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Projectile motion using Mathematica

Dharma R. Paudel
Central Campus of Science and Technology, Mid-Western University, Surkhet, Nepal
avipaudel1983@gmail.com

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Abstract

When anybody is thrown into space at some velocity by making an angle with horizontal, then it moves under the influence of gravity alone is known as a projectile. When air is present, it moves under the combined effect of both air and gravity. In the absence of air, the body traces completely a parabolic path. In this case, the projectile is moving with constant horizontal velocity while vertical velocity is changing due to the gravity. With air, the trace of the projectile is not parabolic. The vertical, as well as horizontal velocities, are affected by the air. This article shows the path of the projectile in both cases by using the Mathematica tool. Mathematica is a modern technical computing tool used for various computing as well as the simulation process. The traces are obtained at various initial velocities by keeping some other parameters constant. Results show that the projectile describes more range without air than with air as expected theoretically.

Keywords: Air drag, gravity, Mathematica, projectile, velocity.

Introduction

A projectile is anybody thrown into the sky with an initial velocity which moves under the effect of gravitational acceleration and air resistance[1]. The path traced by a projectile is called trajectory. Since the acceleration due to gravity acts vertically, a projectile moves in a uniform speed along the horizontal direction while the vertical component of the velocity goes on changing in the rate equal to the acceleration due to gravity[1]. This is only the ideal case. However, in reality, the path of a projectile is affected by the air resistance[2].

When the body is projected into the sky, the friction due to air plays an important role in the body to reach the surface of the earth. Without air resistance, the projectile moves completely a parabolic path[3]. However, due to the air resistance, the projectile does not follow completely the parabolic path. The vertical motion of the projectile is changed and the motion is driven by the collective effect of both gravity and air[4]. Our aim is to draw the path of it is motion in the presence of air and that in the absence of using Mathematica tool.

The opposing force due to the air depends on the velocity of the projectile and air drag coefficient[5]. In terms of the drag coefficient c_d, the drag force (F_{air}) might be written as[6:]

$$F_{air}=1/2\ c_d\ \rho\ v^2{}_A \qquad (1)$$

where ρ = density of the air medium, v = velocity of the projectile, A = area of the body facing in the air medium.

The coefficient of air drag is not constant but depends on different quantities like the shape of the body, Reynold number, etc[7]. The typical value of c_d for the sphere is 0.45 and that of a circular flat plate is 1.12[6].

However, as for limitation, we have taken this coefficient as a constant throughout this paper.

Theory: Suppose a projectile of mass 'm' is launched from ground with the velocity 'v_0'. We first consider the ideal case i.e. no air resistance. Let the angle of projection with the horizontal i.e. X-axis is 'θ', as shown in Figure-1 by the dotted line[1].

The initial velocity v_0 possesses two components, horizontal velocity $v_0 \cos(\theta)$ and vertical velocity $v_0 \sin(\theta)$ along X and Z axes respectively. The horizontal velocity remains unchanged while vertical velocity is affected because of the acceleration (g). The variation of vertical velocity and horizontal velocity with time is shown in Figure-2 and Figure-3 by the blue colored curves[8].

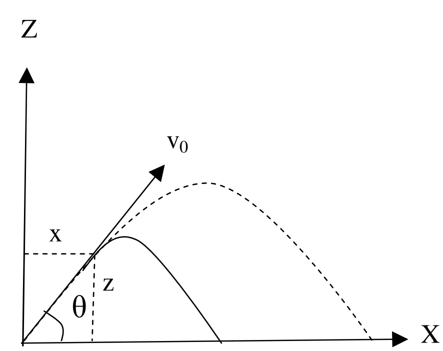

Figure-1: Projectile motion from ground. Solid line represents the path with air resistance while, the dotted lines represents that without air[1].

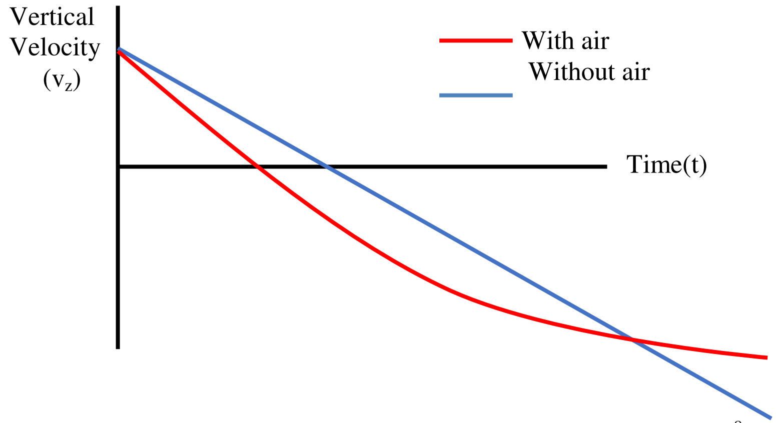

Figure-2: Variation of vertical velocity in the air and its absence[8].

