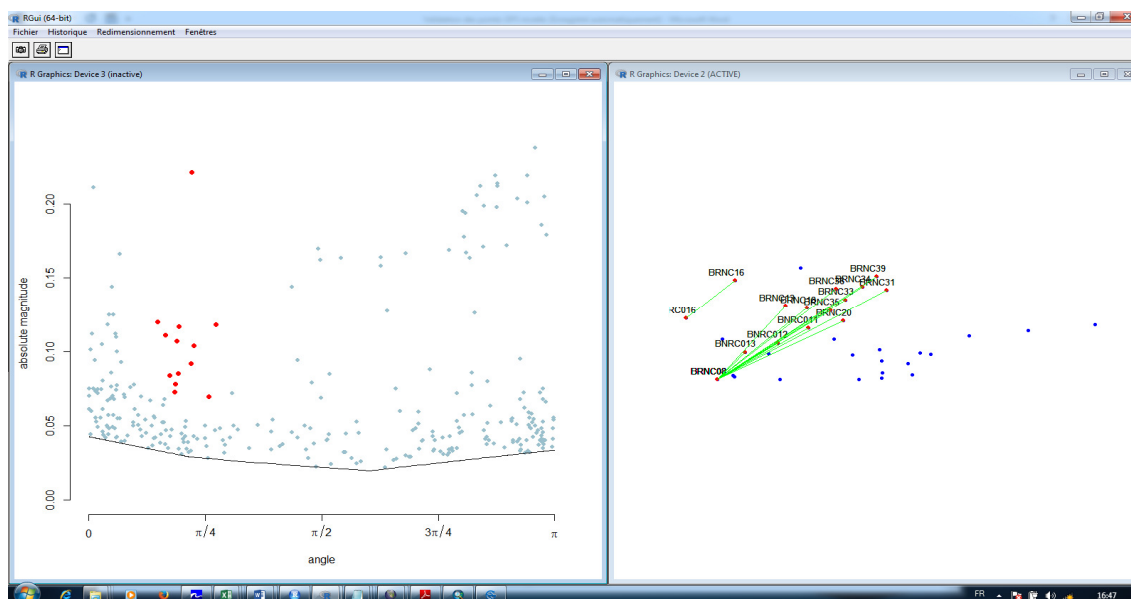
**Table-1:** Results of the assessment of the differences between the EGM96 and EGM08 and the RNGB.

Deviation	Min (m)	Max (m)	Rate (m)	Standard deviation (m)
RNGB-EGM08	-0,16	-0,02	-0,07	0,03
RNGB-EGM96	-0,17	-0,28	-0,21	0,03

**Global trend search:** In Figure-4, the selection of pairs of points (in red) above the angle  $\pi/4$  that strongly deviate from the scatter plot reveals a disparity between the southwestern regions of Cotonou and the northeast from the region.



**Figure-3:** Relative precision of the two global models of geoid used<sup>3</sup>.



**Figure-4:** Selection of points couples<sup>5</sup>.

It has thus been defined to apply a deterministic trend, a function of longitude and latitude on all of these data. Once the trend has been detected as well as its direction, the trend analysis tool in the geostatistical analyst section of arcGis will determine the order of the polynomial regression to be applied to these data. In Figure-5, it can be seen that the polynomial curves which adjust the projected point clouds on the various axes (X, Z) and (Y, Z) are slightly curved respectively upwards and downwards, which leads to choose a second degree polynomial.

**Adaptation of the gravimetric geoid EGM08 to leveled GPS points:** After the trend analysis, a simple second degree polynomial transformation was therefore used to remove the long wavelength errors produced by the global model EGM2008, Table-2 gives the parameters of the polynomial regression of 2<sup>th</sup> degree after adaptation to the leveled GPS points and Table-3 gives the statistical results of the residuals after removal of the bias (7cm) and the trend in the EGM2008 model.

