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Time Dependent Deterioration, Time Dependent Quadratic Demand with Partial Backlogging

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Abstract

An inventory management model considering deterioration rate is time dependent and time dependent quadratic demand is studied. Shortages are allowed with partially backlogging rate. Salvage value is considered for deteriorating items. This model is developed to minimizing Total Cost (TC) of the inventory system. Sensitivity analysis is carried out to check the robustness of this model.

Keywords: Time dependent models, quadratic demand, deterioration rate, salvage value, shortages.

Introduction

Researchers have developed inventory models with exponential demand rate by considering increasing / decreasing demand with respect to time. Inventory models are studied with constant, linear, stock and price dependant demand rate. The phenomenon of exponentially increasing/ decreasing for any product is not always realistic. However deteriorating products like vegetables, medicine, milk products and photo films etc are subject to life time due to deterioration. Hence researchers developed inventory models with deterioration rate considering i. constant deterioration, ii. time dependent deterioration, iii. Weibull deterioration rate etc. Also most of the research papers considering the inventory carrying cost is constant. In this paper the deterioration rate is assumed as function of time and the holding cost is also function of time. Since the demand rate is quadratic function of time one should refer Khanra and Chaudhuri[1]. So, it is reasonable to assume that the demand rate, in certain commodities, due to seasonal variations may follow quadratic function of time [i.e., $D(t) = A + Bt + Ct^2$; $A \geq 0, B \neq 0, C \neq 0$]. Here A is the initial rate of demand, B is the initial rate of change of the demand and C is the acceleration of demand rate. This functional form of quadratic demand rate, explains the accelerated (retarded) growth/decline in the demand patterns which may arise due to seasonal demand rate. This explains different types of realistic demand patterns depending on the signs of B and C. Giri and Goyal (2001) studied the literature survey of deteriorating inventory models[2]. Shital et al (2013) suggested an EOQ model for with linear demand considering shortages under permissible delay in payments and inflation[3]. Bhandari and Sharma (2004) have studied a Single Period Inventory Problem with Quadratic Demand Distribution under the Influence of Marketing Policies[4]. Mohan and Venkateswarlu (2013) developed an inventory management model with linear demand rate incorporating variable holding cost and salvage value[5]. Nita H. Shah et al (2008) derived a time dependent inventory model considering demand rate is exponentially declining[6]. Ajanta Roy (2008) proposed an EOQ model for deteriorating items with price dependent model considering time varying holding cost[7]. Ghosh and Chaudhuri[8] have developed an inventory management model for a deteriorating item having an instantaneous supply, a quadratic time-varying demand. Shortages are allowed. Two-parameter Weibull distribution to represent the time to deterioration is incorporated. Shukla et al developed an EOQ model for deteriorating items with exponential demand rate and shortages[9]. Teng (2002) derived an EOQ model under the condition of permissible delay in payments[10]. Venkateswarlu and Mohan studied an EOQ model for time varying deterioration and price dependent quadratic demand with salvage value[11]. You proposed an Inventory policy for products with time-dependent demands and considering price also[12]. Patra et al suggested an EOQ model for Single Warehouse System with Price Depended Demand in Non-Linear (Quadratic) Form[13]. Mondal et al derived an inventory system for price dependent demand for ameliorating items[14]. Burwell et al proposed an economic lot size model for price-dependent demand model incorporating quantity and freight discounts[15].

In this paper, an inventory model with time-dependent deterioration when the demand rate is a quadratic in nature. Time horizon is infinite and shortages are allowed. The optimal total cost is obtained considering the salvage value for deteriorated items. The sensitivity analysis with numerical example carried out at the end.

Assumptions and Notations: The mathematical model is developed on the following assumptions and notations:
The demand rate $D(t)$ is assumed as $D(t) = A + Bt + Ct^2$, $A \geq 0, B \neq 0, C \neq 0$.

Lead time is zero. S, Lost sale cost per unit. A_1, the ordering cost per order . $\theta(t) = \theta t$ is the deterioration rate, $0 < \theta < 1$. C_1, is the shortage cost per unit. C_3, purchase cost per unit. $I(t)$ is the inventory level at time t. R, is the backlogging rate, $0 \le R \le 1$. The salvage value γ^*C_4, $0 \le \gamma < 1$ is associated with deteriorated units during a cycle time.

