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Analysis of the Sampling in Quality Control Charts in non uniform Process by using a New Statistical Algorithm

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Abstract

One of the errors in preparing control charts is sampling at the same interval time. Even though, the random sampling method is used, no attention is given to sampling time interval and therefore to find out whether the process is in control, the whole process (population) should be examined in all different time intervals. This leads to a considerable increase in quality control cost. In this paper, the attention is given by differentiating sampling time interval and the possibility of detecting trouble areas, therefore would increase. In this way, finding out trouble areas would be easier which leads to a decrease in quality control cost.

Keywords: Process control, control chart, sampling, time interval, test of hypothesis.

Introduction

Many papers have been written about different types of quality control chart. The comparison of the effectiveness and robustness of nine typical control charts for monitoring both process mean and variance have been done[1]. Shewart's control chart for variable known as $\overline{X}$ chart includes finding three design parameters consist of the sampling interval, coefficient of control limits and sample size.The design of control charts may be noticed in three categories: i. heuristic designs, ii. economic designs and iii. statistical designs[2]. The design of $\overline{X}$ chart based on economic conditions has been already examined and a model for finding assignable cause has been designed[3]. A model for the economic design of control charts by implementing different production models has been developed[4]. In this case, a considerable monetary benefit of using an economic design rather than a heuristic design proposed by Ishikawa[5] was exhibited. The economic statistical designs for control charts by considering the statistical and economic objectives have been introduced[6]. These designs are semi economic designs and are more costly when it is compared with pure economic designs. An algorithm for optimizing the design of control charts has been proposed. In this case, control charts based on economic design have been developed[7]. The genetic algorithm to optimize parameters in an economic design has been also used[2]. An economic-statistical model to design acceptance control charts has been developed[8]. An economic model for the synthetic chart has been proposed[9]. A joint economic-statistical design of $\overline{X}$ and R control charts has been reviewed[10].

In recent years, for controlling the process mean and variance simultaneously, some researchers suggested different statistical charts[11,12].

Numerous papers regarding statistical quality control charts by considering statistical and economic design based on variable sampling interval (VSI) have been written[13-21]. The performances of VSI charts near non-normal process distributions have been studied[18]. A VSI multivariate Shewhart chart on the foundation of Hotelling's statistic has been considered[19]. A VSI policy for the steady-state performance has been examined[20]. A modification of the VSI idea called VSIs with sampling at fixed times (VSIFT) control charts, which implement in practice easily has been demonstrated[21].

It is possible that the product quality in one time of a work shift be different with the product quality in another time of the same shift. In some mentioned research, there is no mention of fluctuations in product quality. There was given consideration to this matter in some previous researches. There are two differences with the mentioned algorithm in this paper. First, those researches have been done with taking sample in fix times and our algorithm concentrates on random sampling. Second, in research done, it is assumed the fluctuation in quality of product would be revealed as they are produced. But in this paper, the method introduced would be able to recognize the possible fluctuation in products quality before implementing quality control chart and sample is taken with regarding them.

Material and Methods

In some production systems the process does not operate uniform. For example, a worker may begin the work with higher quality, but by getting close to lunch hours the quality get worse.

This of course is due to worker getting tired. Also this same worker after lunch and break starts his job very good and get worse after a certain period of time. This matter can also be checked in production equipment. A machine may produce better products in the beginning of shift, and get worse after a while because the parts get warm or dusty, or if electronic equipment is used, the temperature goes above standard level and brings down accuracy. Therefore, when we are dealing with a process control chart in such conditions mentioned above, number of sampling and sample size should be able to keep the process in control even in the periods the P.C (process capability) is decreased. Another word, the times the P.C is higher, the number of sampling and sample size are more than required.

It should be emphasized, in some times of the day more defectives are produced. According to reasons mentioned above, this would be true to look at it, as mean. Another word, this can happen in some days and not happen in other days. For example, regarding worker's tiredness, this can happen because of high environment temperature; but in other days the worker is not too tired to produce more defective units. If this happens, trend or falling points up or down will be observed which shows out of control condition. Therefore, in those times the sample means are supervised more carefully, in order to observe points falling out of control limits with higher probability, and make the necessary correction if trend or falling points, upper or lower control limits, are to be created confidently. Then, corrective action should be taken permanently every day and there would be no need for more supervision.

