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Exploring Multiscale variability of monthly runoff time series of Diani River using fractal theory in Southern Guinea

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Abstract

Runoff time series modeling is necessary for hydrological applications, including understanding the evolution of river regimes and forecasting and controlling floods. However, in the Guinea republic, West Africa's water tower, the intrinsic characteristics involved in hydrological variables dynamics remain unknown. This preliminary work aims to explore, for the first time in Guinea, the multifractality and complexity properties of monthly runoff time series measured from 2000 to 2019 on the Diani River, which is one of the largest rivers in Guinea. To this end, the following parameters have been computed: Lyapunov exponent, Hurst exponent, Higuchi fractal dimension, width of the multifractal spectrum and spectrum asymmetry index. Numerical results indicate: i. a clear footprint of persistence and multifractality in the runoff time series irrespective of the time period. However, the persistence and multifractality degree depend on the time period considered ii. a sign of chaotic dynamic systems and predictive instability in runoff variations. iii. the predictive scheme based on the multifractality and persistence could be adapted for Diani river runoff prediction. The conclusions drawn from these results should prove useful for the validation of global and regional climate models.

Keywords: Diani River, Runoff, Long-range dependence, Multifractality, Complexity, Lyapunov exponent, Guinea.

Introduction

Hydrologists have numerous roles, including operational management of hydraulic structures, modeling hydrological phenomena, understanding the evolution of river regimes, forecasting and controlling floods. To reach these goals, the understanding of the intrinsic characteristics of hydrological variables is essential[1]. For these requirements numerous studies related to the invariance regimes, complexity and multifractal analysis of the hydrological variables records have been developed in several countries around the world. For instance, the multifractal characteristics of long term runoff records have been studied by Kantelhardt et al.[2,3]; Koscielny-Bunde et al.[4]; in China by Zhang et al.[5]; Zhang et al.[6]; Li et al.[7]; in France by Labat et al.[8]; in Georgia in the southeastern USA by Hirpa et al.[9]; in Canada by Tan and Gan[10]; in India by Adarsh et al.[11,12], in Brazil by Rego et al.[13].

Even though many studies have focused on the multifractal analysis of runoff time series in different parts of the globe, to our best knowledge, in Guinea, West Africa's water tower, the multifractal properties of hydrological variables remain unknown, and the dynamic of runoff time series recorded from Guinean rivers are not yet investigated in fractal framework. Such gap in research work could be related to three main reasons: the unavailability of hydrological variables over a long period, the presence of a large number of missing values in the hydrological variables time series in most of the country' rivers, and the measurement networks are less densely populated. The first step towards gap filling is to understand the multifractal and chaotic characteristics of the available hydrological variables time series.

This preliminary work aims to explore, for the first time in Guinea, the multifractal and complexity properties of monthly runoff time series measured on the Diani River, which is one of the largest rivers in southern Guinea. In the following section, we describe the data and present the methods. The subsequent section deals with the results analysis, and finally, the conclusions are drawn.

Materials and Methods

Study area and Data description: The Diani River (Figure-1) is one of most important rivers in Guinea. It is located in the forest region, precisely in Macenta's prefecture, sub-prefecture of N'zébéla. The Diani river is the only one between the southern guinea rivers, having a gauging station, which is located at the Diani bridge, 4km from N'zébéla on the national road linking Macenta to N'Zérékoré. It rises in the classified forest along the Milo River, near Vassérédou and delimits Guinea from Liberia along a 50 km stretch. The Diani River serves as a natural border between Liberia and Guinea, and enters Liberian territory near Banié (Youmou Prefecture), where it is known as the Saint Paul River. The Diani River watershed, which is one of the three main watersheds of Guinea Forest, has a surface area of 9333 Km^2 with an average gradient of 3.31m/Km.

From Figure-2, it can be seen that the runoff from 2000 to 2019 at Diani bridge hydrological station in Diani river has certain inconsistent fluctuations, reflecting the strong variability of this runoff series.

