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Short Communication

The Study of the Number of 2-Matchings in Graphs

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Abstract

In this paper we study the 2-matching of finite groups. We use conjugation and character of groups to calculate the number of 2-matchings for some import groups and using them we prove that a finite group G is abelian if and only if the number of its 2-matching is zero.

Keywords: Matching, conjugation, character, non commuting graph.

Introduction

Let G be a non abelian group and $Z(G)$ the center of G. The related graph$\Gamma_G$of G is a graph in which the set of its vertices is $V(\Gamma_G) = G - Z(G)$ and two vertices $x,,y$ are connected if $xy \neq yx$. in another word $E(G) = \{xy|[x,y] \neq 1\}$.

Study the groups using the properties of their graphs is a very important field of study in group theory[1,2],. The theorems used in graph theory can be found in Solmon R.[3], All Simple Groups are Characterized by their Graphs.

If Γ is a graph a matching for Γ is a global subgraph in which components are vertices and edges. A, 2-matching is a matching with two edges. The number of 2- matching is shown by $\rho(\Gamma, 2)$. A two matching for a group G is a 2-matching for the related graph $\Gamma_G$so $\rho(\Gamma_G, 2) = \rho(G, 2)$. For a graph Γ, the $diam(\Gamma)$ is the diameter of Γ and the circumference is shown by $girth(\Gamma)$. If x is an arbitrary member of group G , the degree of x in G is shown by $d_G(x)$. As a direct consequence $d_G(x) = |G - c_G(x)|$. In which $c_G(x)$ is the centralizer of x in G. For more results about the prime group of a graph[4,5,6] and applications of group theory are given in several titurature[7-11].

Matching of graphs

To prove the main theorem of the paper we need some prior propositions:

Proposition1: if G is a finite group then $diam(\Gamma_G) = 20$ and Γ_G is a connected graph[12].

Proposition2: $girth(\Gamma_G) = 3$ in which G is a non abelian and finite group[12].

Proposition3: if G is a non abelian and infinite then Γ_G is Hamiltonian, Proof: see refrence[12].

In the following proposition we obtain a formula that gives us the number of 2- matching's in a graph. This formula is quite simple and based of the centralizer of those members that are not in the center of group.

Proposition:

$$\rho(G,2) = \rho(\Gamma_G, 2) = \binom{\frac{1}{2}\sum_{x \epsilon G - Z(G)} |G - C_G(x)|}{2} - \sum_{x \epsilon G - Z(G)} \binom{|G - C_G(x)|}{2}$$

Proof: according to Usman M. et. al.[10] if Γ_G is a simple graph and m, n the number of vertices and edges and d_i the degree of vertex i in Γ_G then

$$\rho(\Gamma_G, 2) = \binom{m}{2} - \sum_{i=1}^{n} \binom{d_i}{2}$$

But$m(\Gamma_G) = \frac{1}{2}\sum_{x \epsilon G - Z(G)} |G - C_G(x)|$,$n(\Gamma_G) = |G - Z(G)|$and for an element x we know, $d_G(x) = |G - C_G(x)|$so the result follows.

Example: 1: Consider the group:
$G = S_3 = \{I, (1,2), (1,3), (2,3), (1,2,3), (1,3,2)\}$
Then:

$c_G(1,2) = \{I, (1,2)\}c_G(1,3) = \{I, (1,3)\}c_G(2,3) = \{I, (2,3)\}$
$c_G(1,2,3) = \{I, (1,2,3), (1,3,2)\}c_G(1,3,2) = \{I, (1,2,3), (1,3,2)\}$

So the representation of the Γ_{S_3} is figure-1.

Example 2: $\rho(S_3, 12) = 12$

We calculate this using the conjugation class of the group. The conjugation class of S_3 is shown in table-1.

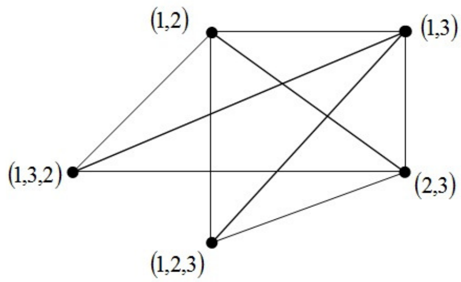

Figure-1
The representation graph of Γ_{S_3}.

Table-1
The Conjugation class of S_3

x	3	12	1^3
$\|C_{S_4}(x)\|$	3	2	6
$\|Class(x)\|$	2	3	1

So according to the table $m = \frac{3(4)+2(3)}{2} = 9$ therefor by the proposition 2:

$\rho(S_3, 12) = \binom{9}{2} - 2\binom{3}{2} - 3\binom{4}{2} = 12$

Example 3: $\rho(S_4, 2) = 21558$, The conjugation class of S_4 is shown in table-2.

Table-2
The conjugation class of S_4

x	4	13	2^2	12^2	1^4
$\|C_{S_4}(x)\|$	4	3	8	4	24
$\|Class\ (x)\|$	6	8	3	6	1

So $m = \frac{6(20)+8(21)+3(16)+6(20)}{2} = 228$ therefor by proposition2:
$\rho(S_4, 2) = 21558$

We can also similarly prove the following:
$\rho(A_4, 2) = 756, \quad \rho(A_5, 2) = 1269765$

We know that if G is an abelian group then $\rho(G, 2) = 0$ here we prove a theorem that shows the converse is true.

Theorem: if $\rho(G, 2) = 0$ then G is abelian. Proof: by contrary let G is non abelian then according to proposition 3, $girth(\Gamma_G) = 3$ and so $|V(\Gamma_G)| \geq 3$. Also by propositions (1) and (3) ,Γ_G is a connected Hamilitonian graph. Now if $|V(\Gamma_G)| \geq 4$ Then $\rho(\Gamma_G, 2) > 0$ that is a contradiction. So $|V(\Gamma_G)| = 3$ consequently $|G - Z(G)| = 3$ and we have:

$|G| - |Z(G)| = 3$ so $k|Z(G)| - |Z(g)| = 3$ this means $(k-1)|3$ so $k = 2\ or\ 4$. If $k = 2$ then $|Z(G)| = 3$ and so $|G| = 6$ and by assumption G is non abelian so $G \cong S_3$ but according to proposition (3), $\rho(G, 2) = \rho(S_3, 2) = 12$ that is a contradiction. If $k = 4$ then $|Z(G)| = 1$ and then $|G| = 4$ therefor G is abelian that is a contradiction as well. So G is abelian.

Conclusion

The study of groups using methods of graph theory is widely used in group theory. One of this tools is the number of matching in graphs. The result of this paper show that a combination of matching with some other concepts in group theory such as conjugation give important properties of groups as we saw the number of 2-matchings somehow shows if group is abelian or not. So it will be a key for further works to use matching's to figure out other properties of finite groups.

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