**Table-2:** Parameters of the adaptation transformation of the determined geoid.

A <sub>00</sub>	A <sub>01</sub>	A <sub>02</sub>	A <sub>10</sub>	A <sub>11</sub>	A <sub>20</sub>
-1398, 126758	365, 99695	-22, 3699	190, 95158	-33, 35705	4, 5196982

**Table-3:** Results of the statistical evaluation of residues after bias and trend suppression in the EGM2008 model.

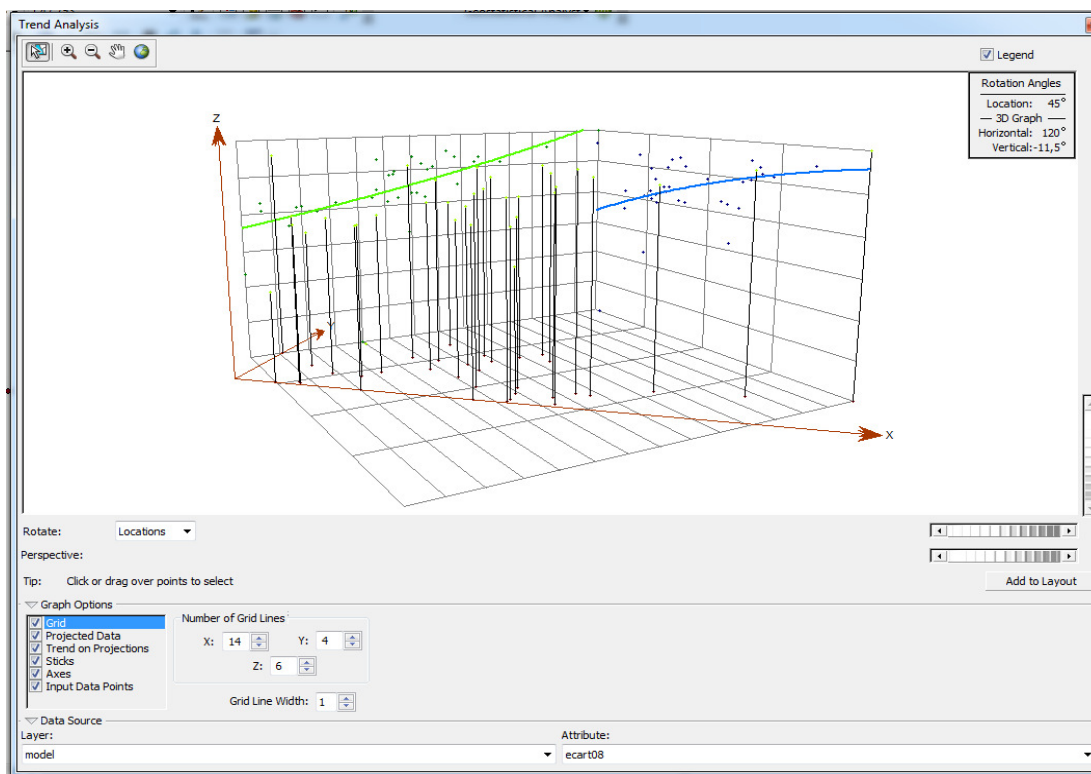
	Min	Max.	average	Standard deviation
Egm2008-through	-0,084	0,060	0,000	0,030
Egm2008-trend	-0,035	0,057	0,000	0,021

**Relative precision:** Table-4 and Figure-6 show the relative accuracy of the EGM2008 model after removal of bias and trend.

**Table-4:** Relative precision.

Average mileage	Egm2008-through (m)	Egm2008-trend
0,5 ~ 1,5 km	0,253	0,250
1,5 ~ 6 km	0,072	0,058
6 ~ 8 km	0,051	0,029

By comparing the standard deviations of the results, we note that the suppression of the trend improves the absolute and relative accuracy of the model.



**Figure-5:** Geostatistical trend analysis tool from arcGis<sup>5</sup>.

**Signal interpolation:** Table-5 shows the standard deviations of the residuals, signals and noise after interpolation by the EGM2008-trend geoid residue kriging method.

