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Practical Ranges of Electrons and Positrons in Intermediate and High Energy Region for Condensed Materials

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Abstract

The present paper presents a simple empirical formula for the practical ranges of electrons and positrons from atomic numbers 1 to 92 in different materials. The formula is existing in the expression of the multiplication of factor related to the continuous-slowing-down approximation (CSDA) range and to multiple scattering detours. The factor being articulated as a parameter of electron energy received by the target (incident) and atomic number of the medium (Z). For CSDA range in the practical-range formula, exact recorded values available or an fairly accurate methodical expression derived as a parameter of electron energy received by the target (incident) and atomic number of the medium, atomic weight and mean excitation energy of medium can be used. The utmost variation of the consequential formula with the other accessible data was 2%. The formula can also be applied to not too heavy compounds and mixtures by using an effectual atomic number and atomic weight. It is quite clear from the results drawn that this method gives better concurrence with the existing data.

Keywords: CSDA ranges, Intermediate energy range, Atomic number, Total energy.

Introduction

The electrons and positrons during their passage through matter interact in elastically as well as elastically with the atoms of the absorber. This fact is to be utilized for the calculations of the practical ranges of these particles in the target material. The path of the electron and positron gets distorted during slowing down due to occurrence of multiple scattering. This distortion in the path of these particles increases with decreasing energy. This is because of enhanced significance of multiple scattering at low energies. Until at some stage, the electron energy and positrons energy is decreased to the extent, then the motion of the motion of these particles becomes random. This random motion results in diffusion of these particles in the absorber. The energy, at which the diffusion sets in, depends upon the incident kinetic of these particles and also on the properties of the absorber material. However, in any case the randomly moving electrons and positrons are left with substantial fraction of their incident kinetic energy. Therefore even after the diffusion sets in, the electrons and positrons can travel on an average, some distance which is a substantial fraction of their practical range in the absorber. Thus, for calculation of practical ranges of these particles in an absorber, both parts of their journey, viz, before and after the diffusion sets in, must be taken into account.

The average path lengths of the charged particles in a boundless homogeneous medium are defined as the range of stimulating particles at a specified energy. The integration of the reciprocal of the stopping power on energy from a ultimate to an preliminary value is an approximation to the evaluated range and is called the continuous-slowing-down approximation (CSDA) range. In a particular direction the projections of the paths of the particles, viz the direction of the normal to the surface of an effectively semi-infinite medium, have more realistic experimental and application significance than the meandering paths. The projected paths are frequently represented by the extrapolated range and the practical range quantities .The extrapolated range may be defined as the point wherever the digression at the sharpest point on the almost straight sliding portion of the number-diffusion curvature coincides the width axis. In a different definition, the curvature of the integral charge cultivation as a parameter of deepness is used as a substitute for the diffusion curve[1,2]. Similarly the depth-dose curve as a replacement for of the diffusion curve is worn in to describe the practical range, and is mostly used for the energy determination of therapeutic electron beams[3,4]. In the present paper the practical range is measured. Thus measure of the penetration depth (practical range) is worn in scheming detectors for electrons and scheduling curative treatment and industrialized exposure to radiation by electron beams.

Theory, Results and Discussions

Tabata et al.[5] proposed a widespread semi empirical formula for the extrapolated range. To build it feasible to assess these curvatures for random absorber matter and incident electron energy[6,7], this formula can be included into investigative expressions for transmission curves and depth-dose curves. Tabata et al.[2] obtained a organized set of values of the extrapolated range for electrons of energies from 0.1 to 100 MeV in elemental media of atomic numbers from 1 to 92 from the charge-deposition distributions calculated by the ITS-3.0

Monte Carlo code system[8,9]. These values are in fine agreement with the investigational information published prior by Tabata et al.[10].

Methodical expressions for the CSDA range have been proposed by many authors[11-19]. With view to widespread formulas that envelop a huge amount of substances and a extensive area of energy, we have prepared a new methodical phrase for the CSDA range to integrate it in the practical-range formula as the error involved in the then available data is 5% which cannot satisfy our need and precision required for the calculation of practical ranges. By using an efficient atomic number (and an effective atomic weight when the methodical phrase for the CSDA range is applied) the present practical range method can also be in corporated to not so heavy compounds and mixtures. Tabata et al have considered that the extrapolated range r_{ex} can be expressed as the multiplication of a feature f_d and the CSDA range r_o; [20]

$$r_{ex} = f_d\, r_o, \tag{1}$$

Here f_d is a parameter of incident electron energy and the atomic number Z. Other parameter of the medium that characterize it does not hold any importance in the above expression. A gauge of various scattering detours of electrons is the quantity given by the reciprocal of $f_{d..}$ Recently some studies[4,24] defined the "detour factor as the proportion of the projected range (the average depth of penetration) to the CSDA range rather than its reciprocal, and its performance is analogous to that of f_d. We have used database values of r_0 in Equation-l and the second factor in the equation, f_d as a parameter of incident electron energy has been established empirically as:

$$f_d = 1 / [\, a_1 + a_2/(\, 1 + a_3/\, \tau_0^{a_4} + a_5\, \tau_0^{a_6}\,)] \tag{2}$$

