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Measurement of volumes in the *Gaṇita-Sāra-Saṅgraha* of Mahāvīrācārya (c.850 AD)

Vanaja V.[1] and Shailaja M.[2*]
[1]Department of Mathematics, Government First Grade College, Yelahanka, Bangalore-560 064, India
[2]Department of Mathematics, Government First Grade College, Vijayanagara, Bangalore-560 104, India
shaila.ac55@yahoo.com

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Abstract

Ancient Indian mathematical texts presented detailed problems in arithmetic, algebra, and geometry, which were essential for daily life. Mahāvīrācārya's treatise, the Gaṇita-Sāra-Saṅgraha (GSS), represents a significant contribution to mathematics. This work comprises nine chapters and approximately 1100 verses. Chapter eight, titled Khātavyavahāraḥ (Calculations regarding excavations), uniquely and thoroughly addresses calculations for the depth of a ditch, the volume of a pyramid and its frustum, the volume of a cone, and the volume of a sphere. He developed formulas for calculating the areas and volumes of both regular and irregular polygons. His contributions represent a significant and distinctive advancement upon the work of earlier mathematicians such as Āryabhaṭa and Bramhagupta. This paper examines issues concerning certain calculations and other Indian mathematical texts, offering modern mathematical interpretations.

Keywords: *GSS,* excavation, volume, *Karmāntia*, *Auṇdra*, depth, equilateral triangle, equilateral quadrilateral, circle, pyramid, sphere.

Introduction

Mahāvīrācārya was a famous mathematician from Karnataka, a southern Indian state, who lived in the ninth century (about 850 AD). He wrote the *Gaṇita-Sāra-Saṅgraha* (GSS), a pure mathematical treatise of our ancient mathematics. Most of the ancient mathematical works are the part of the astronomical text but this treatise is exceptional. In the eleventh century, Pavaluri Mallanna translated this literature, which was primarily in Sanskrit, into Telugu in verse. For about 300 years, this text GSS had the exceptional honor of serving as a textbook in South India[1]. The Madras government published it in 1912 AD after Rao Bahadur Prof. M. Rangācārya of the Presidency College, Madras, translated it into English. Prof. L.C. Jain translated this material into Hindi in 1963, and Prof. Padmavatamma translated it into Kannada in the year 2000. There are nine chapters in this book that cover excellent mathematical ideas in geometry, algebra, and arithmetic[1]. The notion of excavation calculations, volumes and their frustum of certain solids, calculations related to piles (of bricks), and calculations related to saw work (in sawing wood) were all covered in chapter 8, which has a total of 68 verses[2]. Here, we used contemporary computations to find out the ideas associated with specific passages in the text.

Measurement of volume of excavation

Ancient mathematics discusses measuring the cubical contents (volume) of excavations. In his GSS text, Mahāvīrācārya articulated this idea as follows[3]:

Kṣetra phalaṁ vedha guṇaṁ samakhāte vyāvahārikaṁ gaṇitam
Mukhatala yutidalama thasatsaṅkhyāpatṁ syātsamī karaṇam||
(GSS, viii-4)

English Translation: The estimated measure of the cubical contents in a typical excavation is obtained by multiplying area by depth. After adding the halved quantities of the same denominations and dividing the total by the number of the aforementioned (halved quantities), the sums of (all the varied) top dimensions with the corresponding bottom dimensions are halved. This is how the average equivalent value is determined[2].

This verse explains how to calculate the volume of an excavation that is roughly equal to any given irregular excavation using the parameters of a regular excavation.

Regular dimension: i.e., Volume of a regular excavation = Given area × Depth

Irregular dimension: Average equivalent value =

$$\left(\frac{\frac{\text{n number of top dimensions}+\text{n number of bottom dimensions}}{2}}{\text{n}}\right)$$

Concerning the formulas mentioned above, the section of a regular excavation that contains the area of geometrical surfaces like a square, an equilateral triangle, a circle, and a rectangle is represented by several instances provided by Mahāvīrācārya.

Example:

sāṣṭaśatavyāsasyakṣetrasya hipañcaṣaṣṭisahitaśatam |
vedhovṛttasyatvaṁsamakhātekiṁ phalaṁ kathaya||
(*GSS*, viii-7)

The excavation is 165 *hastas* deep and has a diameter of 108 *hastas* in respect to a regular circular region that represents the section of a typical excavation. Determine its volume[2].