Formulation and solution of the model: The differential equation which describes the instantaneous inventory level at time t can be written as

$$\frac{d(I_1(t))}{dt} + \theta(t)\, I_1(t) = -(A + Bt + Ct^2), \quad 0 \le t \le t_1 \tag{1}$$

$$\frac{d(I_2(t))}{dt} = -R(A+Bt+Ct^2), \quad t_1 \le t \le T \tag{2}$$

Where $\theta(t) = \theta t$ with the initial condition $I_1(0) = Q$ and $I_1(t) = 0 = I_2(t)$ at $t = t_1$

The solution of equation (1) using initial conditions is given by

$$I_1(t) = \left\{A(t_1 - t) + \frac{B(t_1^2 - t^2)}{2} + \frac{C(t_1^3 - t^3)}{3}\right\} + \theta\left\{\frac{A(t_1^3 - t^3)}{6} + \frac{B(t_1^4 - t^4)}{8} + \frac{C(t_1^5 - t^5)}{10}\right\} + \theta^2\left\{\frac{A(t_1^5 - t^5)}{40} + \frac{B(t_1^6 - t^6)}{48} + \frac{C(t_1^7 - t^7)}{56}\right\} - \theta\left(\frac{A(t^2 t_1 - t^3)}{2} + \frac{B(t^2 t_1^2 - t^4)}{4} + \frac{C(t^2 t_1^3 - t^5)}{6}\right) - \theta^2\left(\frac{A}{12}(t_1^3 t^2 - t^5) + \frac{B}{16}(t_1^4 t^2 - t^6) + \frac{C}{20}(t_1^5 t^2 - t^7)\right) + \theta^2\left(\frac{A}{8}(t^4 t_1 - t^5) + \frac{B}{16}(t_1^2 t^4 - t^6) + \frac{C}{24}(t_1^3 t^4 - t^7)\right) \tag{3}$$

The solution of equation (2) using above initial conditions is

$$I_2(t) = -R\left[A(t_1 - t) + \frac{B(t^2 - t_1^2)}{2} + \frac{C(t^3 - t_1^3)}{3}\right]$$

Using $I_1(0) = Q$, we obtain

$$Q = \left[\begin{array}{l} At_1 + \frac{Bt_1^2}{2} + \frac{Ct_1^3}{3} + \theta\left(\frac{At_1^3}{6} + \frac{Bt_1^4}{8} + \frac{Ct_1^5}{10}\right) + \\ \theta^2\left(\frac{At_1^5}{40} + \frac{Bt_1^6}{48} + \frac{Ct_1^7}{56}\right) \end{array}\right] \tag{4}$$

The total cost of the inventory system consists of the following cost:

$$\text{Carrying cost per cycle} = C_h * (h + \beta t)\int_0^{t_1} I_1(t)\,dt$$

$$C_h\left[\begin{array}{l} h\left(\begin{array}{l}\frac{At_1^2}{2} + \frac{Bt_1^3}{3} + \frac{Ct_1^4}{4} + \theta\left(\frac{At_1^4}{12} + \frac{Bt_1^5}{15} + \frac{7ct_1^6}{96}\right) + \\ \theta^2\left(A*0.0111\, t_1^6 + B*0.00952\, t_1^7 + C*0.0083\, t_1^8\right)\end{array}\right) \\ \beta\left(\begin{array}{l} A\frac{t_1^3}{6} + \frac{Bt_1^4}{4} + \frac{Ct_1^5}{10} - \theta\left(\frac{At_1^5}{40} + \frac{Bt_1^6}{48} + \frac{Ct_1^7}{56}\right) + \\ \theta^2\left(\frac{At_1^7}{336} + \frac{Bt_1^8}{384} + \frac{Ct_1^9}{432}\right)\end{array}\right) \end{array}\right] \tag{5}$$

$$\text{Ordering cost per cycle} = \frac{A_1}{T} \tag{6}$$

$$\text{Purchase cost per cycle} = \frac{C_3}{T}\left(I_1(0) + \int_{t_1}^{T} RD(t)\,dt\right)$$

$$= \frac{C_3}{T}\left(\begin{array}{l}\left[\begin{array}{l}At_1 + \frac{Bt_1^2}{2} + \frac{Ct_1^3}{3} + \theta\left(\frac{At_1^3}{6} + \frac{Bt_1^4}{8} + \frac{Ct_1^5}{10}\right) + \\ \theta^2\left(\frac{At_1^5}{40} + \frac{Bt_1^6}{48} + \frac{Ct_1^7}{56}\right)\end{array}\right] \\ + R\left(A(T - t_1) + \frac{B}{2}(T^2 - t_1^2) + \frac{C}{3}(T^3 - t_1^3)\right)\end{array}\right) \tag{7}$$

$$\text{Lost sales} = \frac{1}{T}\left[S\int_{t_1}^{T}(1 - R)D(t)\,dt\right]$$

$$= S(1 - R)\left(A(T - t_1) + \frac{B}{2}(T^2 - t_1^2) + \frac{C}{3}(T^3 - t_1^3)\right) \tag{8}$$

$$\text{Shortage cost per cycle} = \frac{C_1}{T}\int_{t_1}^{T} - I_2(t)\,dt$$

$$= C_1 R\left[\left(\frac{A}{2}(T-t_1)^2+\frac{BT}{2}\left(\frac{T^2}{3}-t_1^2\right)+\frac{CT}{3}\left(\frac{T^3}{4}-t_1^3\right)+\frac{B}{3}t_1^3+\frac{C}{4}t_1^4\right)\right] \quad (9)$$

