Research Journal of Physical Sciences ISSN 2320–4796
Vol. **6(2),** 9-20, February **(2018)** *Res. J. Physical Sci.*

Computational study of nonlinear modulation of wave propagation in model media

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Available online at: www.isca.in, www.isca.me
Received 30th December 2017, revised 27th January 2018, accepted 2nd February 2018

Abstract

In this paper, the computational study of nonlinear modulation of wave propagation in model media was carried out. The models studied are the Free Space Model (FSM), the Modified Rojas Model (MRM) and the Square Power Model (SPM). The change in the dielectric constant due to electromagnetic (EM) wave field that propagates through a medium is a typical non-linearity. The basic equations that govern the propagation of electromagnetic waves in nonlinear media were derived using Maxwell's equations. We obtained the numerical solution of the equations for different models of wave-media properties using fourth order Runge-Kutta scheme implemented in MATLAB software. The spatial EM wave profile graphic displays were supplemented by the symmetric spatial Fast Fourier Transform (FFT) analysis. The MRM model is essentially an EM wave attenuator. Complicated as the wave profile may look, the FFT showed attenuated wave of one wave number amidst background noise. However, at the fundamental frequency $f_0 = 47.7 \times 10^6 Hz$, *the SPM is capable of exhibiting solitary propagation without attenuation or steeping. The MRM and SPM support a variety of characteristics. There are attenuation, lossless or solitary propagation, amplitude amplifications or wave steeping and multiplicity of modes for all frequencies* ($f_0, 10f_0$ *and* $25f_0$) *examined. The EM wave propagation characteristics of the SPM showed that materials which could be fabricated according to this model would be very useful as EM wave guides as they could support waves without losses as opposed to the present known commercial optical fibers. On the other hand, applications which require EM wave shielding need material fabrication according to MRM.*

Keywords: Nonlinear modulation, wave propagation, spatial electromagnetic wave and dielectric function.

Introduction

The current advances in nanotechnology and exotic materials suggest that materials can in the nearest future be made in accordance to predetermined properties (models), beyond those of physical, chemical, and thermal ones. Optical fibers are good examples already, where material properties have been manipulated so that it can guide signals with much purity. In the light of this anticipation, models of field dependent materials can be assumed and the nature of EM wave's propagation in them can be studied.

Nonlinear optics constitutes one of the most significant aspects of technology and modern science[1]. Telecommunication is already transformed with the help of nonlinear optics and similar impact is expected very soon on technology that involves computer science. Nonlinear methods cover a wide region of different applications now, such as harmonic generations and frequency sum[2], frequency up converted lasing[3], optical power limiting[4], optical data storage and photodynamic therapy[5].

Components of a light wave comprises of magnetic and electric fields. These fields oscillate with high frequencies by sinusoidal wave form. The atoms, constituting the material, are seen as charge distributions pushed away from their equilibrium state when exposed to the electromagnetic field. Light interacts with material linearly when a field is said to be weak. For a situation of light with high intensity, the electromagnetic field modifies strongly the medium of the optical properties, which of course affects the propagation of the radiation. Frankly speaking, nonlinear matter is a medium whose optical response depends on the intensity of the optical field that propagates through it. The change of the dielectric constant due to Kerr effect is one of the typical nonlinearity. The Kerr nonlinearity enriches optics in general, and, in particular, the light propagation through nonlinear Photonic Crystals[6]. Photonic crystals (PCs) are structures of dielectric in periodic wave form that control light propagation. The degeneracies of the free-photon states at Bragg's planes are being removed by the periodicity of the dielectric constant and it produces a range of forbidden photons energies[7]. Nonlinear PCs and nonlinear PC-based fibers were successfully applied in the fields of harmonic generation and frequency mixing[8], several wavelengths with efficient phase matching[9]; in optical diodes manufacturing[10], optical switchers and limiters[11], to mention just a few examples. Today, a lot of nonlinear effects are being discovered and studied in different materials (gases, liquids and solids). Other examples of them are, like, nonlinear photo-absorption, harmonic generation, self-focusing, nonlinear photoionization, photo-dissociation, phase conjugation, etc. The area of applications of these nonlinear phenomena is vast. They are applied in medicine, biology, telecommunications, military and industries.

The study of nonlinear modulation of wave propagation in model media has been of increasing importance in recent years in plasma physics as well as in optical fibers. The nonlinear propagation of strong interacting electromagnetic fields which is explored by the standard theoretical way is based on the nonlinear solution of the equation for a specific nonlinearity (quadratic, cubic, etc.). But in many experiments the laser field's intensity is very high that the conventional expansion of the nonlinear polarization in a Taylor series over the electric field strength stops to be a good approximation, and the corresponding language of susceptibilities is no longer valid[12]. It is also a common view that for future use more sophisticated understanding of the basic mechanisms underlying non-linear phenomena of this kind will be required. This motivates us to explore nonlinear modulation of wave propagation in model media where details of material properties can been manipulated so that it can guide optical signals with much purity.

It is therefore very important to study computationally how media property will modulate electromagnetic wave propagation in them. If a particular model gives an interesting and useful wave modulation, it will open up experimental challenges on how to realize the materials with such model properties. The problem of how media properties alter or modulate the electromagnetic wave propagation in them remains an open one. The exact solution of the electromagnetic wave equation for model media shall be investigated by numerical techniques in this study.

For an understanding of the nonlinear modulation of wave propagation in model media, it is necessary to consider the theory of electromagnetic wave propagation normal to the plane of propagation with respect to a material medium[13].

Theoretical Consideration

The propagation of electromagnetic waves is determined by Maxwell's equations[14]. That is:

$$\nabla \times \vec{E} = -\frac{\partial B}{\partial t} \quad (1)$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad (2)$$

$$\nabla . \vec{D} = \rho_f \quad (3)$$

$$\nabla . \vec{B} = 0 \quad (4)$$

Where: $\vec{E}$ is the electric field vector and $\vec{H}$ is the magnetic field vector. $\vec{D}$ is the electric flux density and $\vec{B}$ is the magnetic flux density. The current density vector is $\vec{J}$. The flux densities $\vec{D}$ and $\vec{B}$ appear in response to the electric and magnetic fields $\vec{E}$ and $\vec{H}$ propagating inside the medium and are related to them through the constitutive relations given[15] by

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \quad (5)$$

$$\vec{B} = \mu_0 \vec{H} + \vec{M} \quad (6)$$

Where: ε_0 is the vacuum permittivity, μ_0 is the vacuum permeability, $\vec{P}$ is the induced electric polarization and $\vec{M}$ is the induced magnetic polarization.