This can be observed in figure-1 and figure-2. Figure-1 and figure-2 are related to $\overline{X}$ charts respectively for first and second production units, in which the inspection is done periodically. Figure-1 shows the control charts for six consecutive days. In four days we have out of control situations and in two days in control. In figure-2, the inspections are for six consecutive days in second production unit. In this figure, points show a descending trend. In fifth inspection all six days are out of control and corrective action should be taken for this production unit.

In these conditions, the better way would be dividing work shift in equal time of *t* therefore, when the process variation is more significant, a smaller *t* is chosen. Then, in a specific period of time e.g. *n* days %100 inspections are executed and results are maintained.

In the following, the "control chart for average" is used for explanation. Results can be used for other charts.

If in *n* period in time 0-t, a number of products are produced, the selected mean specification obtained by

$$\mu_1 = \frac{\sum_{i=1}^{a} x_i}{a} \qquad (1)$$

Where, Xi is the selected specification for each product and *a* is number of products and μ_1 is the process mean at time 0-t. This is so for μ_2, μ_3, ... at times t-2t, 2t-3t , ... Therefore, specification mean capability of process can be obtained. If the work shift is equal to time *T*, then for the number of μ_is we have the formula below:

$$m = \frac{T}{t} \qquad (2)$$

Where, m is number of μ_is and *T* is the length of work shift and *t* is time interval that μ_is is calculated in which. Afterward, these *m* means are compared and one chart is used for equal means.

Results and Discussion

In many instances the means of more than two populations are chosen to compare. When the experiment includes more than two groups, each time each couple is compared by using test. This increases the number of comparisons required and also increases the possibility that the difference between groups are randomly significant. The test used for comparing more than two population is the "analysis of variance" (ANOVA). The null hypothesis (H_0) states that there is no difference among the population means and the alternative hypothesis (H_1) would be at least two population means show significant difference. If H_0 is rejected there would be difference among the groups and there should be some research to find out where the differences are[22, 23].

H_0: $\mu_1=\mu_2=....\mu_K$

H_1: $\mu_i\neq\mu_J$

At least two population means are different. For ANOVA test F distribution was used. Suppose there are K populations and the number of random sample of *i*th population (i=1,2,.....,k) equal to n_i and if *n* is total number of population samples, we have

$$\sum_{i=1}^{k} n_i = N \qquad (3)$$

Population 1: $x_{11}, x_{12}, \ldots, x_{1n_1}$

Population 2: $x_{21}, x_{22}, \ldots, x_{2n_2}$

Population K: $x_{K1}, x_{K2}, \ldots, x_{Kn_K}$

Assume independent x_{ij} with normal distributions with means μ_i and common variance σ^2 therefore $X_{Ij}=\mu_i+e_{Ij}$.

If μ is total mean and α_i is treatments' effect:

$$\mu_i = \mu + \alpha_i$$

$$(x_{ij} - e_{ij}) = \mu + \alpha_i \Rightarrow x_{ij} = \mu + \alpha_i + e_{ij} \qquad (4)$$

In which the condition $\sum_{i=1}^{k} \alpha_i = 0$ (total deviations from the mean is zero) is not ruled. The null hypothesis to be tested would be all mean populations having equal means $\mu_1 = \mu_2 = ... = \mu_k$ or its equivalence

$$H_0 : \alpha_i = 0, i = 1,2,3,...,k$$

$$H_1 : \alpha_i \neq 0$$

The alternative hypothesis; the difference is at least for one *i* value.

If H_o is rejected there is significant difference among population means. (Sum of square total) SST=SS (Tr) (treatments sum of square) +SSE (sum of square errors)

$$\sum_{i=1}^{k}\sum_{j=1}^{n_i}(x_{ij}-\bar{x}_{00})^2=\sum_{i=1}^{k}n_i(\bar{x}_{i0}-\bar{x}_{00})^2+\sum_{i=1}^{k}\sum_{j=1}^{n_i}(x_{ij}-\bar{x}_{i0})^2 \quad (5)$$

$\bar{x}_{io}$ is the average of *ni* observation of *i*th population and $\bar{x}_{00}$ is the average of all *n* observations[22,23]. The analysis of variance is shown in the table-1.

