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The generalized fuzzy demand and supply transportation problem

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Abstract

The present paper deals with the transportation problem with uncertainty in demand and supply of items. In the previous years these types of problems have been discussed and presented a lot of algorithm to solve such type of problem in deterministic and stochastic environment. But only a limited number of authors have discussed the uncertainty in demand and supply. As practically we see that if there is a variation in demand and supply the cost of the item and transportation cost can be very as the transported vehicle capacity. If there is a variation in the capacity and supply of a vehicle then the cost of the transportation can effect. Here we will discuss such type of problem and make an algorithm to solve such situation. The LR-type fuzzy numbers are used to represent uncertainty in demand, supply and capacity of vehicle.

Keywords: Fuzzy sets, Integer programming, Transportation problem, LR-type fuzzy number and uncertainty.

Introduction

The study of transportation of items is an open field for researchers, planner and managers. A good strategy for optimal cost may help the industries to respond the market demand quickly and run the industries more efficiently and competitively in the market. In conventional approach of transportation problem, the transported cost and optimal time has been taken deterministic while in real world situation, the transported time and cost could be imprecise because of either route or vehicle capacity or different uncertain environment factors.

In the present paper we assume that the sort of uncertainty that comes into play is better representation of time and cost by fuzziness instead of randomness and probability. In these complex situations the concept of fuzziness are very effective. Fuzzy numbers are often used to describe the transportation time and cost are depends on environment and market strategies. In many situation the decision maker could only be estimate the interval in which the transportation cost and time with a certain degree of confidence level. The concept of fuzziness in cost has been studies by various researchers[1-5].

Preliminaries Definitions

λ- CUT: With any fuzzy set A we can associate a collection of crisp sets known as λ- cuts or level sets of A. A λ- cut is a crisp set consisting of elements of A which belong to the fuzzy set at least to a degree λ. As we shall see, λ- cuts offer a method for resolving a fuzzy set in terms of crisp sets (something analogous to resolving a vector into its components). λ- Cuts are indispensable in performing arithmetic operations in fuzzy sets that represent various qualities of numerical data. It should be noted that λ cuts are crisp, not fuzzy sets.

The λ- cut of a fuzzy set 'A' denoted by A_λ, is a crisp set comprised of all the elements $x \in X$ (X is the universe of discourse) for which the membership function $\mu_A(x) \geq \lambda$ i.e.

$A_\lambda = \{x \in X / \mu_A(x) \geq \lambda\}$

Where: λ is a parameter in the range $0 < \lambda \leq 1$.

L-R type fuzzy number[1]: The L-R type fuzzy number is a special case of representations of fuzzy number which is proposed by Dubois[5]. They introduced two functions called L and R, which maps (R^+ [0, 1]) and all of these functions are decreasing shape function define as:

L (0) = 1; ⟶ L (x) < 0, ∀ x<1 ; L (1) = 0 or [L (x) > 0 ∀ 1< x <∞] and L (∞) = 0.

A LR-type fuzzy number can be define as:

$$\mu_m(x) = \begin{cases} L\left[\dfrac{m-x}{\alpha}\right] & for \;\; x \leq m \\ R\left[\dfrac{x-m}{\beta}\right] & for \;\; x \geq m \end{cases}$$

Where: reference functions L (for left) R (for right) and scalars $\alpha > 0$, $\beta > 0$.

Here: m called the mean value of M is a real number α and β are called the left and right spreads respectively.

The membership function $\mu_m(x)$ for the fuzzy number M is an L-R type fuzzy number, can be expressed as $(M_1, M_2, \alpha, \beta)_{LR}$.

If A_i and B_j are L-R type fuzzy numbers define as:

$A_i = (\underline{a}_i, \overline{a}_j, \alpha_{A_i}, \beta_{A_i})_{L_i - R_i}$; $\overline{B_i} = (\underline{a}_i, \overline{a}_j, \alpha_{B_i}, \beta_{B_i})_{S_i - T_i}$

$i = 1,2, \ldots\ldots, m;\; j = 1,2, \ldots\ldots, n$

λ- cuts A_i^λ and B_j^λ are intervals of the form:
$A_i^\lambda = [\, a_i - L_i^{-1}(\lambda)\, a_{Ai},\ a_i + R_I^{\lambda}(\lambda), \beta_{Ai}]$, i = 1, 2, ….., m.
$B_j^\lambda = [\, b_i - S_j^{-1}(\lambda)\, a_{Bj},\ b_j + T_j^{-1}(\lambda), \beta_{Bj}]$, j = 1, 2,….., n.