Assumptions of the model: The basic equations that govern propagation of electromagnetic waves can be obtained from Maxwell's equations which describe the time and space evolution of magnetic and electric fields[16]. Here, time discretization is completely irrelevant since time must disappeared in the final equation as treated in accordance with Helmholtz decomposition. A number of assumptions are necessary to realize the computation that follow and to arrive at a good results without time: i. We shall assume a rectangular symmetry so that Cartesian coordinates x,y,z can be used. ii. The direction of propagation of the EM waves is the x direction. iii. The electric and magnetic vectors of the EM waves are in the y and z directions respectively, and that they vary only in the x direction, i.e. $E = E_y(x)j, H = H_z(x)k$, where j and k are unit vectors in y and z directions respectively. iv. The media are perfect dielectrics and non-magnetic. v. The electric and magnetic fields are harmonic in time. vi. The dielectric properties of the media respond to the spatial component of the electric field only and that it is nonlinear only in the x direction.

Derivation of the working equations: Recall Faraday equation (1)

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

The Faraday equation in rectangular coordinates is given by

$$\nabla \times E = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) i + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) j + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) k$$

$$= -\left(\frac{\partial B_x}{\partial t} i + \frac{\partial B_y}{\partial t} j + \frac{\partial B_z}{\partial t} k\right)$$

This means that

$$\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) i + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) j + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) k = -\left(\frac{\partial B_x}{\partial t} i + \frac{\partial B_y}{\partial t} j + \frac{\partial B_z}{\partial t} k\right) \quad (7)$$

By assumption (c) the surviving terms are:

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \quad (8)$$

Similarly, the Ampere equation (2)

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

Can be expressed in rectangular coordinates as:

$$\nabla \times H = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) i + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right) j + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) k$$

$$= \left(J_x + \frac{\partial D_x}{\partial t}\right) i + \left(J_y + \frac{\partial D_y}{\partial t}\right) j + \left(J_z + \frac{\partial D_z}{\partial t}\right) k$$

Or

$$\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) i + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right) j + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) k = \left(J_x + \frac{\partial D_x}{\partial t}\right) i + \left(J_y + \frac{\partial D_y}{\partial t}\right) j + \left(J_z + \frac{\partial D_z}{\partial t}\right) k \quad (9)$$

By assumption (c) and (d; J=0), the surviving terms are:

$$\frac{\partial H_z}{\partial x} = -\frac{\partial D_y}{\partial t} \quad (10)$$

For a material medium the constitutive relations enables us to express (9) and (10) as:

$$\frac{\partial E_y}{\partial x} = -\frac{\partial \mu H_z}{\partial t} \quad (11)$$

$$\frac{\partial H_z}{\partial x} = -\frac{\partial \varepsilon E_y}{\partial t} \quad (12)$$

By assumption (d), the medium being non-magnetic, we have (11) as

$$\frac{\partial E_y}{\partial x} = -\mu_0 \frac{\partial H_z}{\partial t} \quad (13)$$

And by assumption (f), we can write (12) as

$$\frac{\partial H_z}{\partial x} = -\varepsilon \frac{\partial E_y}{\partial t} \quad (14)$$

Elimination of H_z between (13) and (14) leads to wave-like equation:

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon \frac{\partial^2 E_y}{\partial t^2} \quad (15)$$

Assumption (f) is restricted to x-coordinate only, so$\varepsilon = \varepsilon(E(x))$. In this case the time harmonic assumption (e) for electric fields:

$$E(x,t) = W(x) e^{-i\omega t} \quad (16)$$

Where: $W(x)$ is y-component of electric field that depends on x-coordinate, gives the equation

$$\frac{d^2 W}{dx^2} + \mu_0 \omega^2 \varepsilon(w) W = 0 \quad (17)$$

Mathematically, the solution of (15) consists of finding the solution of (17) and substituting in (16), but spatial modulation of waves is what is of practical importance, so we do not need to go back to $E(x,t)$; $W(x)$ is just what we need.

Models of dielectric function: The behavior of material towards electric field using free space model (FSM), Square Power Model (SPM) and Modified Rojas Model (MRM) is modelled by the following equations:

FSM $\varepsilon(w(x)) = \varepsilon_0$ (18)

MRM $\varepsilon(W(x)) = \varepsilon_0[W + \beta_2 Sin^2(\beta_1 W)]$ (19)

SPM $\varepsilon(w(x)) = \varepsilon_0(1 + \delta W^2)$ (20)

Where: $\beta_1,\ \beta_2\ and\ \delta$ are parameters with dimensions that play the role of making the units consistent. However since our field is EM, $W = W(x)$, hence $\varepsilon(W)$ is functional.

Numerical solution of the models: The solutions of the models were obtained using Runge-Kutta 4th order method. The second order ordinary differential equation was reduced into a system of two first order ordinary differential equation. The ODE45 module in MATLAB which is built based on Runge-Kutta 4th and 5th order method was used to implement Runge-Kutta Algorithm in MATLAB. The equations were converted as follows:

A substitution of FSM into equation (17) gives

$$\frac{d^2 W}{dx^2} + (\mu_0 \omega^2 \varepsilon_0) W = 0 \quad (21)$$

But, $\omega^2 = 4\pi^2 f_0^2$

Equation (21) becomes

$$\frac{d^2 W}{dx^2} + 4\pi^2 f_0^2 \varepsilon_0 \mu_0 W = 0 \quad (22)$$

$$\frac{d^2 W}{dx^2} + k_0 W = 0 \quad (23)$$

Where: $k_0 = 4\pi^2 f_0^2 \varepsilon_0 \mu_0$ (24)

We let $W = P_1(x)$ and $W' = \frac{dW}{dx} = P_2(x)$ then we obtain the system

$$\frac{dW}{dx} = P_2 \Rightarrow \frac{dP_1}{dx} = P_2.$$

This give rise to

$$\frac{d}{dx}\left[\frac{dW}{dx}\right] + k_0 W = 0$$

$$\frac{dP_2}{dx} + k_0 P_1 = 0$$

$$\frac{dP_2}{dx} = -k_0 P_1$$

Hence our new system of equations is given by

$$\frac{dP_1}{dx} = P_2 \quad (25)$$

$$\frac{dP_2}{dx} = -k_0 P_1 \quad (26)$$

A substitution of MRM into equation (17) gives

$$\frac{d^2W}{dx^2} + \mu_0\omega^2\varepsilon_0[W + \beta_2 Sin^2(\beta_1 W)]W = 0 \quad (27)$$

or, $\frac{d^2W}{dx^2} + k_0[W + \beta_2 Sin^2(\beta_1 W)]W = 0$ (28)