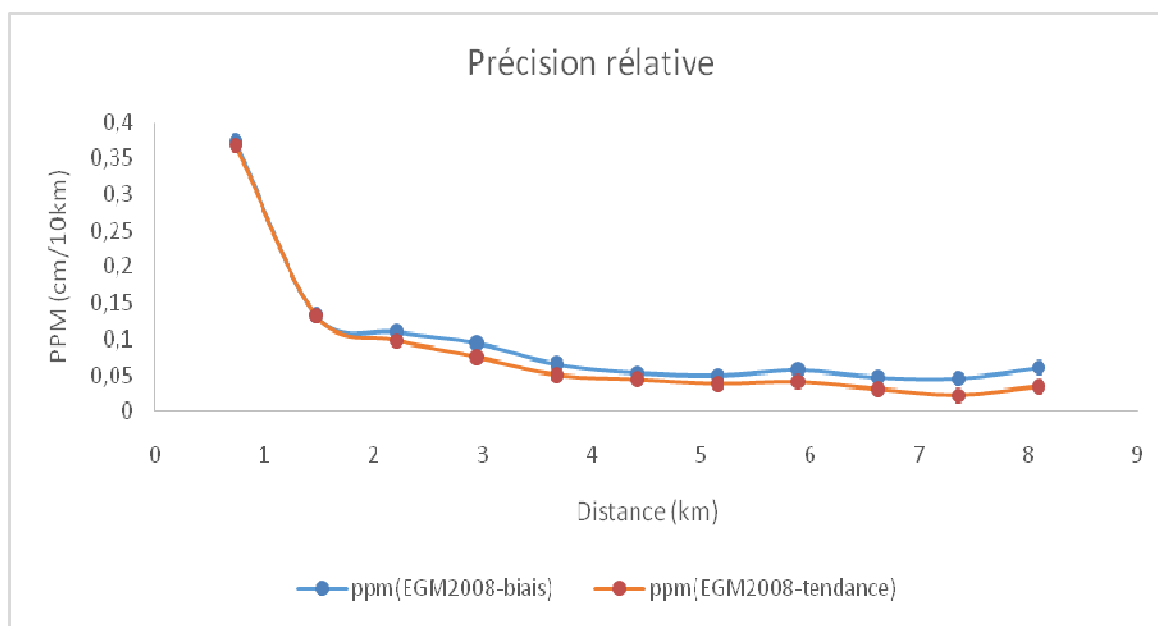
**Table-5:** Evaluation of standard deviations after trend and signal<sup>5</sup>.

Standard deviation	Signification	Value
$\sigma_v$	Standard deviation of residues	0,021
$\sigma_s$	Standard deviation of signals	0,006
$\sigma_n$	Standard deviation of noise	0,020

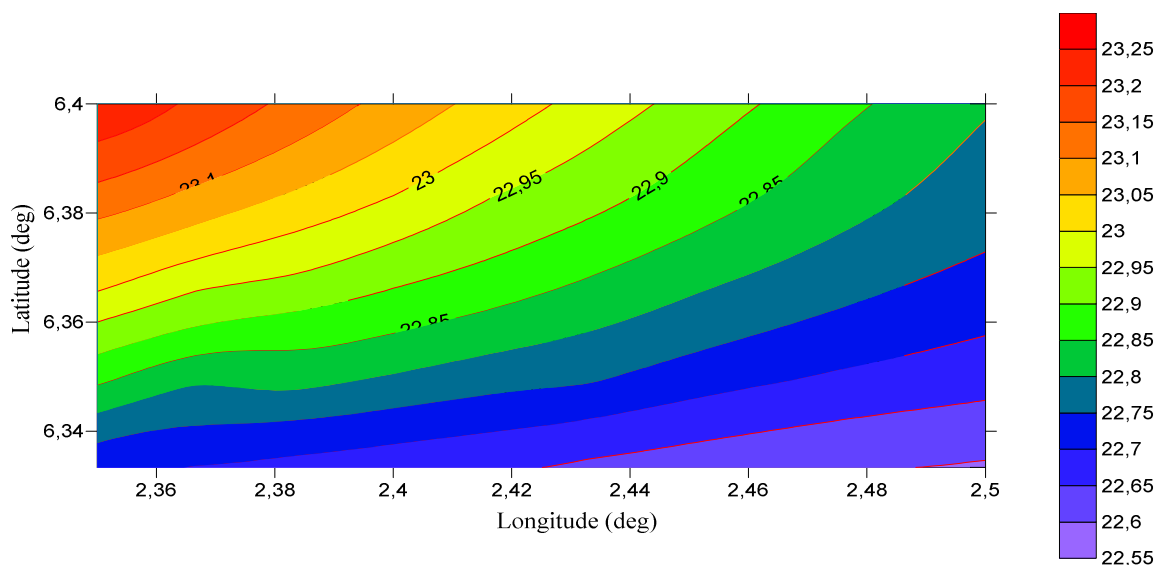
The main part of the residues is uncorrelated. In fact, the noise is almost equal to the residue and this is probably due to faults on the altitudes (displaced benchmarks) or incorrectly positioned GPS antennas.