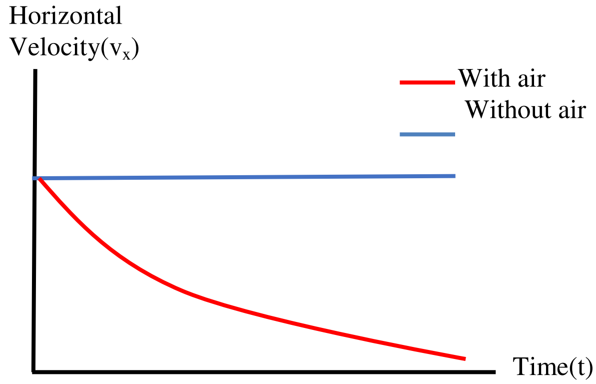

Figure-3: Variation of horizontal velocity in the air and it's absence[8].

At time t, the position of the projectile is given by[9]:

$x = v_0\ t \cos(\theta) = v_x t$ (2)

$z = v_0\ t \sin(\theta) - ½\ gt^2 = v_z t - ½\ gt^2$ (3)

From Equation (2) and (3), we can see that x position is steadily increasing while the z position is parabolic in nature.

When the air resistance is included, then motions in both the directions are affected. The path of the projectile does not become a parabolic in this case (Figure-1) and the variation of vertical and horizontal components of velocities are shown in the Figure-2 and Figure-3 by red curves[8]. The air resistance force is directly proportional to the square of the instantaneous velocity v of the projectile[6]. However, to get the tractable equation of motion, we take the dependency as directly proportional to v and opposite direction to it[5]. Therefore, if c be a constant then force due to the air resistance is given by using Equation (1) as:

$\mathbf{F}_{air} = -c\mathbf{v}$ (in vector form) (4)

Then, the equations of motions are given as:

$m\ dv_x/dt = -c\ v_x$ …(along horizontal) (5)

$m\ dv_z/dt = -cv_z - mg$ ….(along vertical) (6)

where z increases upward.

Suppose, the terminal velocity of the projectile in the air drag is v_t. Then, we can obtain the coefficient of air drag c by[6]:

$cv_t = mg$

Or, $c = mg/v_t$

Then, Equations (5) and (6) are simplified as:

$m\ dv_x/dt = -(mg/v_t)v_x$ (7)

and

$m\ dv_x/dt = -mg - (mg/v_t)v_z$ (8)

Integration of the equations of motion gives:

$x(t) = (v_0 v_t/g)\cos(\theta)\{1 - \exp(-gt/v_t)\}$ (9)

and

$z(t) = (v_t/g)\{v_0 \sin(\theta) + v_t\}\{1 - \exp(-gt/v_t)\} - v_t t$ (10)

Objective: i. To trace out the path of a projectile with and without air drag in earth surface using Mathematica tool.

Methodology

In this study, the simulation of a projectile path is done with the help of Wolfram demonstrations projects in Mathematica[10]. Mathematica can be recognized as a symbolic mathematical computational program, which is used in many scientific, engineering, mathematical and computing fields[11].

The Mathematica function used here to demonstrate is the 'Manipulate' which is of parametric plots of x and z with appropriate language[12]. Here, the parametric plots of x and z in Equation (2) and (3) give the curves without air resistance while that of Equation (9) and (10) give the curves with air resistance. We can vary the angle of projection(θ), initial velocity(v_0), the time allowed to spend for the projectile (t) and terminal velocity(v_t). We can then obtain the plots for the same projectile with and without air drag according to the input of the above variables. Here, we make all the variables constant except velocity (v_0) and obtain the plots.

Result and discussion

We fix angle (θ) equal 1.00895 radians, terminal velocity v_t equals 117.5 m/s, time (t) equals 13.24sec. Now, the plots for the initial velocity of 54.4m/s is shown in Figure-4 and the plots for 59.1m/s, is shown in Figure-5.