The deterioration units per cycle (DC) = $I(0)-\int_0^{t_1}(A+Bt+Ct^2)\,dt$

$$= C_4\left[\theta\left(\frac{At_1^3}{6}+\frac{Bt_1^4}{8}+\frac{Ct_1^5}{10}\right)+\theta^2\left(\frac{At_1^5}{40}+\frac{Bt_1^6}{48}+\frac{Ct_1^7}{56}\right)\right]$$

$$\text{Salvage value} = \gamma C_4\left[\begin{array}{l}\theta\left(\frac{At_1^3}{6}+\frac{Bt_1^4}{8}+\frac{Ct_1^5}{10}\right)+\\ \theta^2\left(\frac{At_1^5}{40}+\frac{Bt_1^6}{48}+\frac{Ct_1^7}{56}\right)\end{array}\right] \quad (10)$$

Total Cost = Ordering cost +Purchase Cost+ Inventory Holding cost + Shortage Cost +Lost Sales-Salvage value

$$TC = \frac{A_1}{T}+\frac{C_3}{T}\left(I_1(0)+\int_{t_1}^{T}RD(t)\,dt\right)$$
$$+\frac{1}{T}\left[\int_0^{t_1}(h+\beta t)I_1(t)\,dt+\frac{1}{T}C_1\int_{t_1}^{T}-I_2(t)\,dt+\frac{1}{T}S\int_{t_1}^{T}(1-R)D(t)\,dt\right]$$

$$TC = \left\{\begin{array}{l}\frac{A_1}{T}+\frac{C_3}{T}\left(\left[\begin{array}{l}At_1+\frac{Bt_1^2}{2}+\frac{Ct_1^3}{3}+\theta\left(\frac{At_1^3}{6}+\frac{Bt_1^4}{8}+\frac{Ct_1^5}{10}\right)+\\ \theta^2\left(\frac{At_1^5}{40}+\frac{Bt_1^6}{48}+\frac{Ct_1^7}{56}\right)\end{array}\right]+R\left(A(T-t_1)+\frac{B}{2}(T^2-t_1^2)+\frac{C}{3}(T^3-t_1^3)\right)\right)\\ +\frac{C_h}{T}\left[\begin{array}{l}h\left(\begin{array}{l}\frac{At_1^2}{2}+\frac{Bt_1^3}{3}+\frac{Ct_1^4}{4}+\theta\left(\frac{At_1^4}{12}+\frac{Bt_1^5}{15}+\frac{7Ct_1^6}{96}\right)+\\ \theta^2\left(A*0.0111\,t_1^6+B*0.00952\,t_1^7+C*0.0083\,t_1^8\right)\end{array}\right)\\ \beta\left(\begin{array}{l}\frac{At_1^3}{6}+\frac{Bt_1^4}{4}+\frac{Ct_1^5}{10}-\theta\left(\frac{At_1^5}{40}+\frac{Bt_1^6}{48}+\frac{Ct_1^7}{56}\right)+\\ \theta^2\left(\frac{At_1^7}{336}+\frac{Bt_1^8}{384}+\frac{Ct_1^9}{432}\right)\end{array}\right)\end{array}\right]-\\ +\frac{C_1R}{T}\left[\left(\begin{array}{l}\frac{A}{2}(T-t_1)^2+\frac{BT}{2}\left(\frac{T^2}{3}-t_1^2\right)+\\ \frac{CT}{3}\left(\frac{T^3}{4}-t_1^3\right)+\frac{B}{3}t_1^3+\frac{C}{4}t_1^4\end{array}\right)\right]\\ +\frac{1}{T}\left(\begin{array}{l}S(1-R)\left(A(T-t_1)\right)+\\ \frac{B}{2}(T^2-t_1^2)+\frac{C}{3}(T^3-t_1^3\end{array}\right)\end{array}\right\}$$

$$\gamma C_4\left[\theta\left(\frac{At_1^3}{6}+\frac{Bt_1^4}{8}+\frac{Ct_1^5}{10}\right)+\theta^2\left(\frac{At_1^5}{40}+\frac{Bt_1^6}{48}+\frac{Ct_1^7}{56}\right)\right] \quad (11)$$

The necessary condition for minimizing the total cost is $\frac{\partial(TC)}{\partial t_1}=0$, $\frac{\partial(TC)}{\partial T}=0$

provided $\frac{\partial^2(TC)}{\partial t_1^2}>0$, $\frac{\partial^2(TC)}{\partial T^2}>0$ and

$\frac{\partial^2(TC)}{\partial t_1^2}*\frac{\partial^2(TC)}{\partial T^2}-\frac{\partial^2(TC)}{\partial t_1\partial T}>0$.