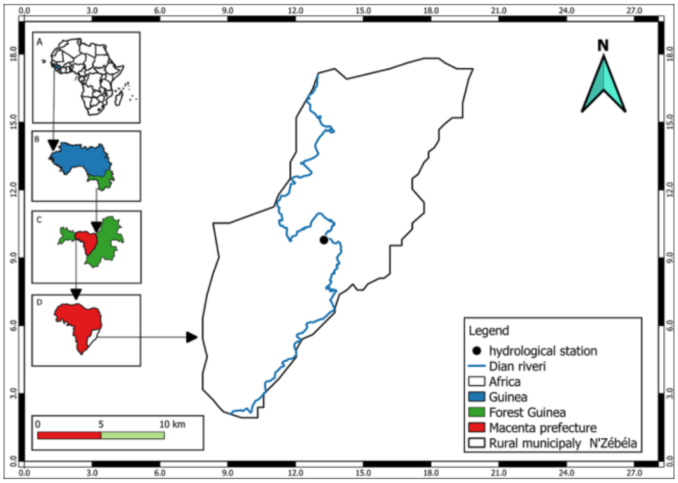

Figure-1: Geographic location (a) of Guinea in Africa, (b) of forest Guinea in Guinea, (c) Macenta in forest Guinea, (d) N'zébéla in Macenta, (e) Diani river in N'zébéla.

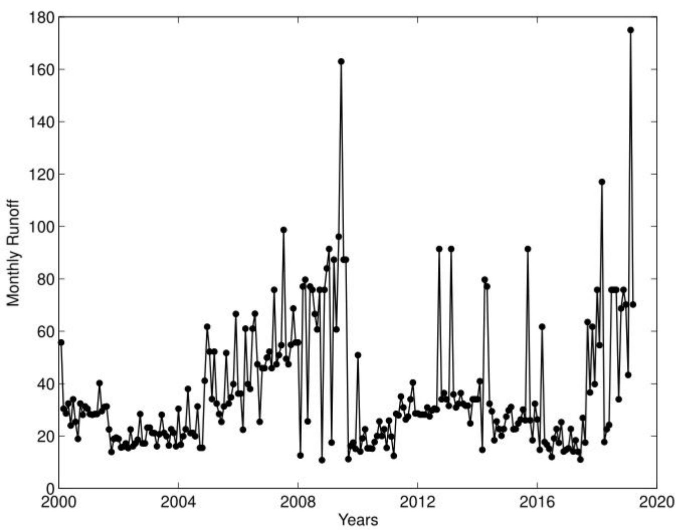

Figure-2: Temporal variation of monthly runoff measured at Diani Pont hydrological station from 2000-2019.

Methods: Rescaled-Range (R/S) method: The one-dimensional R/S method is briefly described by the following steps[14-18]:

One computes the subsets of runoff time series mean, $\bar{y}_\tau = \frac{1}{\tau}\sum_{k=1}^{\tau} y_k$, where $\{y_t | t = 1:1:N\}$ are studied records and $1 \le \tau \le N$.

One computes the range (R_τ) and standard deviation (S_τ) respectively by (1) and (2):

$$R_\tau = \left[\max_{1\le i\le\tau} \sum_{k=1}^{i}(y_k - \bar{y}_\tau) - \min_{1\le i\le\tau} \sum_{k=1}^{i}(y_k - \bar{y}_\tau)\right] \quad (1)$$

$$S_\tau = \left[\frac{1}{\tau}\sum_{k=1}^{\tau}(y_k - \bar{y}_\tau)^2\right]^{1/2} \quad (2)$$

where, $1 \le \tau \le N$

One computes the rescaled range as $(R/S)_\tau = R_\tau / S_\tau$.

One determines the Hurst exponent ($H_{R/S}$) by plotting the $log(R/S)_\tau$ versus $\log(\tau)$as:

$$log(R/S)_\tau = loga + H_{R/S}.log(\tau) \quad (3)$$

Fractal Dimension Analysis: If X is a time series, with length N, noted as $(X_k)_{k\epsilon 1;N}$, then Higuchi's method can be briefly outlined as follows[19-21]: Reconstruct time sequence and calculate the average length as:

$$L_m(k)\frac{1}{k^2}\left(\sum_{i=1}^{\left[\frac{N-1}{k}\right]}|X(m+ik) - X(m-(i-1)k)|\right)\left(\frac{N-1}{\left[\frac{N-m}{k}\right]}\right) \quad (4)$$

Where the symbol [y] represents the bigger integer part of y, $k\epsilon\{1,\dots,k_{max}\}$ and $m\epsilon\{1,\dots,k\}$. In this work, $k_{max} = 8$.

Compute the total fractal length as follows:

$$L_k = \frac{1}{k}\sum_{m=1}^{k} L_m(k) \quad (5)$$

Deduce the Higuchi fractal dimension as the opposite of the slope from equation (6) in log-log plot

$$L_k \propto k^{-D} \quad (6)$$

Multifractal Analysis: To reveal the multi scaling properties involved in runoff time series, the multifractal detrended fluctuation analysis, MFDFA developed by Kantelhardt et al.[22] is adopted. In this method, the generalized Hurst exponen t$h(q)$ is deduced through the non linear relationship between $F_q(s)$ and the timescale as follows:

$$F_q(s) \propto s^{h(q)} \quad (7)$$

$F_q(s)$ is the function of fluctuations. $H_{MFDFA} = h(2)$ is the Hurst exponent obtained from MFDFA.