We let $W = y_1(x)$ and $W' = \frac{dW}{dx} = y_2(x)$ then we obtain the system

$$\frac{dW}{dx} = y_2 \Longrightarrow \frac{dy_1}{dx} = y_2.$$

and,

$$\frac{d}{dx}\left[\frac{dW}{dx}\right] + k_0[W + \beta_2 Sin^2(\beta_1 W)]W = 0$$

$$\frac{dy_2}{dx} + k_0[y_1 + \beta_2 Sin^2(\beta_1 y_1)]y_1 = 0$$

$$\frac{dy_2}{dx} = -k_0[y_1 + \beta_2 Sin^2(\beta_1 y_1)]y_1 \quad (29)$$

Hence our new system of equation is given by

$$\frac{dy_1}{dx} = y_2 \quad (30)$$

$$\frac{dy_2}{dx} = -k_0[y_1 + \beta_2 Sin^2(\beta_1 y_1)]y_1 \quad (31)$$

Also a substitution of SPM into equation (17) gives

$$\frac{d^2W}{dx^2} + \mu_0\omega^2\varepsilon_0(1 + \delta W^2)W = 0 \quad (32)$$

We let $W = v_1$ and $W' = \frac{dW}{dx} = v_2$ then we obtain the system

$$\frac{dW}{dx} = v_2 \Longrightarrow \frac{dy_1}{dx} = v_2.$$

which means

$$\frac{d}{dx}\left[\frac{dW}{dx}\right] + k_0(1 + \delta W^2)W = 0 \quad (33)$$

$$\frac{dv_2}{dx} + k_0(1 + \delta v_1^2)v_1 = 0$$

or, $\frac{dv_2}{dx} = -k_0(1 + \delta v_1^2)v_1$

Hence our new system of equations are given by

$$\frac{dv_1}{dx} = v_2 \quad (34)$$

$$\frac{dv_2}{dx} = -k_0(1 + \delta v_1^2)v_1 \quad (35)$$

Table-1: Parameters Used in the Simulation.

Parameter	Value
Fundamental EM wave, f_0	$47.7 \times 10^6 Hz$
Permittivity of free space, ε_0	$8.854 \times 10^{-12} Fm^{-1}$
Permeability of free space, μ_0	$12.566 \times 10^{-7} NA^{-1}$
Wave frequency, ω	$2\pi f_0$
Beta 1, β_1	100
Beta 2, β_2	300
Delta, δ	0.05
Angle in radians	$(\pi/_{180})$
$Pi(\pi)$	3.142

Results and discussion

The solution of equations (25) and (26) are found in the interval $0 \leq x \leq 100$, with step size of 0.001(FSM). The solutions of equations (30), (31), (34) and (35) are considered in the interval $0 \leq x \leq 200$ (MRM and SPM) with step size of 0.001. The EM wave propagation is varied by $f = f_0, 10f_0, 25f_0$ *and* $50f_0$ for our entire models.

Propagation of EM waves from free- space medium to other media models is considered with a boundary region from $(0 \leq x \leq 100)$ to $(100 \leq x \leq 300)$, with the step size of 0.001.

In general, the electric field of the EM wave propagation is

$$W(x) = P_1(x) = y_1(x) = v_1(x) \quad (36)$$

The output of the ODE45 solver (Electric field W(x)) is introduced into FFT-codes as we transform from wave position domain to wave number domain.

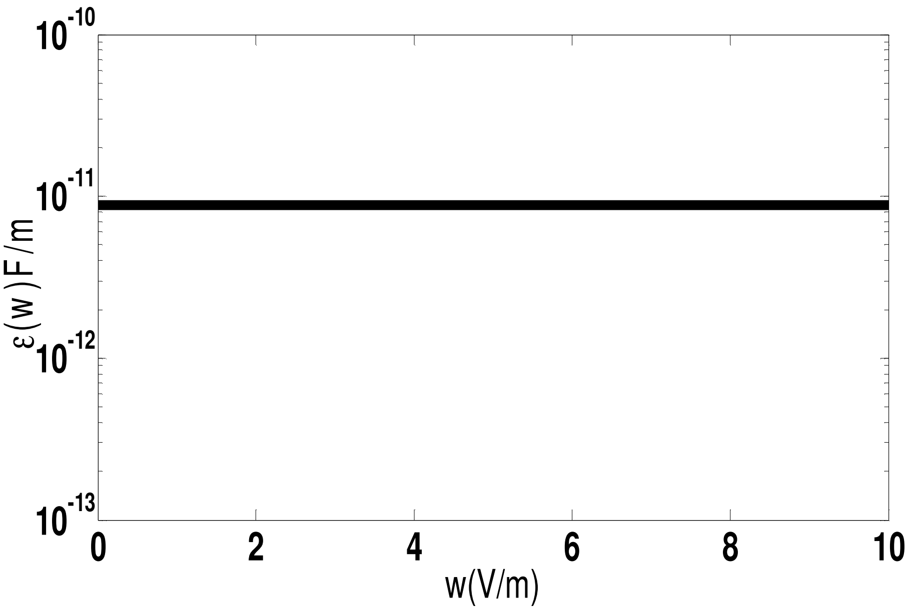

Figure-1: Sketch of free space model.

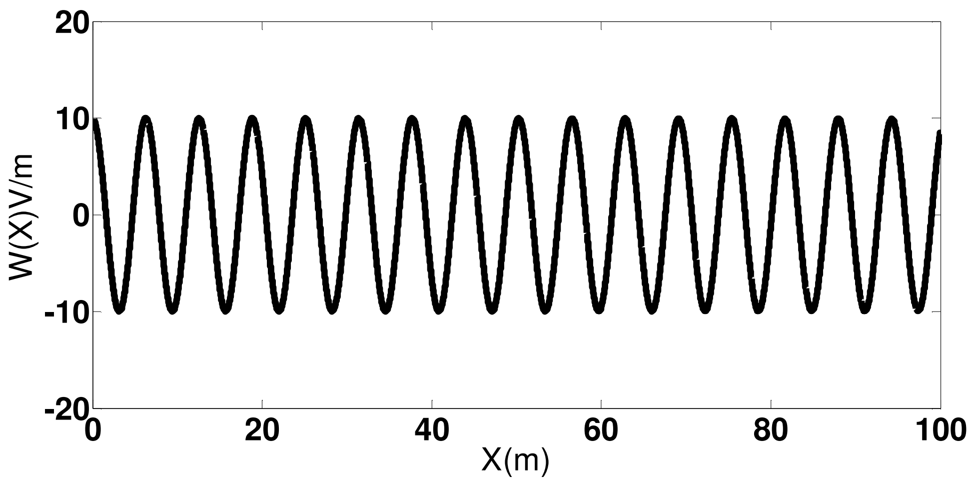

Figure 2(a): Electric field of EM waves W(x) along x-direction in free-space model for f= f_0, h=0.001 L=100.