Shape function: F is the shape function such that: i. F is a continuous function which is non- increasing on the half plane [0, ∞]. ii. F (0) = 1, iii. F is strictly decreasing in the domain on which it is positive.

And left or right spread can be one of the following particular case i.e.: i. Linear = $F(y) = max\ \{0, 1-y\},\ y \in R^+ \cup \{0\}$, ii. Exponential = $F(y) = e^{-py},\ p \geq 1,\ y \in R^+ \cup \{0\}$, iii. Power $F(y) = \{0, 1-y^p\},\ p \geq 1,\ y \in R^+ \cup \{0\}$, iv. Rational $F(y) = \frac{1}{1+y^p}\ p \geq 1,\ y \in R^+ \cup \{0\}$

Proposed problem

In the present paper we have proposed an algorithm to solve the transportation problem with fuzzy demand and supply as well as transported items taken as fuzzy numbers[6]. In the present work we have taken both the cases which have taken earlier in the previous papers. Here we have suggested a solution algorithm which has been described infinite steps for solving the generalized transportation problem. In this algorithm we have formulated the problem in fuzzy non-linear programming which has been converted into a non-fuzzy problem and find an integer solution to the given problem. These types of problems can be written as,

Table-1: Types of problems.

Source i *Destination-j*	*1*	*2*	*3*	.	.	*n*	a_i
1	C_{11} n_{11}	C_{12} n_{12}	C_{13} n_{13}	.	.	C_{1n} n_{1n}	a_1
2	C_{21} n_{21}	C_{22} n_{22}	C_{23} n_{23}	.	.	C_{2n} n_{2n}	a_2
3	C_{31} n_{31}	C_{32} n_{32}	C_{33} n_{33}	.	.	C_{3n} n_{3n}	a_3
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
M	C_{m1} n_{m1}	C_{m2} n_{m2}	C_{m3} n_{m3}	.	.	C_{nm} n_{nm}	a_m
d_j	d_1	D_2	d_3	.	.	d_n	

Where: n_{ij}'s are L-R type fuzzy numbers.

n_{ij} is the transportation capacity of the vehicle from the i^{th} source to the j^{th} destination. C_{ij} is the transported cost for a vehicle from the i^{th} source to the j^{th} destination. In one cycle and a_i 's and d_j's are also L-R type fuzzy numbers.

Where: $a_i = (a_i^1, a_i^2, a_i^3)$ is a triangular* fuzzy number and $b_j = (b_j^1, b_j^2, b_j^3)$ is also a triangular fuzzy number.
And $\Sigma a_i^1 = \Sigma b_j^1,\ \Sigma a_i^2 = \Sigma b_j^2,\ \Sigma a_i^3 = \Sigma b_j^3$.
*Here we are taking a particular case which can be easily generalized for other type of fuzzy numbers.

Let us suppose x_{ij}'s are the number of cycles for a vehicle from i^{th} source to the j^{th} destination, and then we have

$$\sum_{j=1}^{n} n_{ij}\, x_{ij} \leq a_i \quad i = 1,2,\ldots,m$$

The unsatisfied demand (p_j) for destination j can be defined as

$$p_j = d_j - \sum_{i=1}^{m} n_{ij}\, x_{ij} \quad j = 1,2,\ldots,n \tag{2}$$

Then the revenue lost by an unsatisfied demand of destination j can be written as,

$$\sum_j k_j\, p_i \quad j = 1,2,\ldots,n \tag{3}$$

Now the total cost will be,

$$\sum_{i,j} k_{ij}\, p_{ij} + \sum_j k_j\, p_i \quad \begin{matrix} i = 1,2,\ldots\ldots,m \\ j = 1,2,\ldots\ldots,n \end{matrix} \tag{4}$$

Therefore the fuzzy integer generalized transportation problem can be formulated as[3,7],

$$\text{Minimize} \sum_{i,j} k_{ij}\, p_{ij} + \sum_j k_j\, p_i \quad \begin{matrix} i = 1,2,\ldots\ldots,m \\ j = 1,2,\ldots\ldots,n \end{matrix} \tag{5}$$

$$\sum_{j=1}^{n} n_{ij}\, x_{ij} \leq a_i \quad i = 1,2,\ldots,m$$

$$p_j = d_j - \sum_{i=1}^{m} n_{ij}\, x_{ij} \quad j = 1,2,\ldots,n$$

P_j, x_{ij} are integer values
$p_j,\ xij > 0 \quad \forall i = 1,2,\ldots,m;\ i = 1,2,\ldots,n$

Algorithm: Step-1: Convert the transported problem to equivalent non-linear programming problem in mathematical from then we convert the fuzzy L-R type numbers n_{ij} in the interval from by introducing λ– cut to these fuzzy numbers. Taking variation in the value of λ between 0 and 1 and initialize λ equal to zero and convert the fuzzy numbers n_{ij} , a_i , d_j in interval from.