Here the symbols a_i (i = 1, 2,. . . , 6) represent constants for a given medium, and τ_0 is the kinetic energy of the incident electron in units of the rest energy of the electron. The term with a_5 governs the behavior of f_d at higher energies, and the term with a_3, governs its behavior at lower energies. From the scrutiny of the outcome the equations for a_i as a factor of atomic number (Z) have been resoluted as follows:

$$a_1 = b_1 Z^{b_2} \tag{3}$$
$$a_2 = b_3 + b_4\, Z \tag{4}$$
$$a_3 = b_5 Z^{b_6 - b_7 \ln z} \tag{5}$$
$$a_4 = b_8 / Z^{b_9} \tag{6}$$
$$a_5 = b_{10} Z^{b_{11} - b_{12} \ln z} \tag{7}$$
$$a_6 = b_{13} Z^{b_{14}} \tag{8}$$

Equations (5) and (6) have been chosen to be analogous to Equations (7) and (8), and are rather uncertain. The technique of least squares, in which the sum of squares of the relative deviations of the formula from the facts has been reduced, is used to seek after the values of b_j. To reduce the maximum deviation, larger weights are provided to a few of the data in the final determination of b_j.

Here the symbols b_j (j = 1, 2,. . . , 14) represent constants autonomous of medium.

Table-1
Values of constants b_j (j= 1, 2,. ., 14) in the expression for f_d

j	b_j	J	b_j
1	0.3879	8	14.03
2	0.2178	9	0.7406
3	0.4541	10	$4.294X\ 10^{-3}$
4	0.03068	11	1.684
5	$3.326X\ 10^{-l6}$	12	0.2264
6	13.24	13	0.6127
7	1.316	14	0.1207

Recently, we have been calculated CSDA ranges for the materials by the following relation[21]

$$R^{\pm}_{CSDA}(E_0) = \left(-\frac{1}{\rho}\left(\frac{dE}{dS}\right)^{\pm}_{Tot} \right) dE + R^{\pm}(E_0)$$

$$-\frac{1}{\rho}\left(\frac{dE}{dS}\right)^{\pm}_{Tot} = (MZ + C)\left(\frac{\gamma^{A^{\pm}Z + B^{\pm}}}{\gamma^{k^{\pm}-1}}\right)$$

The equation for stopping power in the energy region 20 KeV to 50000 KeV for the absorbers of atomic number 1 to 92 is

$$-\frac{1}{\rho}\left(\frac{dE}{dS}\right)^{\pm}_{Tot} = (MZ + C)\left(\frac{\gamma^{A^{\pm}Z + B^{\pm}}}{\gamma^{k^{\pm}-1}}\right) \tag{9}$$

By using eqn.(9) the CSDA ranges of electron and positrons are given as:-

$$R(T_0) = \frac{m_e c^2}{(M \times Z + C)}\left[\frac{\gamma^{K-(A^{\pm}Z + B^{\pm})+1}}{K-(A^{\pm}Z + B^{\pm})+1} + \frac{\gamma^{-(A^{\pm}Z + B^{\pm}-1)}}{-(A^{\pm}Z + B^{\pm}-1)} \right]^{\gamma_0}_{1.02} \tag{10}$$

Where M, C and K are constants, the total energy of electron or positron in electron mass unit is given by γ, and Z is the atomic number of the material.

Table-2
Values of constants (M & C)

Range (Atomic Number)	M	C
1≤Z≤10	-0.3215	3.675
10≤Z≤36	-0.0132	1.677
Z>36	-0.00655	1.4177

Table-3
Value of constant (K)

Energy	K-	K+
20KeV<E≤1000KeV	3.95	3.75
10000KeV<E≤50000KeV	3.86	3.62

Table-4
Value of constant for biomedical materials (K)

Energy	K-
20KeV<E≤1500KeV	2.8
1500KeV<E≤50000KeV	3.8

Comparison between Proposed and Reported Values

With the assistance of equation (1) practical ranges of electrons and positrons in different elements for a variety of energies in their given energy regions[22] are evaluated. Comparison is then made with the values thus obtained with other available data[20,23]. In order to match up to calculated values of practical ranges of positrons, electrons, air and, water subsequent process has been adopted. Tables-4 to 9 represents the evaluated values of practical ranges. This empirical formula is convincing for lower atomic numbers. We can therefore conclude that the result of practical ranges derived by projected formula is in close conformity with the published data as compared to the values reported by prior investigators so far.