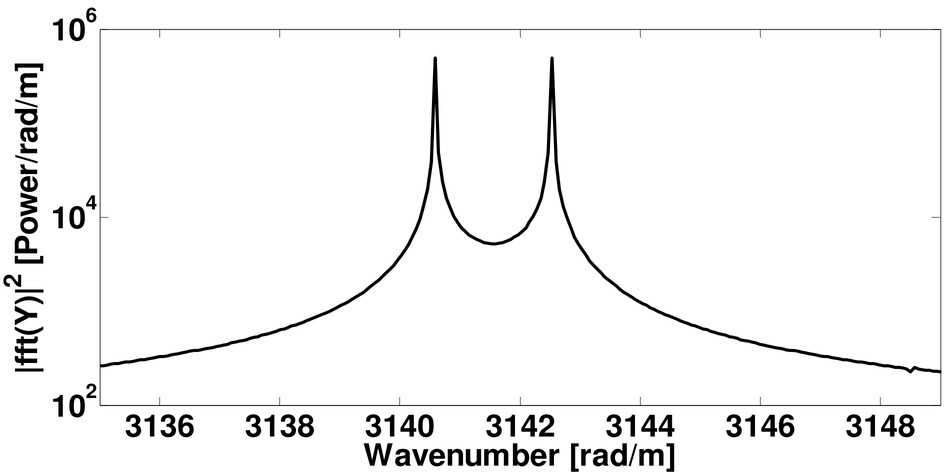

Figure-2(b): Transformation from wave position (using FFT) to wave number for free space model for $f = f_0$.

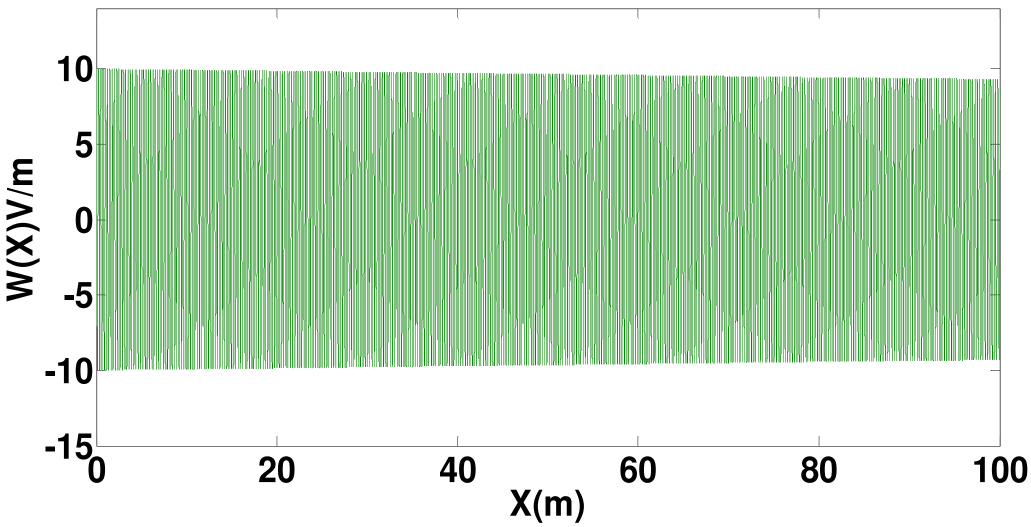

Figure-3(a): Electric field of EM waves $W(x)$ along x-direction for free-space model for f= $25f_0$, h=0.001, L=100.

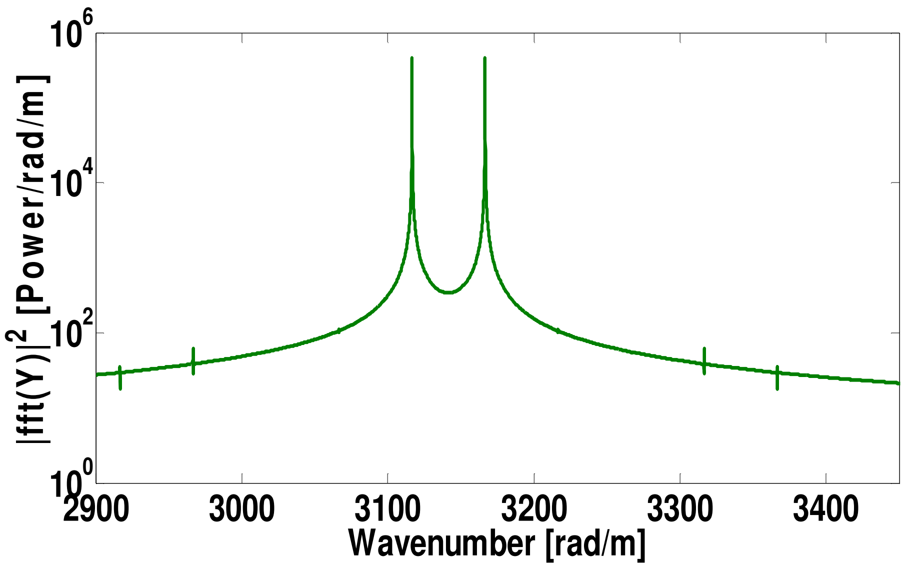

Figure-3(b): Transformation from wave position (using FFT) to wave number for free space model for $f = 25f_0$.

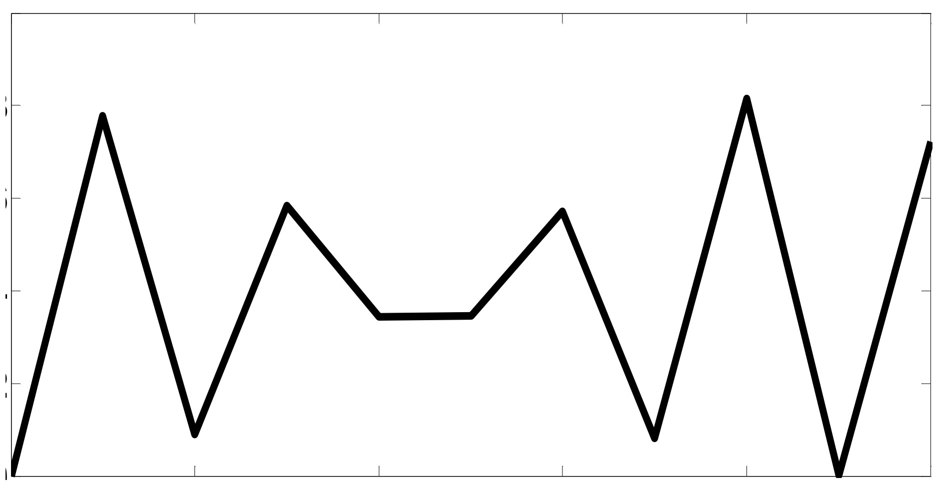

Figure-4: Sketch of Modified Rojas Model.$\beta_1 \& \beta_2 \gg 0$.