Then the above problem can be written as[7],

$$Minimize \sum_{i,j} k_{ij}\, p_{ij} + \sum_j k_j\, p_i \quad \begin{matrix} i = 1,2,\ldots\ldots,m \\ j = 1,2,\ldots\ldots,n \end{matrix}$$

$$\sum_{j=1}^{n} n_{ij}\, x_{ij} \leq a_i \quad i = 1,2,\ldots,m \tag{6}$$

$$p_j = d_j - \sum_{i=1}^{m} n_{ij}\, x_{ij} \quad j = 1,2,\ldots,n$$

$n_{ij} \equiv [n_{ij}, H_{ij}]\ \forall \quad i = 1,2,\ldots,m;\ j = 1,2,\ldots,n$
$a_i \equiv [a_i^1, a_i^2]\ \forall \quad i = 1,2,\ldots,m;\ j = 1,2,\ldots,n$
$d_i \equiv [d_i^1, d_i^2]\ \forall \quad i = 1,2,\ldots,m;\ j = 1,2,\ldots,n$

P_j, x_{ij} are integer values.
$p_j,\ xij > 0\ \ \forall\ \ i = 1,2,\ldots,m;\ \ j = 1,2,\ldots,n$

Step-2: Now the problem can be written as,

$$Minimize \sum_{i,j} c_{ij}\, x_{ij} + \sum_j k_j\, p_i \quad \begin{matrix} i = 1,2,\dots\dots..,m \\ j = 1,2,\dots\dots..,n \end{matrix}$$

$$\sum_{j=1}^{n} n_{ij}\, x_{ij} \le t_i \quad j = 1,2,\dots.,m \tag{7}$$

$$p_j + \sum_{i=1}^{m} n_{ij}\, x_{ij} = m_j \quad j = 1,2,\dots.,n$$

$h_{ij} \le n_{ij,} \le H_{ij}$ $\forall$ $i = 1,2,\dots\dots.,m;$ $j = 1,2,\dots\dots\dots,n$

$a_i^1 \le t_{i,} \le a_i^2$ $\forall$ $i = 1,2,\dots\dots.,m;$ $j = 1,2,\dots\dots\dots,n$

$d_j^1 \le m_{j,} \le d_j^2$ $\forall$ $i = 1,2,\dots\dots.,m;$ $j = 1,2,\dots\dots\dots,n$

P_j, x_{ij} are integer values.

p_j, $xij > 0$ $\forall$ $i = 1,2,\dots\dots.,m;$ $j = 1,2,\dots\dots\dots,n$

Where: h_{ij} and H_{ij} are lower and upper bounds on n_{ij} respectively.

a_i^1 and a_i^2 are lower and upper bounds of a_i respectively.

d_j^1 and d_j^2 are lower and upper bounds of d_j respectively.

The above problem can also be written as,

$$Minimize \sum_{i,j} c_{ij}\, x_{ij} + \sum_j k_j\, p_i \quad \begin{matrix} i = 1,2,\dots\dots..,m \\ j = 1,2,\dots\dots..,n \end{matrix}$$

$$\sum_{i=1}^{m} n_{ij}\, x_{ij} - t \le 0 \quad j = 1,2,\dots.,m \tag{8}$$

$$for\ all\ i = 1,2,\dots.,m$$

$$p_j + \sum_{i=1}^{m} n_{ij}\, x_{ij} - m_j = 0 \quad j = 1,2,\dots.,n$$

$h_{ij} < n_{ij,} < H_{ij}$ $\forall$ $i = 1,2,\dots\dots.,m;$ $j = 1,2,\dots\dots\dots,n$

$a_i^1 \le t_{i,} \le a_i^2$ $\forall$ $i = 1,2,\dots\dots.,m;$ $j = 1,2,\dots\dots\dots,n$

$d_j^1 \le m_{j,} \le d_j^2$ $\forall$ $i = 1,2,\dots\dots.,m;$ $j = 1,2,\dots\dots\dots,n$

P_j, x_{ij} are integer values.

p_j, $x_{ij} > 0$ $\forall$ $i = 1,2,\dots\dots.,m;$ $j = 1,2,\dots\dots\dots,n$

Solve the problem by integer non-linear programming with the help of mathematical software [MATLAB].