**Adapt the world geoid grid to the Benin altimetric reference system:** World geoid are mentioned in Figure-7, 8 and Table-6.

**Evaluation of the geoid accuracy Ngic\_2017:** This evaluation is made from 10 pre-selected points. It allowed to have the results contained in the following Table-6. Table-7 shows the different differences obtained.

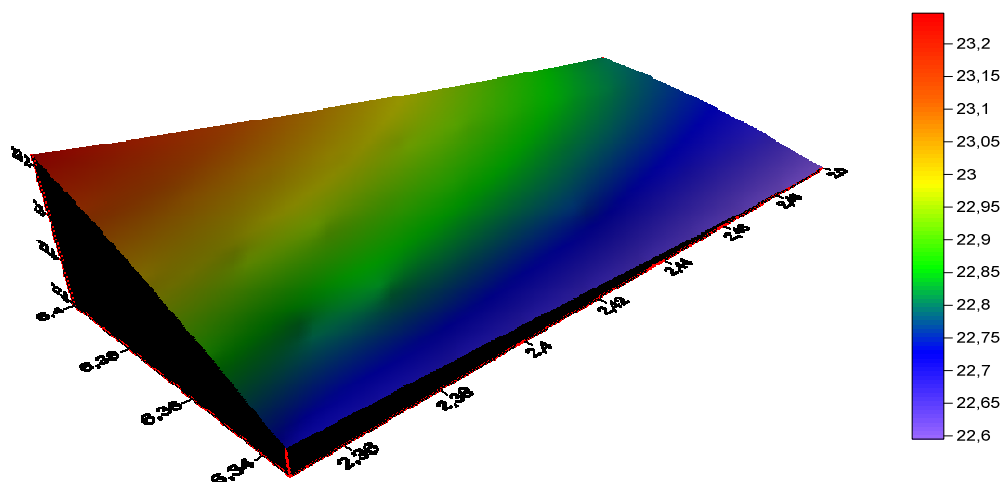


**Figure-6:** Relative precision (ppm = mm / 1km)<sup>5</sup>.



**Figure 7:** Local geoid of the city Cotonou (EGM2008-trend-signal), the Ngic\_2017<sup>5</sup>.





**Figure 8:** Local geoid of the city Cotonou (Nglc\_2017) in 3D and meter<sup>5</sup>.

**Table-6:** Difference at control points between leveled GPS points and geoid Nglc\_2017.

ID	Lon	Lat	Nglc-Control Point Difference
BRNC15	2,378	6,383	-0,016
BRNC37	2,402	6,390	0,001
BNRC015	2,360	6,370	-0,028
BRNC11	2,378	6,355	0,033
S	2,423	6,353	-0,031
BNRC006	2,445	6,361	-0,021
BNRC004	2,465	6,367	-0,019
BRNC19	2,405	6,375	0,011
BRNC05	2,397	6,350	0,023
IGN204	2,395	6,367	-0,037

**Table-7:** Statistical analysis of the difference obtained.

ID	min (m)	max (m)	rate (M)	standard deviation (m)
Nglc_2017	-0,037	0,023	-0,015	0,020

The standard deviation is 2cm which allowed to confirm the obtained accuracy of the noises after withdrawal of the trend and the signal. The geoid Nglc\_2017 is therefore precisely two centimeters (2cm).

**Discussion:** Figures-7 and 8 show the final geoid model determined on the Cotonou zone called Nglc\_2017. Table-6 compares the Nglc\_2017 to leveled GPS points that did not

participate in its development called control points and then allows to confirm its accuracy which is 2cm.

The geoid Nglc\_2017 was therefore calculated from the world geoid EGM2008 after subtracting from the NEGM2008 ripple grid: i. Quadratic correction of deviations (second order trend), ii. The signal part of the residues of the neighboring points (kriging).

## Conclusion

The development of satellite measurement techniques has led several Beninese measurement and localization professionals to become interested in GNSS devices. The third component given by GNSS, the ellipsoidal height is often confusing.

Several development projects require knowledge of terrain relief. In order to optimize the terrain survey time, and to allow all users to no longer confuse the ellipsoidal height with the altitude, it is important to define a Cotonou geoid model, study area. The results of the studies were obtained after adequate field data processing and statistical analysis that eliminated some off-tolerance results. The accuracy of the resulting geoid ripples can be improved by determining the geoid deflection and improving the accuracy of the RNGB with the gravimetric data.

## References

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