The optimal value of t_1 and T and the total cost(TC) is obtained from equation (11) using MATHCAD software.

Since the demand is quadratic in nature the following four demand models are existing. C > 0 and *B* > 0 gives accelerated growth in demand model (M-1). C> 0 and *B* < 0 gives retarded growth in demand model (M-2). *C* < 0 and *B* > 0, we have retarded decline in demand model (M-3). *C* < 0 and C < 0, we have accelerated decline in demand model (M-4)

However in this model the accelerated growth model (M-1) exists and other three models (M-2, M-3, M-4) are not satisfying required condition. Hence an example of accelerated growth model is as follows.

Numerical Example: A = 50, B = 20, C = 2, γ = 0.1
S =10 A_1 = 2000, C_1= 4, C_3= 8 ,
R = 0.7, β = 0.8
t_1= 1.263 T = 3.278 TC = 1217.054

Sensitivity Analysis: Changing the values of parameters A, B, C, S, C_1, C_3, Ө and A by -25% to +25%, the optimal cycle time and total cost of the existing model (M-1) is given in table-1.

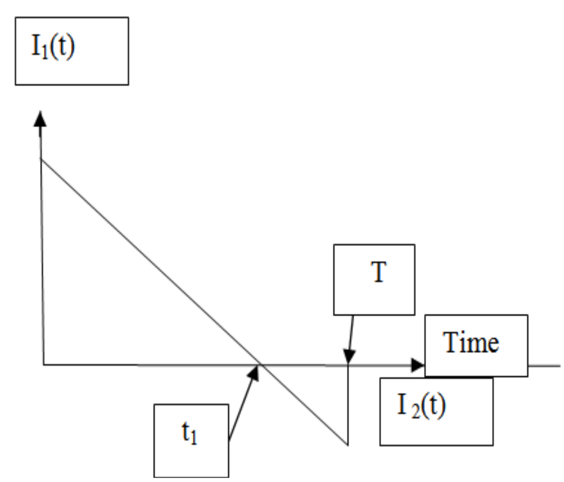

Figure-1

Table-1
(M-I): (A > 0, B > 0 and C > 0)

Parameter	% change	t_1	T	TC
A	-25	-1.66271	2.379499695	-6.89074
	-10	-0.63341	0.91519219	-2.7431
	10	0.554236	-0.884685784	2.726748
	25	1.346002	-2.165954851	6.788277
B	-25	5.621536	5.918242831	-4.95327
	-10	2.058591	2.226967663	-1.92194
	10	-1.74188	-2.043929225	1.852917
	25	-4.038	-4.881025015	4.517384
C	-25	6.967538	2.287980476	-1.7189
	-10	2.53365	0.884685784	-0.65396
	10	-2.21694	-0.823672971	0.618214
	25	-5.22565	-1.952410006	1.490238
C3	-25	9.580364	-0.488102502	-2.92518
	-10	3.562945	-0.183038438	-1.10028
	10	-3.32542	0.183038438	1.024605
	25	-7.91766	0.457596095	2.442044
β	-25	0.791766	0.030506406	-0.1304
	-10	-18.6857	-0.823672971	2.90817
	10	-0.23753	0	0.051271
	25	-0.71259	-0.030506406	0.12711
θ	-25	0.316706	0.030506406	-0.04141
	-10	0.158353	0	-0.01668
	10	-0.31671	0	0.042069
	25	-4.27553	6.497864552	-3.79449
C1	-25	-4.27553	6.497864552	-3.79449
	-10	-1.50435	2.379499695	-1.42911
	10	1.425178	-2.135448444	1.328618
	25	3.246239	-4.942037828	3.158611
A_1	-25	-3.95883	-10.15863331	-13.1941
	-10	-1.42518	-3.8438072	-5.11062
	10	1.346002	3.599755949	4.923857
	25	3.167063	8.633312996	12.00941

From the above table-1 we observed the increasing and decreasing values of the parameters including when all parameters are considered in the same pattern using suitable mathematical software.

Conclusion

In this paper the demand pattern is quadratic function of time and the deterioration rate is time dependent with shortages and backlogging rate. Salvage value is incorporated for the deteriorating items. Suitable numerical example and sensitivity analysis carried out.

Scope for further research: This paper further extended using Weibull rate of deterioration, Pareto distribution with/without salvage value.

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