Step-3: Again for $\lambda^* = \lambda + \delta\lambda$ where $\delta\lambda \in [0,1]$ s.t. $0 < \lambda^* < 1$ and go to step 1 and repeat the above procedure until the interval [0,1] is fully exhausted, then we find the values of objective function for different values of λ.

Step-4: Now we will choose the minimum value of the objective function determine above and the corresponding solutions that will be the optimal value of the above problem are taken as answer.

Numerical Example: Let us take a numerical example to clarify the algorithm and the suggested solution procedure. This example solve, how the fuzzy generalized transportation problem with fuzzy demand and supply as well as numbers transported items as fuzzy numbers, involves integer variables can be solved.

The problem with four sources and three destinations, is given in Table-2.

Table-2: Problem with four sources and three destinations.

i/j	1	2	3	a_i
1	$C_{11}=2$ $n_{11}=$ (8,8,4,1)	$C_{12}=5$ $n_{12}=$ (8,9,2,2)	$C_{13}=2$ $n_{13}=$ (6,8,3,2)	(150,200, 20,70)
2	$C_{21}=5$ $N_{21}=$ (7,8,5,1)	$C_{22}=4$ $N_{22}=$ (8,10,2,5)	$C_{23}=6$ $N_{23}=$ (15,20,13,6)	(225,450, 15,30)
3	$C_{31}=2$ $N_{31}=$ (7,8,5,8)	$C_{32}=6$ $N_{32}=$ (10,12,6,4)	$C_{33}=12$ $N_{33}=$ (9,12,5,6)	(150,200, 25,55)
4	$C_{41}=3$ $N_{41}=$ (15,18, 11, 1)	$C_{42}=14$ $N_{42}=$ (9,10,3,4)	$C_{43}=8$ $N_{43}=$ (13,16,2,5)	(100,220, 86,40)
d_j	(150,340, 50,40)	(225,420, 85,100)	(350,510,51, 125)	

Since the above problem is unbalanced transportation problem, therefore we add a row for a dummy source with zero cost. Then the problem converted as given in Table-3.

Table-3: Problem converted.

i/j	1	2	3	a_i
1	$C_{11}=2$ $n_{11}=$ (8,8,4,1)	$C_{12}=5$ $n_{12}=$ (8,9,2,2)	$C_{13}=2$ $n_{13}=$ (6,8,3,2)	(150,200, 20,70)
2	$C_{21}=5$ $N_{21}=$ (7,8,5,1)	$C_{22}=4$ $N_{22}=$ (8,10,2,5)	$C_{23}=6$ $N_{23}=$ (15,20,13,6)	(225,450, 15,30)
3	$C_{31}=2$ $N_{31}=$ (7,8,5,8)	$C_{32}=6$ $N_{32}=$ (10,12,6,4)	$C_{33}=12$ $N_{33}=$ (9,12,5,6)	(150,200, 25,55)
4	$C_{41}=3$ $N_{41}=$ (15,18,11,1)	$C_{42}=14$ $N_{42}=$ (9,10,3,4)	$C_{43}=8$ $N_{43}=$ (13,16,2,5)	(100,220, 86,40)
5	$C_{51}=0$ $N_{51}=$ (14,15,2, 11)	$C_{52}=0$ $N_{52}=$ (16,18,2,2)	$C_{53}=0$ $N_{53}=$ (10,15,6,3)	(100,200, 40,70)
d_j	(150,340, 50,40)	(225,420, 85,100)	(350,510, 51,125)	

In the above problem we will take n_{51}, n_{52}, n_{53} as randomly taken fuzzy parameters for the unsatisfied demand. Without loss of generality we will take $k_j = n_j$, where k_1, k_2, k_3, are revenue lost by the unsatisfied demand.