Conclusion

It is quite apparent from the above mentioned results derived using the proposed empirical relation (1), that energy and atomic number of the material are the parameters used for expressing the practical ranges of different materials. The most important conclusion drawn from the calculation of practical ranges is that CSDA range of the material is an important factor for its calculation. It is also significant that the values obtained from the proposed empirical relation are in better harmony with the hypothetical data as compared to the empirical relations anticipated by preceding researchers[20,23]. The above relation is also much simpler and widely relevant.

Table-5
Values of Practical Ranges (gm/cm2) for the electrons of materials

E (keV)	Carbon			Aluminum		
	Cal	Sta	%diff	Cal	Sta	%diff
30	0.684	1.63	0.420	1.371	1.41	0.972
40	1.496	2.77	0.540	2.366	2.39	0.990
50	2.583	4.14	0.624	3.591	3.59	1.000
60	3.922	5.72	0.686	5.026	4.99	1.007
70	5.490	7.46	0.736	6.653	6.49	1.025
80	7.270	9.39	0.774	8.455	8.14	1.039
90	9.242	11.47	0.806	10.418	9.99	1.043
100	11.392	13.66	0.834	12.530	11.93	1.050
250	61.461	52	1.182	55.944	53.8	1.040
400	80.700	125	0.646	110.390	110	1.004
770	286.124	306	0.935	257.754	296	0.871
1000	345.734	442	0.782	351.800	390	0.902
1530	637.784	706	0.903	55.944	50	1.119
1710	721.534	826	0.874	642.525	814	0.789
1880	800.653	952	0.841	711.826	933	0.763
2000	623.830	1014	0.615	760.655	928	0.820
4000	856.503	2134	0.401	1564.535	2037	0.768
10000	4557.176	_	_	3907.422	_	_
20000	7122.589	_	_	6006.743	_	_
50000	16484.216	_	_	13553.456	_	_

Table-6
Values of Practical Ranges (gm/cm2) for the electrons of air and water

E (keV)	Air			Water		
	Cal	Sta	%diff	Cal	Sta	%diff
100	0.011	0.013	0.871	0.012	0.012	0.968
200	0.034	0.041	0.825	0.034	0.037	0.917
500	0.108	0.163	0.663	0.109	0.148	0.739
1000	0.204	0.407	0.502	0.207	0.370	0.559
2000	0.914	0.925	0.988	0.925	0.852	0.739
5000	2.195	2.482	0.885	2.222	2.347	0.947
10000	4.099	4.987	0.822	4.149	4.853	0.855
20000	7.515	9.692	0.775	7.605	9.720	0.782
50000	16.479	22.430	0.735	16.672	23.320	0.715

Table-7
Values of Practical Ranges (gm/cm2) for the electrons of materials

E (keV)	**Copper**			**Silver**			**Indium**		
	Cal	**Sta**	**%diff**	**Cal**	**Sta**	**%diff**	**Cal**	**Sta**	**%diff**
250	45.5	45	1.01	40.1	40	1.0	39.5	35	1.1
770	204.7	286	0.72	175.6	264	0.7	172.5	257	0.7
1530	442.0	622	0.71	370.0	580	0.6	362.5	574	0.6
1710	497.0	720	0.69	414.2	705	0.6	405.5	697	0.6
1880	548.6	881	0.62	455.3	829	0.5	445.6	-	-

Table-8
Values of Practical Ranges (gm/cm2) for the electrons of materials

E (keV)	Ytterbium			Lead			Gold			Holmium		
	Cal	Sta	%diff	Cal	Sta	%diff	Cal	Sta	%diff	Cal	Sta	%diff
250	35.3	30	1.18	34.1	-	-	34.4	-	-	35.7	32	1.12
770	149.6	241	0.62	142.0	231	0.61	143.6	235	0.61	152.0	242	0.63
1530	305.5	526	0.58	285.6	510	0.56	289.9	541	0.54	311.7	536	0.58
1710	340.0	636	0.53	317.0	590	0.54	322.0	606	0.53	347.1	652	0.53
1880	371.9	763	0.49	345.8	727	0.48	351.5	734	0.48	379.9	771	0.49

Table-9
Values of Practical Ranges (gm/cm2) for the positrons of materials

E (keV)	Aluminum			Silver			Lead		
	Cal	Sta	%diff	Cal	Sta	%diff	Cal	Sta	%diff
159	25.53	25.8	0.99	18.52	17.80	1.04	15.97	16.50	0.97
250	51.51	54	0.95	36.92	37.50	0.98	31.43	32.70	0.96
336	78.81	85.4	0.92	55.91	59.00	0.95	47.10	51.70	0.91

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