If calculated $F \succ F_{\alpha}$ the null hypothesis is rejected[22, 23]. It should be revealed which one of these *m* means are different. First *a* value for α is selected, then by using two tailed test of H_0: $\mu_1=\mu_2$ and H_1: $\mu_1 \neq \mu_2$ the means are compared. It should be noted the determining α value is not supporting H_0. This shows that by increasing α, the tendency goes toward rejecting H_0. To choose a value for α, the following items should be noted; i. If *m* is a big number, α should be small because there is no desire to increase the number of charts used. ii. If the cost of testing sample is high, α should be large because the points which need less sampling are recognized. Another word, when costs increase there is more desire to use more charts. The greater α value raise the probability of rejecting H_0 which leads to result of unequal means. Therefore, the charts needing less sampling would do so. iii. If process percent defective is high α should be small, otherwise depending on defective level, α should increase. So if the percent defective is high this mean there is no need for control accuracy to be high. But if percent defective is low, it means we need to pay more attention in controlling the process. Therefore, the H_0 should be rejected in order to get to planned result. iv. When production level is low α should be selected small, and when the process is producing in large quantity the sample taken from the lot is larger, therefore, if more charts are to be used as much as we can get information from the process, the sample is brought to the chart. But if the production is low the number of sampling on the chart is also low. Therefore, enough information may not be obtained from the process. Thus, when production is low a few number of charts are used and this means accepting H_0 in a situation where α is small. After assigning α, $Z_{\alpha/2}$ is obtained from the table[22, 23].

The z value (standardized variable) is calculated by using:

$$Z = \frac{\mu_1 - \mu_2}{\frac{\sigma}{\sqrt{n}}} \quad (6)$$

Where, μ_1 and μ_2 are respectively the first and second population, σ is population standard deviation and *n* is sample size.

The value of μ_1 and μ_2 are compared and difference of μ_1 and μ_2 are considered according to the following items:

If $Z \succ Z_{\frac{\alpha}{2}}$ or $Z \prec -Z_{\frac{\alpha}{2}}$ two populations are significantly different. If $-Z_{\frac{\alpha}{2}} \prec Z \prec Z_{\frac{\alpha}{2}}$ two populations are not different[22,23], then, the above procedure for (μ_1 and μ_3), ($\mu_2$and μ_3) and so forth is done. We need $\binom{m}{2}$ comparisons. After making all comparisons, μ_is that show no difference with each other, are brought on one chart. If none of the μ_is show any statistical difference, then they are brought on one chart because the process of production is uniform. Therefore, one chart would be enough for the whole process. Now those different means are specified. In this way, it is clear the means are related to which time interval. For example, if μ_1 with μ_3 are equal and μ_2 with μ_4 are equal then the process in time interval. 0-t and 2t-3t have same capability and in the interval of t- 2t and 3t- 4t also have the same capability. Therefore, for the times 0- t and 2t- 3t one chart for the average and in times t- 2t and 3t- 4t another chart should be provided. Then, number of sampling done in each of those time interval should be different and consider the following items: i. In charts where process average is closer to the desired one, the number of sampling should be kept low, and when it is further from process average the number of sampling should be increased. ii. If random sampling method is used in charts where data show longer period of process time, number of sampling should be larger and in charts represent shorter period of process, number of sampling should be small. If time interval inspection is done, only the time interval between two small inspections for the period between two inspections in charts with desired average and time interval between two large inspections for undesired charts should be specified.

In figure-3, excel window is demonstrated. For the purpose of random sampling, this software is used with the design of following procedure: first the noted time intervals for sampling purpose enter column A. Then, in column B, C, and D numbers are given to each time interval and if the importance of time interval for sampling is less, then only in column B a number is given to it. Otherwise in columns C and D numbers are also assigned for selecting in random function. For example, because of time interval a row from 8 to 13 are twice as important as to time interval in rows 1 to 7 and 14 to 19 in the figure shown, numbers are also given to them in column C. The importance of rows 20 to 25 is 1.5 times greater than rows 8 to 13. Afterward, in cell H1, the following function is defined: = INT (RANDBFTWEEN (1, 44))

Then, according to the created random number in column E, the following function is defined. = OR ($ H $ 1 = Bi) i=1, 2…26

If the true revised of that time interval is considered as sampling interval, then for simplifying, we have saved the time gained in G1 cell.