Now the above problem can be converted in the fuzzy non linear problem as follows,

$MINZ = 2x_{11}+2x_{12}+2x_{13}+2x_{21}+2x_{22}+2x_{23}+2x_{31}+2x_{32}+2x_{33}+2x_{41}+2x_{42}+2x_{43}+k_1p_1+k_2p_2+k_3p_3$

Subject to,

$n_{11}x_{11}+n_{12}x_{12}+n_{13}x_{13} \le (150,200,20,70)$
$n_{21}x_{21}+n_{22}x_{22}+n_{23}x_{23} \le (325,450,15,30)$
$n_{31}x_{31}+n_{32}x_{32}+n_{33}x_{33} \le (150,200,25,55)$
$n_{41}x_{41}+n_{42}x_{42}+n_{43}x_{43} \le (200,220,86,40)$
$n_{51}x_{51}+n_{52}x_{52}+n_{53}x_{33} \le (150,200,40,70)$

$p_1+ n_{11}x_{11} + n_{21}x_{21}+ n_{31}x_{31}+ n_{41}x_{41}+ n_{51}x_{51} =(150,180,50,20)$

$p_2+ n_{12}x_{12}+ n_{22}x_{22}+ n_{32}x_{32}+ n_{42}x_{42} +n_{52}x_{52} =(225,250,25,50)$

$p_3+ n_{13}x_{13} + n_{23}x_{23} + n_{33}x_{33}+ n_{43}x_{43} +n_{53}x_{53} =(350,380,50,120)$

$p_1 > 0,\ x_{ij} > 0\ \ i = 1,2,3,4,5 \quad j=1,2,3$

Convert all the fuzzy numbers n_{ij}'s, supply a_i and demand dj values in interval from, by introducing λ – cut to these fuzzy numbers.

$if\ n_{ij} = \left(\underline{n}_{ij}, \overline{n}_{ij}, \alpha_{ij}, \beta_{ij}\right)$ then $n_{ij}^{\lambda} = [n_{ij} - L_{ij}^{-1}(\lambda)\alpha_{ij}, n_{ij}, + L_{ij}^{-1}(\lambda)\alpha\beta_{ij}]$ $i=1,2,....,m;\ j=1,2,....,n$ *and* L_{ij} is the linear shape function.

Now start the value of λ with zero and by the help of Mathematical Software [MATLAB] and solve the above problem to find the value of objective function as well as variables.

Now we will take an increment in the value of λ by 0.01 and repeat the above procedure for λ=0+0.01. Continue in this way until we get λ=1.

And hence we will choose the minimum value of the objective function determine as above for different λ and the corresponding solutions that will be the optimal value of the above problem which is as follows.

By the help of MATLAB software we find that optimal value of the above problem is at λ = 0 and that has been finding as below.

Now the above problem converted into a non-linear programming problem with fuzzy demand and supply and bounded variables as follows:

$MINZ=2x_{11}+5x_{12}+2x_{13}+5x_{21}+4x_{22}+6x_{23}+2x_{31}+6x_{32}+12x_{33}+3x_{41}+14x_{42}+8x_{43}+k_1p_1+ k_2p_2+ k_3p_3$

Subject to,

$n_{11}x_{11}+n_{12}x_{12}+n_{13}x_{13} \le t_1$
$n_{21}x_{21}+n_{22}x_{22}+n_{23}x_{23} \le t_2$
$n_{31}x_{31}+n_{32}x_{32}+n_{33}x_{33} \le t_3$
$n_{41}x_{41}+n_{42}x_{42}+n_{43}x_{43} \le t_4$
$n_{51}x_{51}+n_{52}x_{52}+n_{53}x_{33} \le t_5$
$p_1+ n_{11}x_{11}+n_{21}x_{21}+ n_{31}x_{31}+ n_{41}x_{41}+ n_{51}x_{51} = m_1$
$p_2+ n_{12}x_{12}+ n_{22}x_{22}+ n_{32}x_{32}+ n_{42}x_{42} +n_{52}x_{52} = m_2$
$p_3+ n_{13}x_{13}+ n_{23}x_{23}+ n_{33}x_{33}+ n_{43}x_{43} +n_{53}x_{53} = m_3$
$p_1 > 0,\ x_{ij} > 0\ \ i = 1,2,3,4,5 \quad j=1,2,3$; where $n_{5j} = k_j$ $(j=1,2,3)$

And

$4 \le n_{11} \le 10$	$6 \le n_{12} \le 11$	$3 \le n_{13} \le 10$
$2 \le n_{21} \le 9$	$6 \le n_{22} \le 15$	$2 \le n_{23} \le 26$
$2 \le n_{31} \le 16$	$4 \le n_{32} \le 16$	$4 \le n_{33} \le 18$
$4 \le n_{41} \le 19$	$6 \le n_{42} \le 14$	$11 \le n_{33} \le 21$
$12 \le n_{51} \le 26$	$14 \le n_{52} \le 20$	$4 \le n_{53} \le 18$
$130 \le t_1 \le 270$	$240 \le t_2 \le 480$	$175 \le t_3 \le 255$
$186 \le t_4 \le 260$	$110 \le t_5 \le 270$	
$100 \le M_1 \le 380$	$140 \le M_2 \le 520$	$299 \le m_3 \le 635$