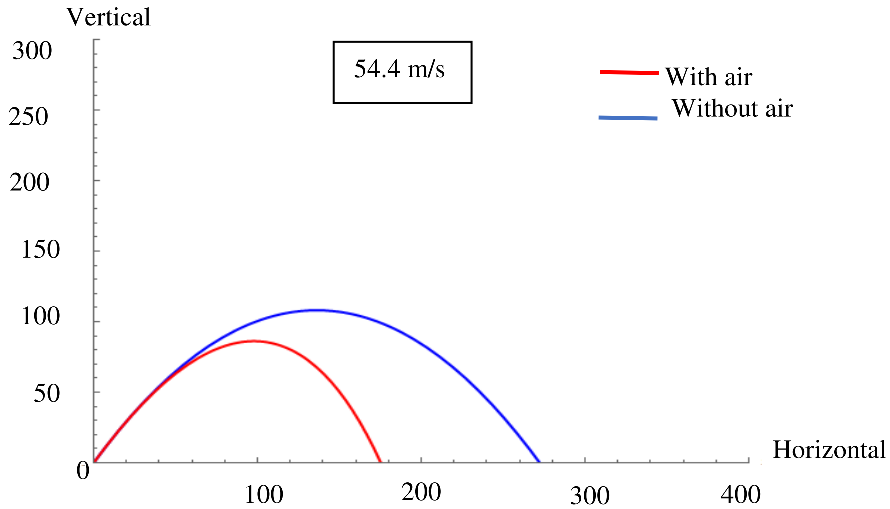

Figure-4: Path of the projectile at speed 54.4 m/s[10].

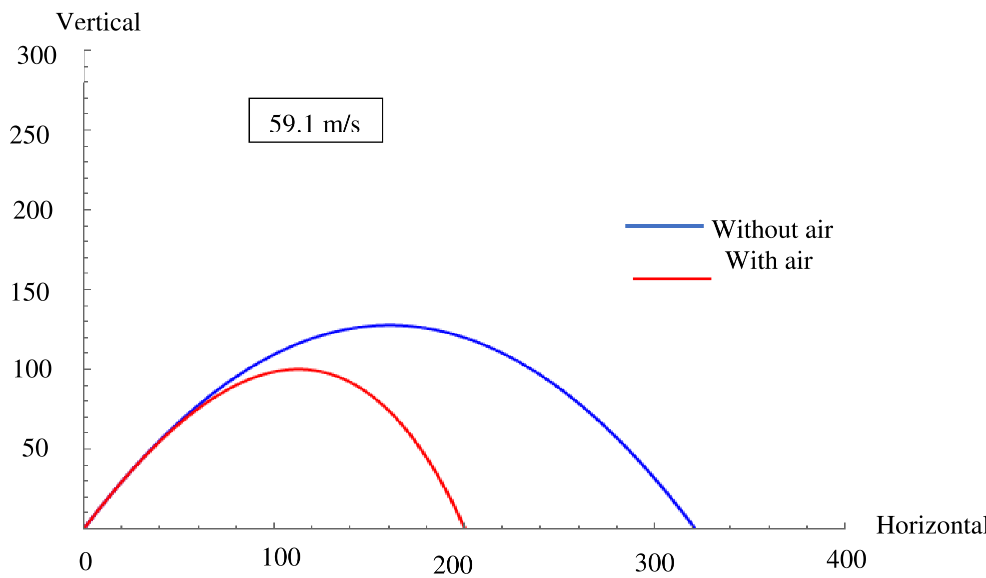

Figure-5: Path of the projectile at speed 59.1 m/s[10].

For the initial velocity 61.2m/s, we get the plots in Figure-6. Similarly, for the initial velocity of 65.4 m/s, we have the plots in Figure-7. Here, the plots show the greater ranges for the larger velocity both in air and without air.