If sampling from the process is done in a situation which process average is far from the desired one, in this case, the times the process average is close to desired average, the number of sampling would be more than required. This of course increases the cost. After performing the steps mentioned, sampling frequency for the times when process average is far from the desired one, would be greater. But for the rest of the time less number of sampling is performed, therefore total number of sampling is reduced.

If it is accepted that before the actions are taken, those times the weak process capability is due to normality of population, another word the undesired condition lies further from the mean and average chart with $\pm 3\sigma$ has normal distribution with 99.73% confidence intervals, then this is also accepted that by this act the chart has more than 99.73% even though the control limits have not changed but in this condition sample mean's population is not normal and normality of population in desired condition would not be kept. Therefore, we have higher probability of process being out of control.

The suggestion would be for every chart that process average is far from desired average, one label is prepared and the reasons of point falling outside limits are noted (in the beginning of paper a number of them was mentioned). In this case if an average sample falls outside limits, for instance, that is due to worker fatigue which is noted on the table. Then we know the worker need some time to rest. In fact, in big organization this act would facilitate quality management, because in some cases, finding trouble shooting roots, take time.

If 100% inspections are not possible in the process, the times in which the possibility of producing defectives is high, the higher number of sampling frequency and more samples would be suggested. For the times the possibility of producing defectives is low, the lower sampling frequency and less sample would be enough and one chart may be used for whole process. In such condition it would not be possible to break down the process.

Conclusion

As discussed in this paper, by dividing time into different equal time interval, first the errors in different time interval should be compared with each other. If there is no difference, we can draw the control charts as usual and it can be examined by the test of analysis of variance. If in this test the inequality of mean errors in different time interval is recognized, in order to better realize the conditions, tests for all combinations of two by two are used. Assigning α value and its effect on accepting or rejecting H_0 was also examined. Finally, it can be inferred that using different sampling time interval can facilitate process control which leads to a quite significant decrease in quality costs.

In the future research, we can use this algorithm for the deviation chart. Furthermore, presenting this algorithm for sampling, in preparing mean and deviation charts simultaneously, will be an interesting subject for future research.

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References

1. Ou Y., Wen D., Wu Z. and Khoo M.B.C, A Comparison Study on Effectiveness and Robustness of Control Charts for Monitoring Process Mean and Variance, *Qual. Reliab. Engng. Int*, **28,** 3-17 **(2012)**

2. Vommi V.B. and Seetala M.S.N, A new approach to robust economic design of control charts, *Applied Soft Computing*, **7**, 211–228 **(2007)**

3. Duncan A.J., The economic design of $\overline{X}$ charts used to maintain current control of a process, *J. Am. Stat. Assoc,* **51**, 228–242 **(1956)**

4. Lorenzen T.J. and Vance L.C., The economic design of control charts: a unified approach, *Technometrics*, **28**, 3–11 **(1986)**

5. Ishikawa K., Guide to Quality Control, Asian Productivity Organization, Tokyo **(1976)**

6. Saniga E.M., Economic statistical control chart designs with an application to $\overline{X}$ and R charts, *Technometrics*, **31**, 313–320 **(1989)**

7. Shamsuzzaman M. and Wu Z., control chart design for minimizing the proportion of defective units, *journal of manufacturing systems*, **25**, 269-278 **(2006)**

8. Mohammadian F. and Amiri A., Economic-Statistical Design of Acceptance Control Chart, *Qual. Reliab. Engng. Int*, **(accept for publish)**

9. Yeong W. C., Khoo M. B. C., Wu Z. and Castagliola P., Economically Optimum Design of a Synthetic $\bar{X}$Chart, *Qual. Reliab. Engng. Int* **(accept for publish)**

10. Chen H. and Pao Y., The joint economic-statistical design of *X* and *R* charts for nonnormal data, *Qual. Reliab. Engng. Int*, **27**, 269–280 **(2011)**

11. Lin Y.C. and Chou C.Y., Adaptive X control charts with sampling at fixed times, *Qual. Reliab. Engng. Int*, **21**, 163-175 **(2005)**

12. Costa A.F.B. and Rahim M.A., Joint X and R charts with two-stage samplings, *Qual. Reliab. Engng. Int*, **20**, 699-708 **(2004)**

13. Reynolds M.R. Jr, Amin R.W., Arnold J.C. and Nachlas J.A., X charts with variable sampling intervals, Technometrics, **30(2)**, 181-192 **(1988)**