The multifractal spectrum $f(\alpha)$ and Hölder exponent$\alpha(q)$ are related to $h(q)$ by the means of the first-order Legendre transforms as follows:

$$\alpha = h(q) + q\frac{h(q)}{dq} \leftarrow\text{Legendre}\rightarrow f(\alpha) = q[\alpha - h(q)] + 1 \quad (8)$$

$$\Delta\alpha = \alpha_{max} - \alpha_{min}.$$

$\Delta\alpha$ denotes the multifractal spectrum width. The larger the $\Delta\alpha$, the stronger is the multifractality of the time series. α_0 is the corresponding to the maximum $f(\alpha)$

$$SAI = \frac{(\alpha_0 - \alpha_{min}) - (\alpha_{max} - \alpha_0)}{(\alpha_0 - \alpha_{min}) + (\alpha_{max} - \alpha_0)}, -1 \le R \le 1. \quad (9)$$

SAI is the spectrum symmetry index. SAI is equal to 0, <0 and >0for symmetric, left-skewed and right-skewed shapes, respectively.

$$H_u = \frac{1}{2}(H_{R/S} + H_{MFDFA}) \quad (10)$$

Where: $H_{R/S}$ and H_{MFDFA} are the Hurst expoenent obtained from rescaled-Range (R/S) and MFDFA method, respectively.

H_u value between 0 and 0.5 indicates that the runoff series are anti-persistent, thus, two adjacent events have an inverse correlation ; H_u value between 0.5 and 1, implies that the runoff series are persistent ; H_u value equal to 0.5 signifies that there are no changes and runoff series are uncorrelated and random[3,14,15]. The uncertainty degree on the predictability of the runoff time series is quantified by means of the Lyapunov exponents, which is computed according to the algorithms proposed by Lai, D., & Chen, G.[23] and Sprott, J.C.[24].

Results and Discussion

The temporal variation of the Hurst exponent (H_u) of the monthly runoff time series recorded during 2000-2019 period for Diani River is shown in Figure-3a and the 95% confidence limits of Pearson coefficent related to Hurst exponent computation are shown in Figure-3b for each sub period. From the results (Figure-3a) it is observed that whatever the time-period (2000-2009, 2010-2019 and 2000-2019), the Hurst exponent is estimated by Pearson coefficient limits between 0.975 and 0.985, indicating the robustness of the linear used to fit the fluctuation functions. From the results (Figure-3b) it is noted that the Hurst exponent values exceed 0.5 for the both sub-periods (2000-2009 and 2010-2019) and 2000-2019 period. During 2000-2019, the Hurst exponent (H_u) can be classified as follows $H_u(2010-2019) < H_u(2000-2009) < H_u(2000-2019)$.

Moreover, independently of the time period considered, the Hurst exponent values vary between 0.86 and 0.96, which is great than 0.73, reported by Kantelhardt et al.[1] as universal value. These findings indicate clearly a long-term persistence in the runoff time series irrespective of the time period. However, the persistence severity decreases from the first sub period to the second. Thus, if a decrease (increase) is observed in the runoff levels during a period of time, the similar decrease (increase) is expected to continue during a similar period of time. Overall, this results suggest that the prediction schemes based on the trends of the preceding elements will be appropriate for Dianiriver's runoff timeserie prediction.

Figure-4 presents the temporal variation of the first positive Lyapunov exponent (λ_1) and Higuchi fractal dimension (HFD) of runoff time series. From the results (Figure-4a) it is noted that the Lyapunov exponent values are between 0.025 and 0.15, with $\lambda_1(2000-2019) < \lambda_1(2000-2009) < \lambda_1(2010-2019)$, suggesting a predictive instability in runoff dynamics and compared to others, 2010-2019 is the time period in which the largest erroneous in long-term predictions of runoff time series is could be related on the starting values uncertainties.

From the results (Figure-4b) it is noted that all values of HFD are non-integer (vary between 1.5 and 2), clear footprint of chaotic dynamic systems with fractal characteristics in runoff variations during 2000-2019 period. The results obtained above indicated that the fractal framework is insufficient to better understand the multi scaling properties of the runoff series. Therefore, the multifractal characterization of the runoff series is necessary.