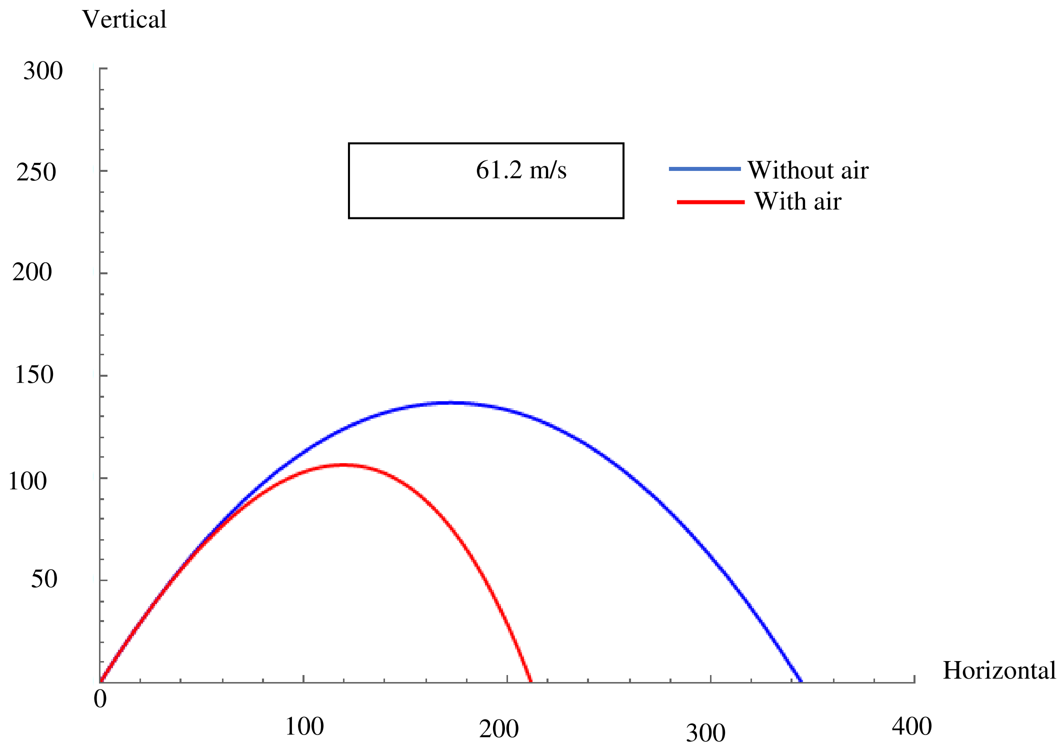

Figure-6: Path of the projectile at speed 61.2 m/s[10].

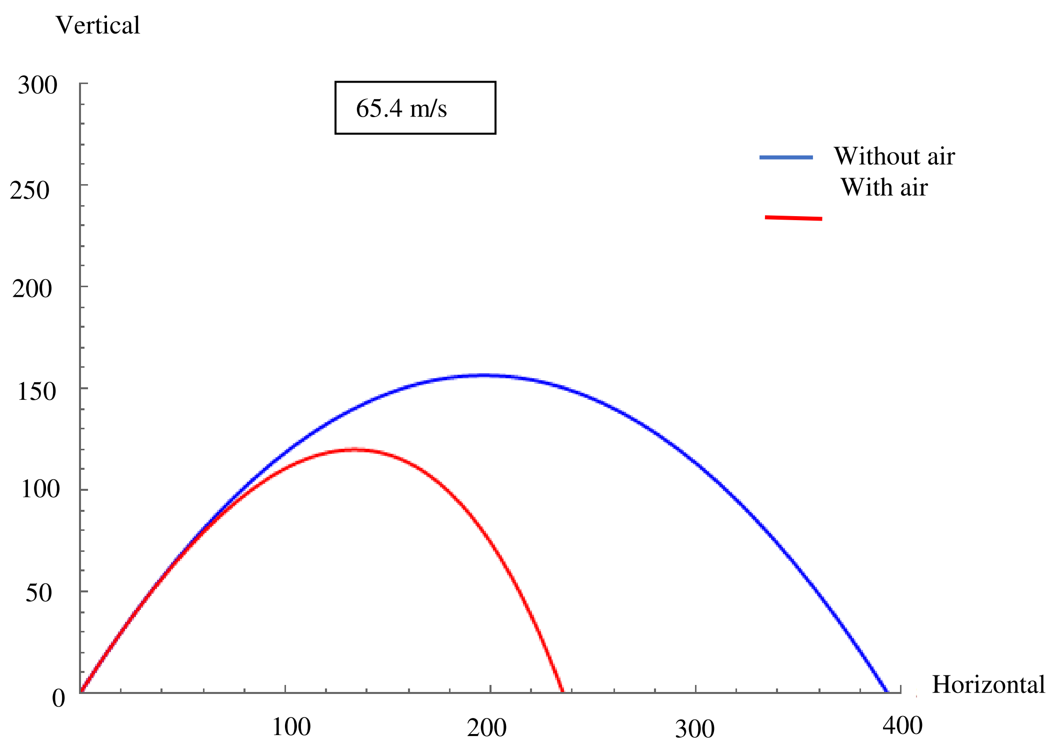

Figure-7: Path of the projectile at speed 65.4 m/s[10].

These figures are drawn using Mathematica tool found on Wolfram demonstrations project[10]. The red color of each plot represents the trace of the projectile with air drag while blue color represents that without air drag. Thus, the program in Mathematica can be used effectively to trace the path of the missile. From these plots, it is obvious that as we increase the initial velocity of the projectile, the horizontal range also increases as expected theoretically. Also, we can see that the range of a projectile is large without air than with air, as expected.

Conclusion

The projectile motion is largely affected by the air resistance on earth. Air resistance is the opposing force for the projectile. When the projectile is launched at an angle, it acquires a greater range without air than with air. The concept of the projectile in the air is largely used in football, javelin throw, shot put, shooting and missile launching. From this study, we can approximate the range, height, etc. of any projectile projected in the earth.

Recommendation for further work: We have previously mentioned that the air drag coefficient is not a constant quantity. Therefore, it is recommended for further study when this constant vary.

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