14. Zhang Y., Castagliola P., Wu Z. and Khoo M.B.C., The Variable Sampling Interval .X Chart with Estimated Parameters, *Qual. Reliab. Engng. Int*, **28**, 19-34 **(2012)**

15. Tagaras G., A survey of recent developments in the design of adaptive control charts, *Journal of Quality Technology*, **30(3)**, 212-223 **(1998)**

16. Wu Z. and Luo H., Optimal Design of the Adaptive Sample Size and Sampling Interval np Control Chart, *Qual. Reliab. Engng. Int*, **20**, 553–570 **(2004)**

17. Reynolds MR Jr and Arnold J.C., Optimal one-sided Shewhart charts with variable sampling intervals, *Sequential Analysis*, **8(1)**, 51-77 **(1989)**

18. Amin R.W. and Miller R.W., A robustness study of *.X* charts with varible sampling intervals, *Journal of Quality Technology,* **25(2)**, 36-44 **(1993)**

19. Aparisi F. and Haro C.L., Hotelling's *T*2 control chart with variable sampling intervals, *International Journal of Production Research*, **39(14)**, 3127-3140 **(2001)**

20. Runger G.C. and Montgomery D.C., Adaptive sampling enhancements for Shewhart control charts, *IIE Transactions*, **25(3)**, 41-51 **(1993)**

21. Reynolds MR Jr, Shewhart and EWMA variable sampling interval control charts with sampling at fixed times, *Journal of Quality Technology*, **28(2)**, 199-212 **(1996)**

22. Freund J., Mathematical statistics (5th edn), Prentice hall, **(1992)**

23. Neter J., Wasserman W. and Whitmore G.A., Applied statistics (3th edn), Allyn and Bacon Inc, **(1988)**

Table-1
Analysis of variance

Source of variation	Degree of freedom	SST	SSR	F statistic
Treatments	K-1	$SS(T_r)$	$MS(T_r) = \frac{SS(Tr)}{K-1}$	$F = \frac{MS(T_r)}{MSE}$
Error	n-K	SSE	$MSE = \frac{SSE}{n-K}$	Degree of freedom
Total	n-1	SST	$MST = \frac{SST}{n-1}$	$\partial_F = (K-1, N-K)$