Now the problem is as follows,

$MINZ = 2x_{11}+ 5x_{12} + 2x_{13} + 5x_{21} + 4x_{22} + 6x_{23} + 2x_{31} + 6x_{32} + 12x_{33} + 3x_{41} + 14x_{42} + 8x_{43} + k_1p_1 + k_2p_2 + k_3p_3$

Subject to,

$n_{11}x_{11}+n_{12}x_{12}+n_{13}x_{13} - t_1 \le 0$
$n_{21}x_{21}+n_{22}x_{22}+n_{23}x_{23} - t_2 \le 0$
$n_{31}x_{31}+n_{32}x_{32}+n_{33}x_{33} - t_3 \le 0$
$n_{41}x_{41}+n_{42}x_{42}+n_{43}x_{43} - t_4 \le 0$
$n_{51}x_{51}+n_{52}x_{52}+n_{53}x_{33} - t_5 \le 0$
$p_1+ n_{11}x_{11}+ n_{21}x_{21}+ n_{31}x_{31}+ n_{41}x_{41}+ n_{51}x_{51} - m_1 = 0$
$p_2+ n_{12}x_{12}+ n_{22}x_{22}+ n_{32}x_{32}+ n_{42}x_{42} +n_{52}x_{52} - m_2 = 0$
$p_3+ n_{13}x_{13}+ n_{23}x_{23}+ n_{33}x_{33}+ n_{43}x_{43} +n_{53}x_{53} - m_3 = 0$

$4 \le n_{11} \le 10$	$6 \le n_{12} \le 11$	$3 \le n_{13} \le 10$
$2 \le n_{21} \le 9$	$6 \le n_{22} \le 15$	$2 \le n_{23} \le 26$
$2 \le n_{31} \le 16$	$4 \le n_{32} \le 16$	$4 \le n_{33} \le 18$
$4 \le n_{41} \le 19$	$6 \le n_{42} \le 14$	$11 \le n_{33} \le 21$
$12 \le n_{51} \le 26$	$14 \le n_{52} \le 20$	$4 \le n_{53} \le 18$
$130 \le t_1 \le 270$	$240 \le t_2 \le 480$	$175 \le t_3 \le 255$
$186 \le t_4 \le 260$	$110 \le t_5 \le 270$	
$100 \le M_1 \le 380$	$140 \le M_2 \le 520$	$299 \le m_3 \le 635$
$p_1 > 0$, $x_{ij} > 0$	$i = 1,2,3,4,5$	$j=1,2,3$

Solution is as follows,

MINZ= **141** and variables are

$x_{11} = 0$	$x_{12} = 0$	$x_{13} = 13$
$x_{21} = 0$	$x_{22} = 16$	$x_{23} = 0$
$x_{31} = 11$	$x_{32} = 0$	$x_{33} = 0$
$x_{41} = 0$	$x_{42} = 0$	$x_{43} = 0$
$x_{51} = 0$	$x_{52} = 0$	$x_{53} = 74$
$n_{11} = 7$	$n_{12} = 11$	$n_{13} = 10$
$n_{21} = 2$	$n_{22} = 15$	$n_{23} = 04$
$n_{31} = 16$	$n_{32} = 09$	$n_{33} = 04$
$n_{41} = 19$	$n_{42} = 06$	$n_{43} = 13$
$n_{51} = 12$	$n_{52} = 19$	$n_{53} = 04$
$p_1 = 3$	$p_2 = 0$	$p_3 = 0$
$t_1 = 130$	$t_2 = 240$	$t_3 = 175$
$t_4 = 186$	$t_5 = 140$	
$m_1 = 364$	$m_2 = 240$	$m_3 = 427$

Conclusion

In the present paper we have suggested an algorithm to solve the integer fuzzy transportation problem under uncertainty. The proposed algorithm which is described in finite steps, has been developed for solving the fuzzy generalized transportation problem in which demand, supply, transported item in one cycle had been taken as a fuzzy number. An illustrated example has been given to clarify the theory and the solution algorithm. Certainly there are many other aspects and questions should be exploded and answered in the area of fuzzy transportation problem. However there remain several open problems which should be solved in future.

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