Solution: Given a normal circular region that represents a piece of a regular excavation, with a diameter of d = 108 *hastas* and a depth of h = 165 *hastas*
Therefore, Volume of regular excavation = Area × Depth=$\pi \frac{d^2}{4} \times 165 = 481140\pi$ cubical *hastas*
According to the modern calculations considering the value of $\pi = 3.14159265359$ the above result of volume is equal to 1511545.889348193[4].
Note: The value of π in several Indian mathematical literature. Ārybhata I states that π=62,832/20,000=3.1416 (Ā,ii-10). The value of π in Jain's edition (suppl.p.33) almost equals 3, although Mahāvīrācārya (GSS, vii-60) took π=√10=3.16. The Greek value of π=22/7 (ratio of two integers) can be found in the works of Ārybhaṭa II (MSi, xv-92i), and Bhāskara II (c.1150) uses it as a practical approximation. In 1761, Lambert demonstrated that π is irrational; nevertheless, in 1882, Lindemann demonstrated that π is also transcendental[5].

Method to find the Accurate Volume of excavation

To get the precise value of the volume of excavation the processes of *Karmāntika, Auṇḍra* are detailed in the following verse.
Bāhyābhyantarasaṁsthitatattatkṣetrasthabāhukoṭibhuvaḥ |
svapratibāhusametā bhaktāstatkṣetragaṇanayānyonyam ||
Guṇitāścavedhaguṇtāḥ karmāntikasaṁjñagaṇitaṁ syāt |
tadbāhyāntarasaṁsthitatattatkṣetrephalaṁ samāniya ||
Saṁyojyasaṅkhyayāptaṁ kṣetrāṇāṁ vedhaguṇitaṁ ca, auṇḍraphalaṁ |
Tatphalayorviśeṣakasyatribhāgena, saṁyuktaṁ karmāntikaphalameva hi bhavatisūkṣmaphalam ||
(*GSS*, viii-9-11$\frac{1}{2}$)

English Translation: The sides of the ground section are added to all the comparable sides of the other sections and divided by the total number of sections of the outer (i.e., at the ground) and different interior sections (of the excavation). In line with the procedure for calculating the area of a figure of that shape, multiply these sides of the average section by each other. The result, when multiplied by the depth, will be the *Karmāntika* volume. Determine the areas of those sections numerous times, add them up, and divide the result by the total number of sectional areas. This quotient, when multiplied by the depth, will yield the *Auṇḍra* volume. The truly exact volume will be one-third the difference between those two volumes plus the *karmāntika* volume[1].

In these verses we get the procedure to find the accurate volume of an excavation.
Let the number of sections be 'n' and $f(a, b, c, \ldots)$ denote the area of a section as a function of its defining (independent) linear dimensions a,b,c….let…$a_1,b_1,c_1,\ldots a_2,b_2,c_2,\ldots$ a_3,b_3,c_3,…….let be these linear dimension or sides of the 'n' sections.
Then the *karmāntika* volume = $P = h \times f(a, b, c, \ldots)$.
where: h is the depth of the excavation and
$a = (a_1 + a_2 + \cdots + a_n)/n$; $b = (b_1 + b_2 + \cdots + b_n)/n$; and so on.
Auṇḍra volume= $G = \frac{h(A_1+A_2+A_3+, \ldots +A_n)}{n}$
Where: $A_1 = f(a_1, b_1, c_1, \ldots)$; $A_2 = f(a_2, b_2, c_2, \ldots)$ etc., are the areas of the 'n' secrions.
Exact Volume = $V = P + \frac{(G-P)}{3} = \frac{2P+G}{3}$
The process will be the same as that described by Brahmagupta in his mathematical treatise, the *Brahmaspuṭa siddhānta*, if we simply take into account the two extreme regions, top and base. In this case, "n" will equal two. It will be observed that Mahāvīra considers multiple parallel portions of the solid rather than just two (top and base) while calculating average volumes[6].
Example:
vāpī samatribāhuviṁśatiruparīhaṣoḍaśaiva tale |
vedhonavakiṁgaṇitaṁ karmāntikamauṇḍramapi ca sūkṣmaphalam. || (*GSS*,viii-13$\frac{1}{2}$)
There is a well that has an equilateral triangle as its (sectional) area. Each side of the top (sectional area) has a value of 20 *hastas*, while each side of the bottom (sectional area) has a value of 16; the depth is 9 *hastas.* What is the precise volume measure here, as well as the value of the *karmāntika* and *auṇḍra* volumes?
Solution: Let the area of an equilateral triangle of side 'a' be taken as ka^2. Then we have
Karmāntika volume

$$= P = k\left(\frac{20+16}{2}\right)^2 \times 7 = 2916k \ \ cubical\ hastas$$

Auṇḍra volume

$$= G = k\left(\frac{20^2+16^2}{2}\right) \times 7 = 2952k\ cubical\ hastas$$

$$\text{Accurate volume} = V = P + \left(\frac{G-P}{3}\right)$$
$$= 2928k\ cubical\ hastas$$

We know that k equals $\frac{\sqrt{3}}{4}$ for exact area, which is also evident from the GSS verse vii-50-51. Thus, $P = 729\sqrt{3}$; $G = 738\sqrt{3}$; $V = 732\sqrt{3}$ will be the necessary outcomes.