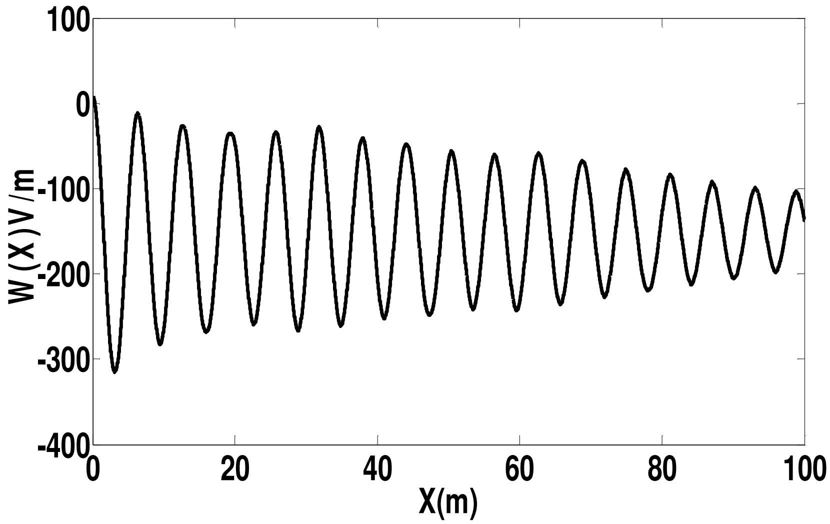

Figure-5(a): Electric Field of EM Waves W(x) Along x-Direction for Modified Rojas Model for f= f_0, h=0.001, L=200.

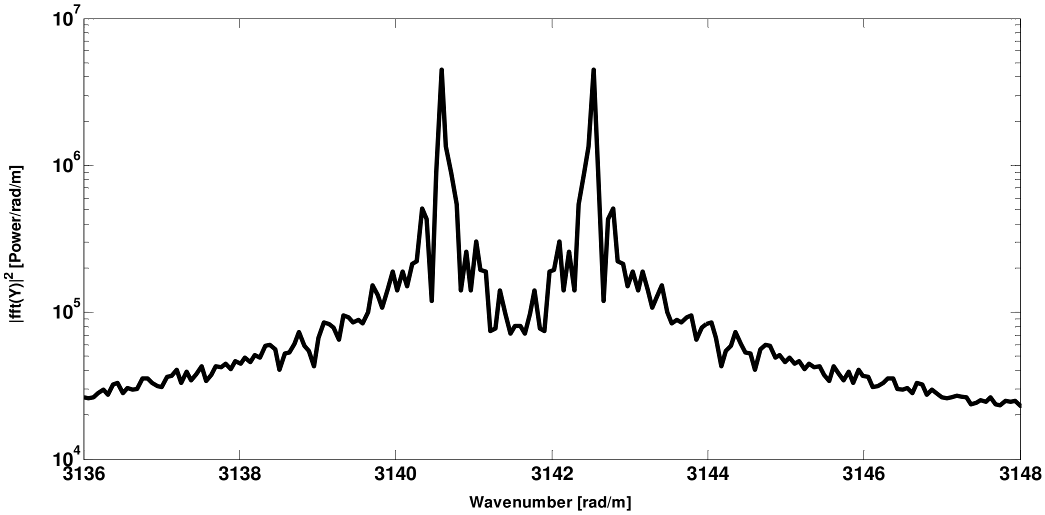

Figure-5(b): Transformation from Wave Position (using FFT) to Wave Number for Modified Rojas Model for $f = f_0$.

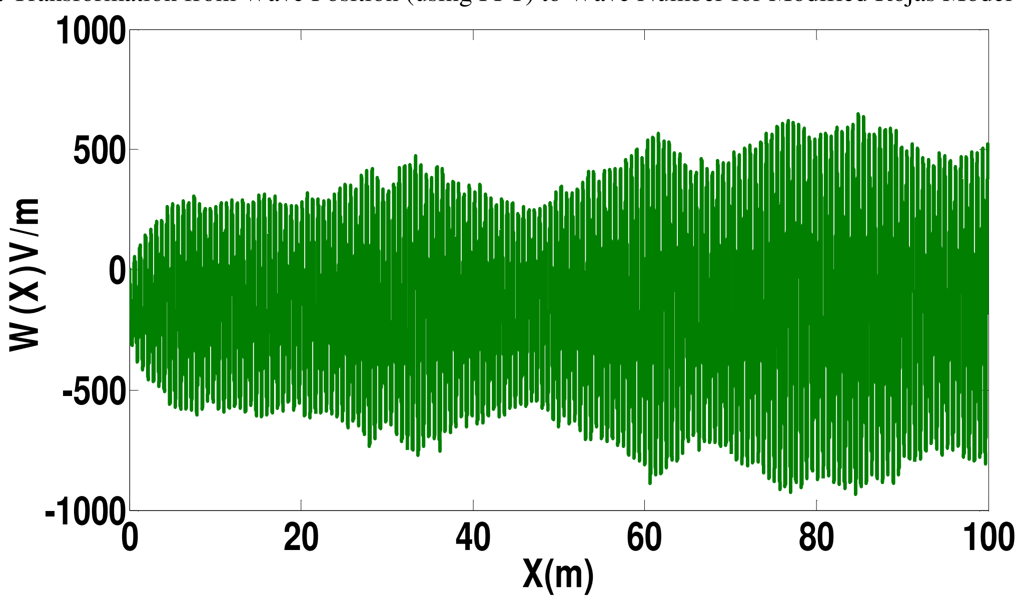

Figure 6(a): Electric Field of EM Waves $W(x)$ Along x-Direction for Modified Rojas Model for $f= 10f_0$, h=0.001, L=200.

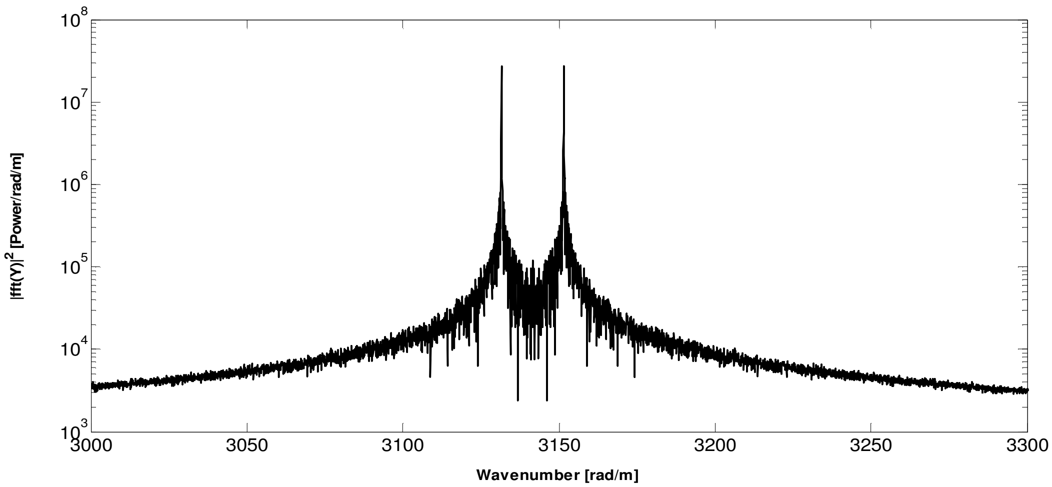

Figure-6(b): Transformation from Wave Position (using FFT) to Wave Number for Modified Rojas Model for $f = 10f_0$.