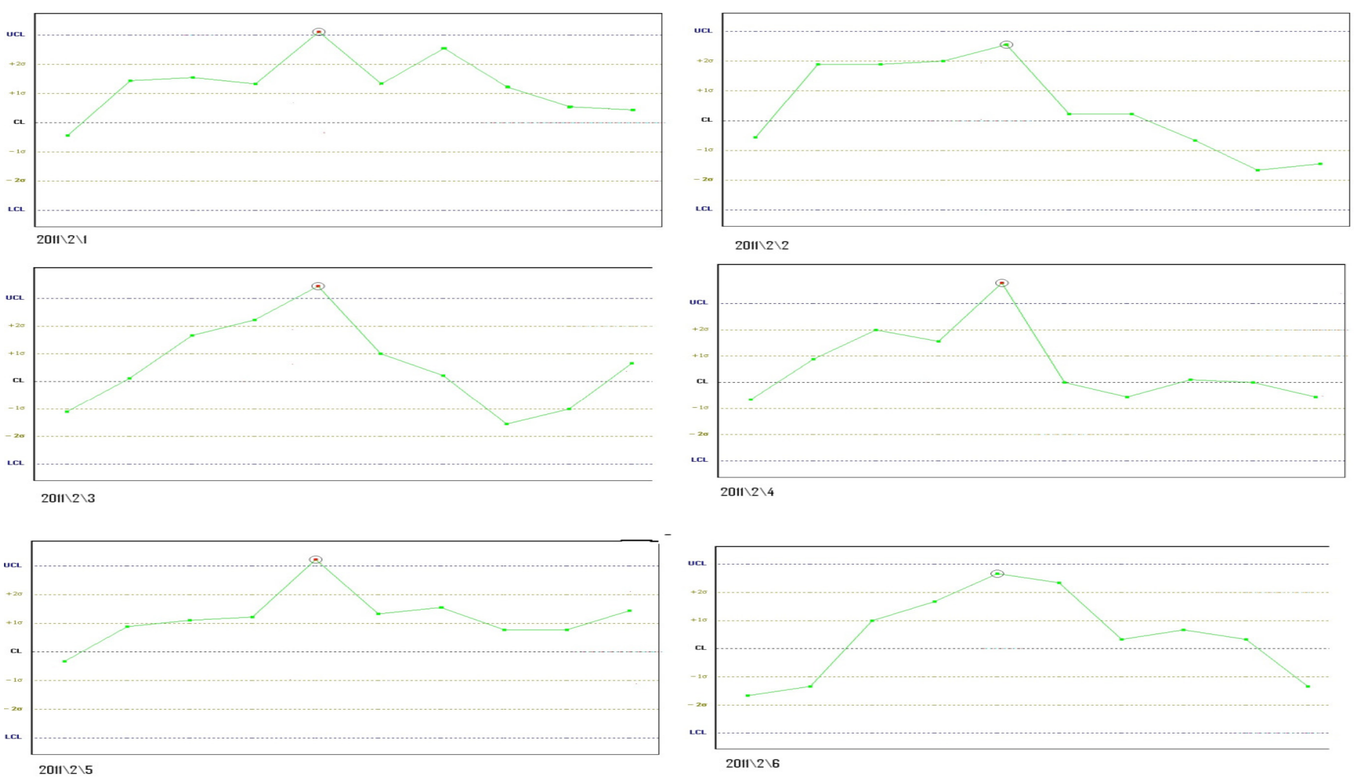

Figure-1
$\overline{X}$ chart for first production unit in six consecutive days

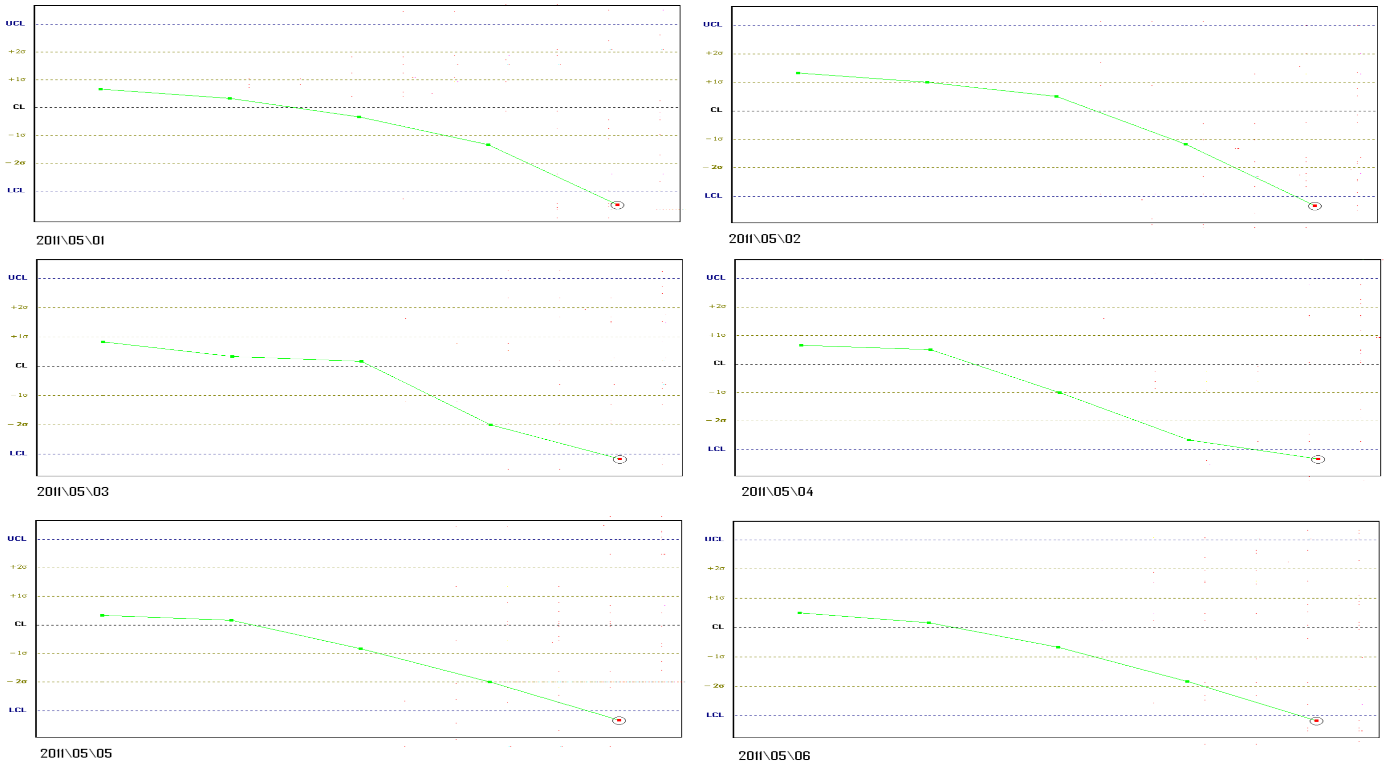

Figure-2