Nonetheless, the answers provided in Jain's edition (suppl. p. 33) match the approximate number $k = \frac{1}{2}$ suggested in GSS vii-7. The actual volume of the supplied prismoid is $732\sqrt{3}$., therefore our approximate selection does not do justice with Mahāvīrācārya's value
Example:
samavṛttāsauvāpī viṁśatiruparīhaṣodaśalva tale |
vedhodvādaśadaṇḍāhkiṁ syātkarmāntikauṇḍrasūkṣmaphalaṁ.||
(*GSS*, viii-14$\frac{1}{2}$)
A well has a sectional area that is regularly circular. The top (sectional area) has a diameter of 20 *daṇḍas* whereas the bottom (sectional area) has a diameter of just 16 *daṇḍas*. There is a

depth of 12 *daṇḍas*. What may be the precise volume measure of *karmāntika*, the *auṇḍra* in this situation?
Solution: Here the extreme sections are circular of diameters 20 and 16 respectively and the depth is 12 so that we have
Karmāntika volume= $P = \frac{\pi}{4}\left(\frac{20+16}{2}\right)^2 \times 12 = 972\pi$;
$Auṇḍra$ volume $= G = \frac{\pi}{4}\left(\frac{20^2+16^2}{2}\right) \times 12 = 984\pi$and
Accurate volume $= V = 972\pi + 4\pi = 976\pi$
The solution depends on the value of π which is clearly explained in the previous section.

Example of frustum of wedges discussed in *GSS* viii is
Navtiraśītiḥ saptatirāyāmaścordhvamadhyamūleṣu |
vistārodvātriṁśatṢoḍaśadásasaptavedho'yam.‖
(*GSS*, viii-16$\frac{1}{2}$)

The top, middle, and bottom lengths of a well with rectangular sections are 90, 80, and 70, respectively; the breadths are 32, 16, and 10. Determine its volume when 7 is its depth[4].
Solution: Here $P = \left(\frac{90+80+70}{3}\right)\left(\frac{32+16+10}{3}\right) \times 7 = \frac{32480}{3}$;
$G = \left(\frac{90\times32+80\times16+70\times10}{3}\right) \times 7 = 11340;$
$$V = P + \left(\frac{G-P}{3}\right) = \frac{98980}{9}$$
Given the printed dimensions and the assumption that the reservoir is an inverted double-frustum of wedges connected at a shared central section, the precise volume of the reservoir can be calculated by summing the volumes of the two components, which are computed independently as follows:
The precise volume can be calculated by summing the two sections, assuming that it is a double frustum of two truncated cones.
$P_1 = \left(\frac{90+80}{2}\right)\left(\frac{32+16}{2}\right) \times \frac{7}{2} = 7140; G_1 = \left(\frac{90\times32+80\times16}{2}\right) \times \frac{7}{2} = 7280;$
$$V_1 = 7140 + \left(\frac{140}{3}\right) = \frac{21560}{3}$$

For lower part,

$P_2 = \left(\frac{80+70}{2}\right)\left(\frac{16+10}{2}\right) \times \frac{7}{2} = \frac{6825}{2}; \quad G_2 = \left(\frac{80\times16+70\times10}{2}\right) \times \frac{7}{2} = 3465;$
$$V_2 = \frac{6825}{2} + \frac{105}{6} = 3430$$
Hence the exact volume $= V = V_1 + V_2 = \frac{31850}{3}$

A few problems pertaining to various types of frustum of solids of shapes on square, rectangular, circular, or equilateral triangle bases can be found in GSS chapter viii[4].