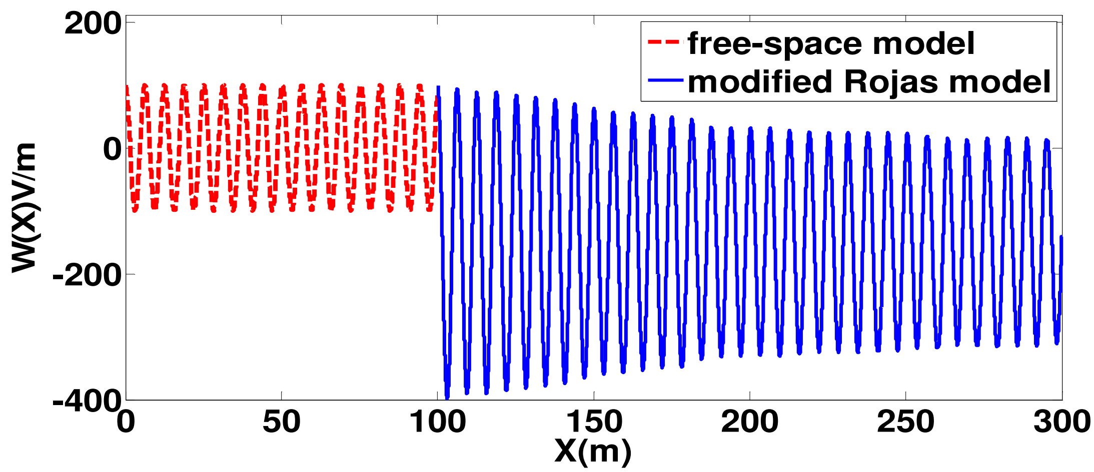

Figure-7: Propagating from Free-Space Medium to Modified Rojas Medium at Same Frequency$f = f_0$. $(from\ 0 \leq x \leq 100\ to\ 100 \leq x\ \ 300.)$ with step of h=0.001.

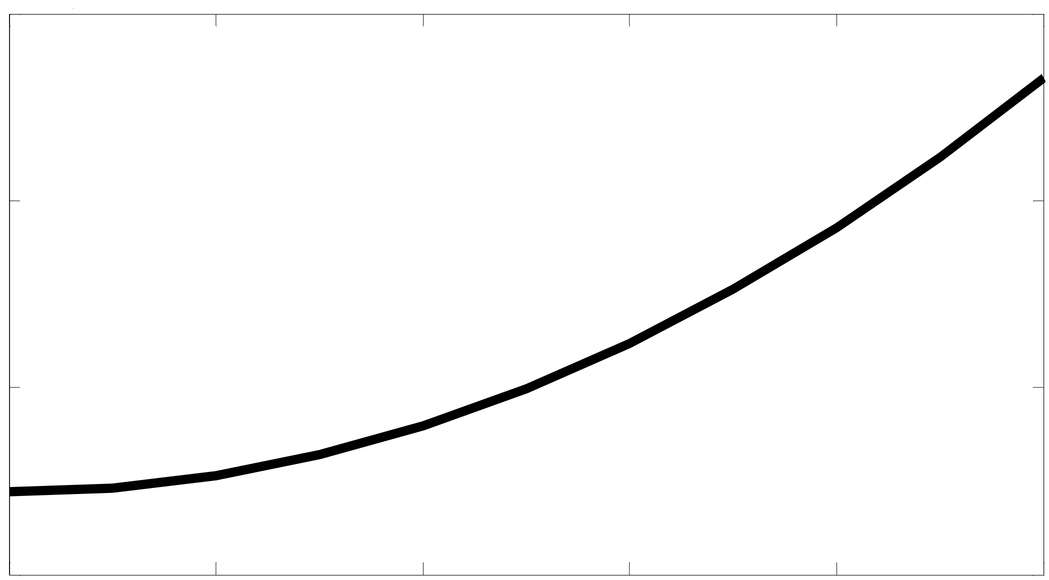

Figure-8: Sketch of Square Power Model.

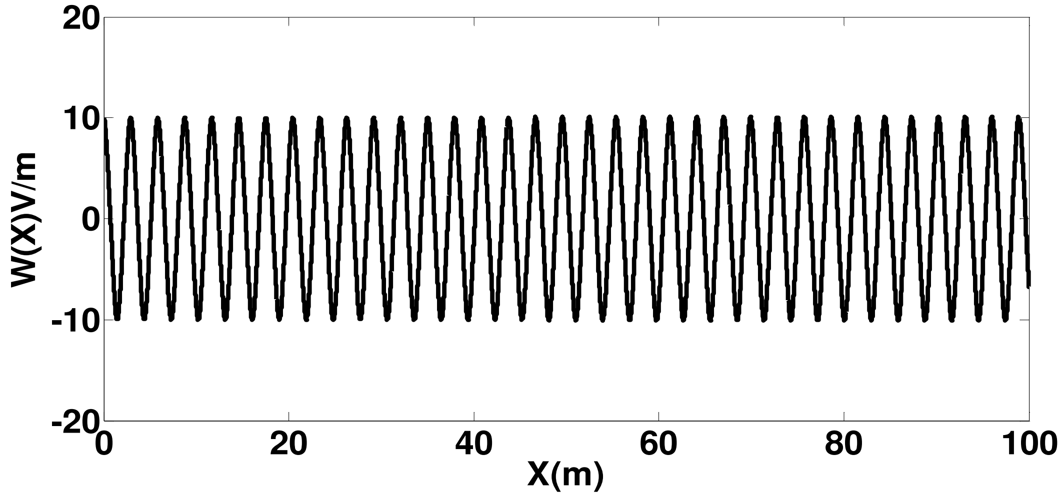

Figure-9(a): Electric field of EM Waves $W(x)$ Along x-Direction for Square Power Model for f = f_0, h=0.001, L=200.