$\overline{X}$ chart for second production unit in six consecutive days

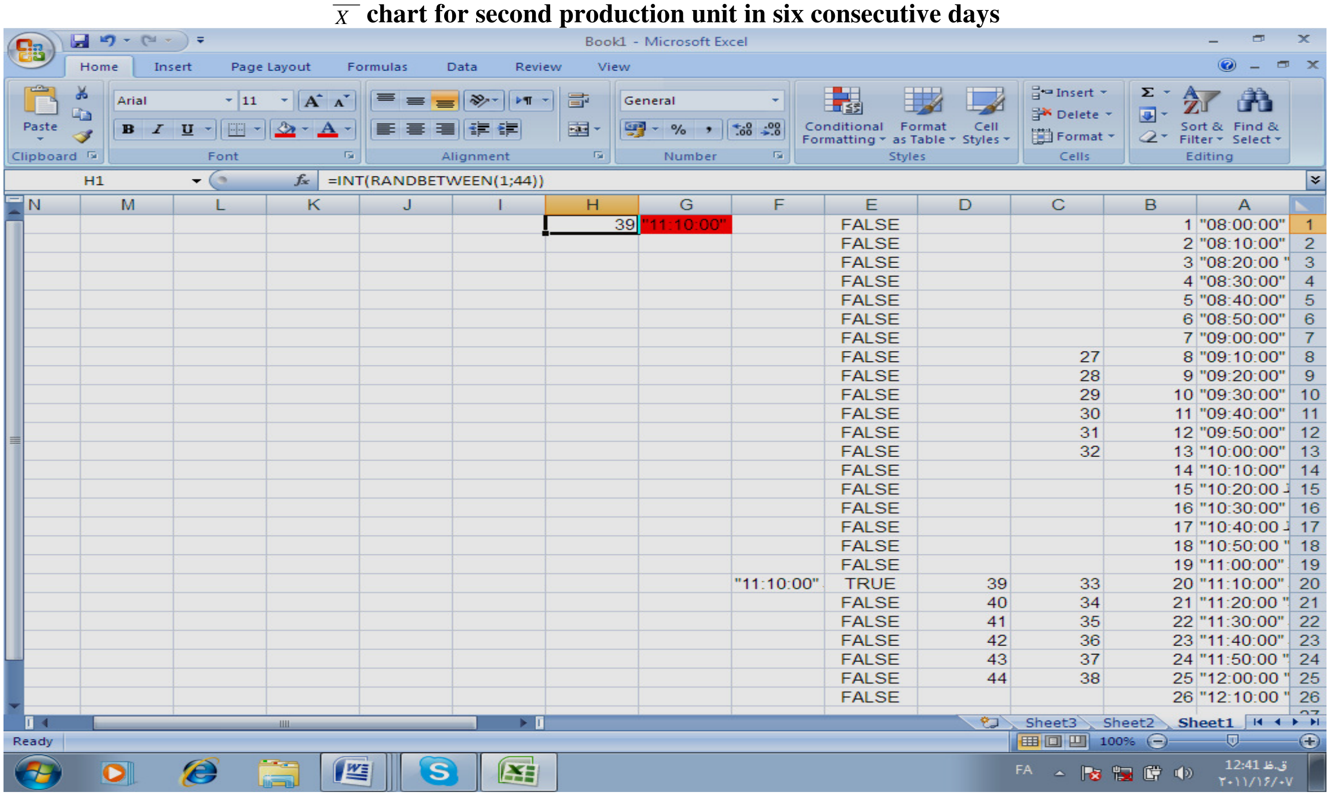

Figure-3
Defined function to specify hypothetical sampling time