Measurement of Volume of a Sphere

The method for arriving the volume of a sphere is explained in the following verse.
Vyāsārdhaghanārdhaguṇā navagolavyāvahārikaṁ gaṇitam|
taddaśamāṁśaṁ navaguṇamśeṣasūkṣmaṁ phalaṁ bhavati.‖
(*GSS*,viii-28$\frac{1}{2}$)

English translation: The estimated value of the cubical contents of a sphere is obtained by multiplying the half of the cube of half the diameter by nine. The precise value of the cubical measure is obtained by multiplying this (approximate value) by nine and dividing the result by ten while ignoring the rest.
The mathematical expressions are as follows;

Approximate Volume of a sphere $= V_a = \frac{\left(\frac{d}{2}\right)^3}{2} \times 9 = \left(\frac{d}{2}\right)^3 \times \frac{9}{2}$
Accurate Volume of a sphere= $V = V_a \times \frac{9}{10}$
We are aware that the current formula for a sphere's volume is$\frac{4}{3}\pi r^3$.
The current GSS formula is relatively good compared to modern formula which is based on earlier Indian mathematics literature. This can be seen by looking at the example that follows[3].
Ṣoḍaśaviṣkambhasya ca golakavṛttasyaviganyya, |
kiṁvyāvahārikaphalaṁ sūkṣmaphalaṁ cāpi me kathaya.‖
(*GSS*,viii-29$\frac{1}{2}$)

Determine the approximate value and precise volume of a sphere with a diameter of 16.
Solution: Here, diameter = d = 16
Approximate Volume of a sphere $= V_a = \left(\frac{16}{2}\right)^3 \times \frac{9}{2} = 2304$
Accurate Volume of a sphere= $V = V_a \times \frac{9}{10} = 2304 \times \frac{9}{10} = 2073.6$
In modern calculations, Volume of a sphere $= V = \frac{4}{3}\pi r^3 = \frac{2048}{3}\pi$

Measurement of Volume of a Pyramid

The text *GSS* describes finding the approximate and accurate value of the volume of an equilateral triangular base pyramid with mathematical expressions in the following verse.
Bhujakṛrtidalaghanaguṇadaśapadanavahṛdvyāvahārikaṁ gaṇitam,|
triguṇaṁ daśapadabhaktaṁ śṛṅgāṭakasūkṣmaghanagaṇitam. ‖
(*GSS*, viii-30$\frac{1}{2}$)

English translation: The square root of the resultant product is divided by nine after the cube of half the square of the side (of the basal equilateral triangle) is multiplied by ten. This results in the roughly computed value (needed). The precisely determined contents of the pyramidal excavation are obtained by multiplying this approximate amount by three and dividing the result by the square root of ten[7].
It is given explicitly for an equilateral triangular base pyramid and discussed two parts of volumes such as approximate volume and accurate volume

Approximate Volume of a pyramid $= V_a = \frac{\sqrt{\left(\frac{a^2}{2}\right)^3 \times 10}}{9} = \frac{a^3}{18}\sqrt{5}$
Accurate Volume of a pyramid $= V = V_a \times \frac{3}{\sqrt{10}} = \frac{a^3}{12}\sqrt{2}$

Where: 'a' provides a side of the basal equilateral triangle and the height of the pyramid.
Let's verify these computations using the example below.
tryaśrasya caśṛṅgāṭakaṣaḍbāhughanasyagaṇayitvā|
kiṁvyāvahārikapahalaṁ gaṇtaṁ sūkṣmaṁ bhavetkathaya.||
(*GSS*, viii-31$\frac{1}{2}$)

Determine the approximate and precise values of the cubical measure of a triangular pyramid whose side is 6 in length.
Solution: Given side of the triangular pyramid = a=6

Approximate Volume of a pyramid $= V_a = \frac{a^3}{18}\sqrt{5} = \frac{6^3}{18}\sqrt{5} = 12\sqrt{5}$

Accurate Volume of a pyramid $= V = \frac{a^3}{12}\sqrt{2} = \frac{6^3}{12}\sqrt{2} = 18\sqrt{2}$

The volume of a pyramid in modern formula is expressed as $V = \frac{1}{3} \times area\ of\ a\ base\ \times height$

An equilateral triangle's area with side'a' is $\frac{\sqrt{3}}{4}a^2$ then area with side 6 is $9\sqrt{3}$ and height of and equilateral triangle $h = \frac{\sqrt{6}}{3}a = \frac{\sqrt{6}}{3} \times 6 = 2\sqrt{6}$

Therefore, required Volume= $V = \frac{1}{3} \times 9\sqrt{3} \times 2\sqrt{6} = 18\sqrt{2}$

In this instance, the volume of a pyramid in GSS and contemporary computations yields the same outcome. This is also one of Mahāvīrācārya's outstanding accomplishments, as noted in his mathematical treatise *GSS*[2].

Conclusion

In the preceding sections, we went into great length on how to measure the volumes of various excavations using mathematical techniques, how to articulate the text *Gaṇita-Sāra-Saṅgraha* and how to analyse it using earlier mathematical studies. The values derived from the provided verses (problems) are also contrasted with contemporary mathematical formulas.

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