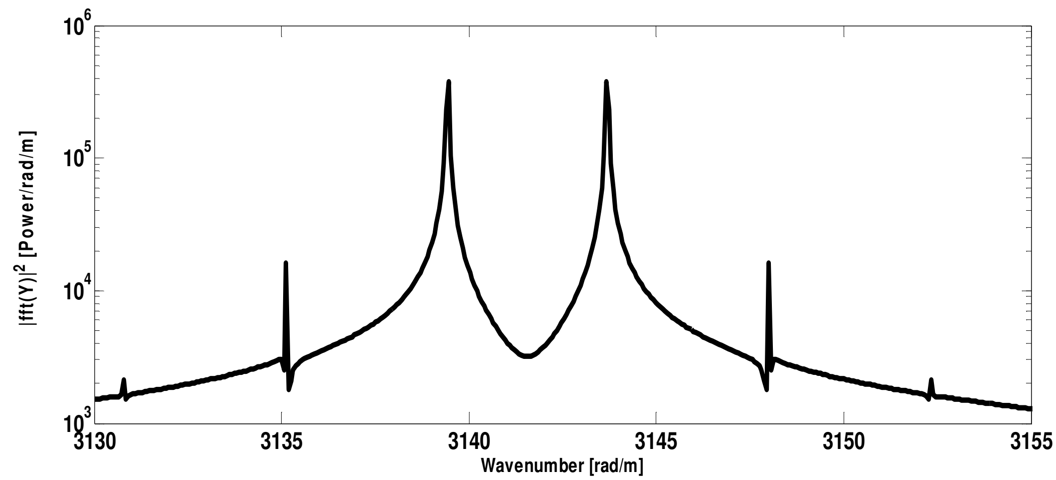

Figure-9(b): Transformation from Wave Position (using FFT) to Wave Number for Square Power Model for $f = f_0$.

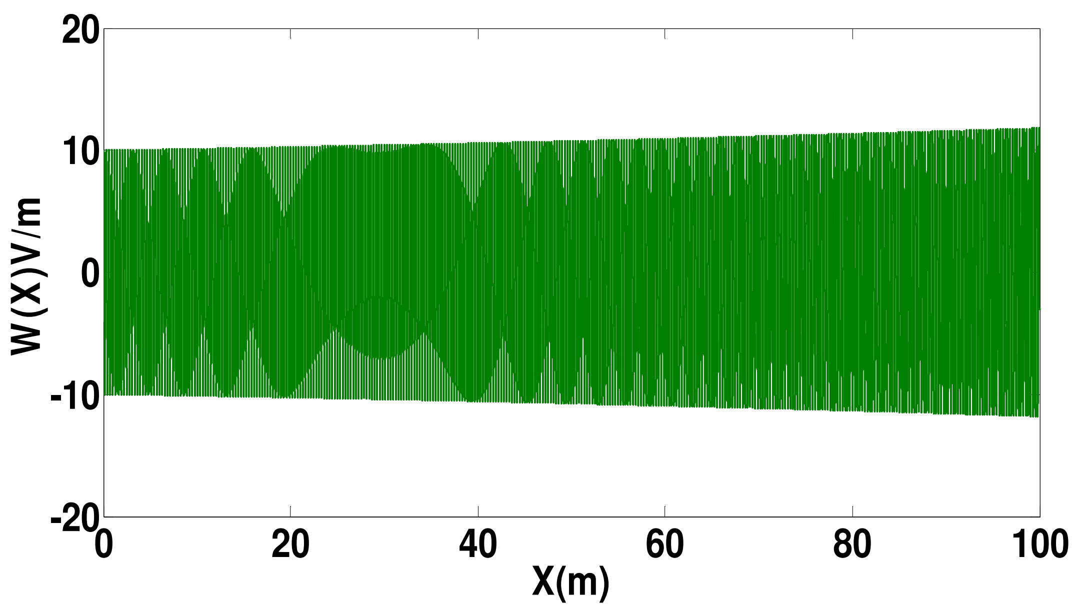

Figure-10(a): Electric Field of EM Waves $W(x)$Along x-Direction for Square Power Model for f = $10f_0$, h=0.001, L=200.

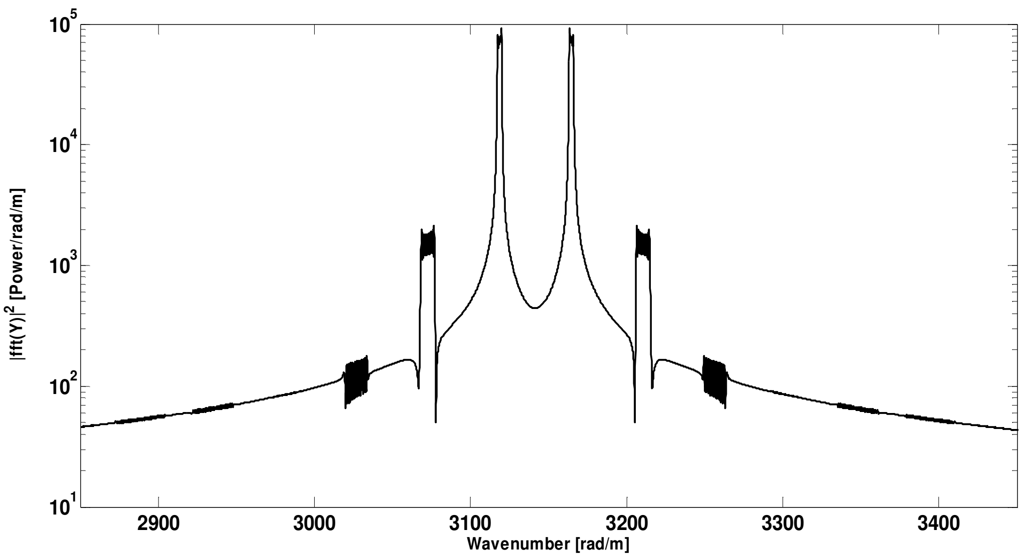

Figure-10(b): Transformation from Wave Position (using FFT) to Wave Number for Square Power Model for $f = 10f_0$.

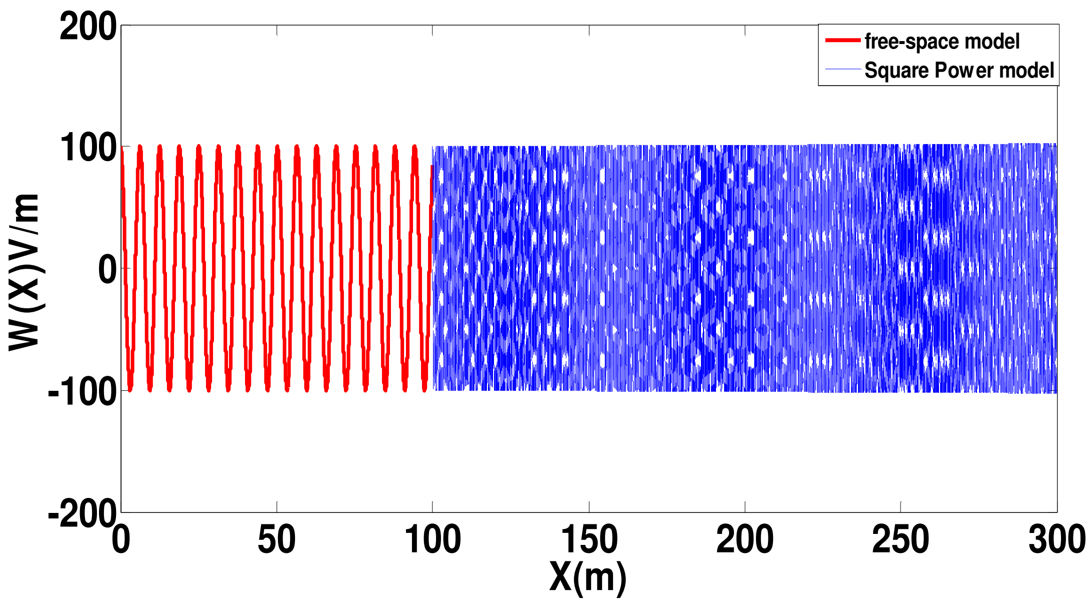

Figure 11: Propagation from Free-Space Medium to Square Power Medium at same Frequency, $f = f_0$. $(from\ 0 \leq x \leq 100\ to\ 100 \leq x \leq 300.)$, h=0.001.

Figure-1 shows the behavior of material modelled by equation (18) towards electric field. The sketch tells us that, no matter the value of electric field, the permittivity of the medium remains unchanged. This means the permittivity is independent of electric field. The amplitude variation of the EM wave in such a medium is expected to be periodic in space. This corresponds to the well-known straight line propagation in the geometric optic approximation. The symmetric spatial Fourier transform of such a wave ought to show two peaks which are just mirror image of each other. Our simulation results reproduce these well-known facts. Figure-2(a) is a sinusoidal EM wave of one wavelength, λ. The periodic nature is of the same amplitude all through along the direction of propagation. In Figure-2(b), the two peaks represent two half-cycle that are projected as mirror image, when viewing from one side. The single peak represents a single mode. Figures-3(a) shows a sinusoidal wave that over crowds at the middle of the medium with the same amplitude as a result of periodic distribution at the value of $25f_0$. Figures-3(b) show single peak reflected as mirror images without obstruction along the line of propagation of waves. In a nutshell, the free space propagation is periodic characterized by a single wavelength and the periodicity maintained respective of the frequency of incident EM. The results serve as a bench-mark to interpret the results of other models that are wave amplitude dependent.

The behavior of a material which is modelled by equation (19) towards electric field (Figure-4) indicates that, at constant value of $\beta_1 \& \beta_2 \gg 0$, the dielectric property of the material changes irregularly with respect to electric field. However the behavior of EM wave propagation in this medium governed by equation (17) when an incident wave of frequency f_0 is propagated (Figure-5(a)) shows a periodic propagation that die down to the lowest on propagating towards the boundary region. The Fourier transform (Figure-5(b)) shows some irregularities from the boundary region and 8 distinct peaks starting from 3139Rad/m to 3141Rad/m are projected as mirror images. These peaks in turn imply that there are 8 spatial periodic waves distribution in the original sample when the value of incident wave is f_0. The amplitude of propagating wave increases along x-axis as it approaches 100m (Figure-6(a)) with constant value of $\beta_1 = 100$ and $\beta_2 = 300$, and also the wave position becomes compactible when the value of EM waves becomes $10f_0$. Many spikes in Figure-6(b) occur at $k_0, 3k_0$, $5k_0, 7k_0$,..., and these modes are said to be odd harmonics of fundamental wave number, k_0 governed by the expression $k_0 = \frac{2\pi}{\lambda}$. This in turn show that numerous periodic distribution of waves are been accommodated by this sample. Uniform sinusoidal periodic wave amplitude is observed in free-space medium (Figure-7) as it propagate into modified Rojas medium. The behavior in the modified Rojas medium shows that the amplitude of the propagating waves dies down toward the boundary region at 300m. This implies that MRM is an attenuator. Because of the high attenuation rate with this type of model material, applications which require EM wave shielding need material fabrication according to MRM.

The model equation (20) which is the result of the sketch in Figure-8 describes the behavior of the dielectric towards electric field as part of parabola by telling us that at constant δ, the permittivity of the medium changes with respect to electric field. The periodic nature of the EM waves in this medium by simulation result (Figure-9(a)) shows sinusoidal with the same amplitude along the direction of propagation when the value is $f_0\, at$ constant value of $\delta = 0.05$. The transformation from wave position (Figure-9(b)) to wave number shows three distinct spikes projected as mirror image when viewed from one side. The three spikes are in turn correspond to three spatial periodic wave distributions in the original spatial sample when the incident wave is f_0. In Figure 10(a), the number of different periodic wave distributions contained in the display multiplies along the medium and the geometry is not fully known. Three peaks projected as mirror image are observed (Figure-10(a)). The three peaks in turn is the three spatial wave distributions that is more of harmonic fundamental wave number k_0 at the incident wave of $10f_0$. The behavior of EM waves in the Free-Space medium is sinusoidal with equal wave amplitude and becomes compactiblewith the same amplitude in Square Power medium at the same frequency f_0 (Figure-11). This means the SPMmedium is lossless at that frequency or it supports soliton propagation at that frequency. This means this material model can support many wave number than that of free space model. In practical sense, the dielectric property of this model material is capable of exhibiting lossless as well as multiplicity of modes that are useful as electromagnetic wave guides with much purity.

Conclusion

The propagation of waves in nonlinear media governed by the nonlinear wave equation was derived from Maxwell's equations for an inhomogeneous, dielectric media with some basic assumptions. A typical non-linearity is the change in the dielectric constant due to electromagnetic (EM) wave field that propagates through a medium. An investigation of the three models studied using non-linear wave equation reveals that dielectric properties of the media respond to spatial component of EM wave's propagation in them and they are inhomogeneous only in x-direction (the direction of propagation). Apart from FSM, the MRM and SPM supports a variety of characteristics. There are lossless or solitary propagation, attenuation and multiplicity of modes for all frequencies examined. The EM wave propagation characteristics of the SPM, showed that materials which could be fabricated according to this model would be very useful as EM wave guides and EM wave shielding as they could support waves without losses as opposed to the present known commercial optical fibers. Also because of the high attenuation rate with this type of model material (MRM), applications which require EM wave shielding need material fabrication according to MRM.

Acknowledgement

The authors wish to thank Mr. Emmanuel Tikyaa for assisting valuably with the different types of numerical implementation in MATLAB.

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