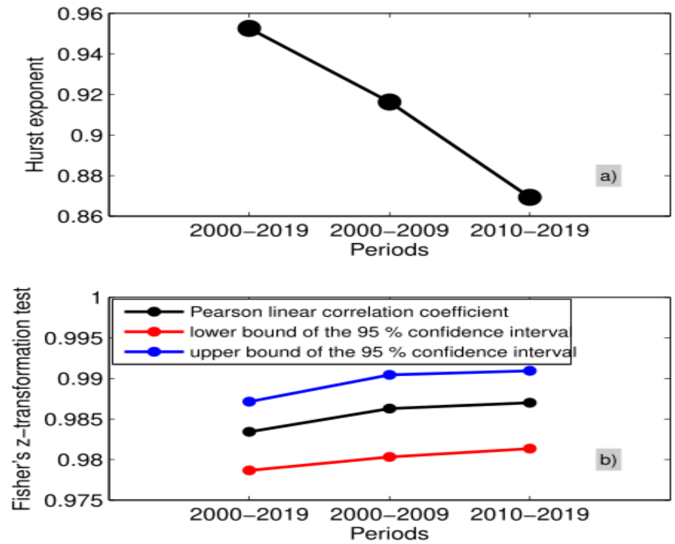

Figure-3: (a) Temporal variation of Hurst exponent of runoff time series, (b) 95% confidence limits of Pearson coefficient.

Figure-5 depicts the multifractal spectrum of the runoff time series during 2000-2009; 2010-2019 and 2000-2019. The runoff time series exhibit multifractal spectrum with distinct shape depending on the time period. Moreover, the multifractal spectrum shapes are all convex parabolas, confirming the multifractal behaviour and greater complexity of runoff time series for Diani River.

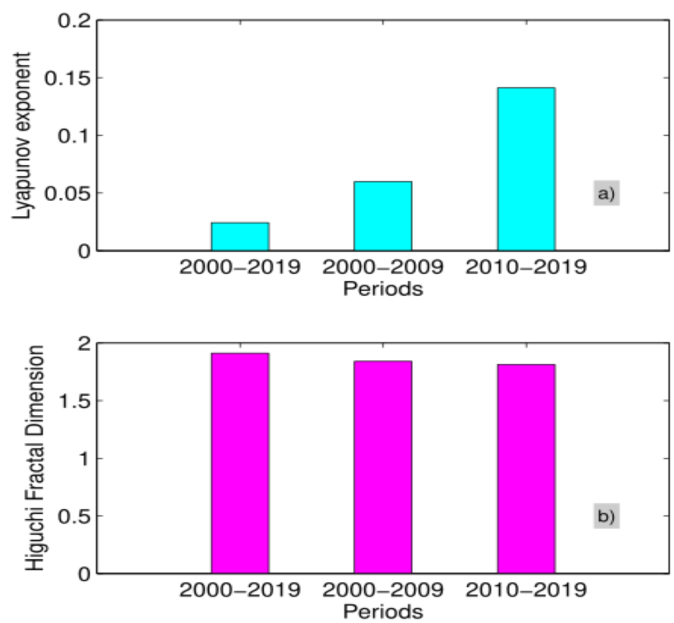

Figure-4: Temporal variation of: (a) Lyapunov exponent, (b) Higuchi fractal dimension (HFD) of runoff time series.

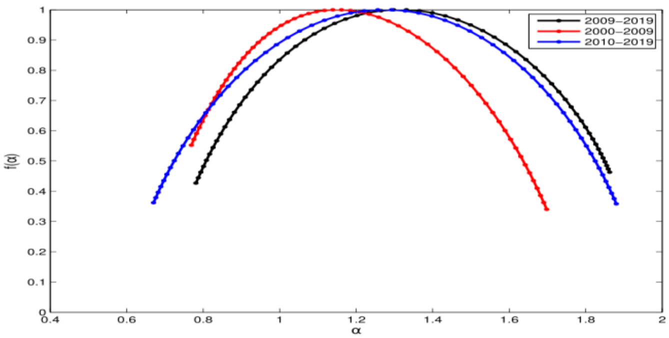

Figure-5: Multifractal spectrum of runoff time series during: 2000-2009; 2010-2019 and 2000-2019.

The temporal variation of the multifractal spectrum width ($\Delta\alpha$) and the spectrum asymmetry index (SAI) during 2000-2019 period forrunoff time series is shown in Figure-6. From Figure-6a, it is noted that whatever the time period considered, $\Delta\alpha$ values are nonzero and greater than zero, with $\Delta\alpha(2000-2009) < \Delta\alpha(2000-2019) < \Delta\alpha(2010-2019)$. Therefore, scaling features and multifractality are present in runoff time series for Diani River, however, to a varying degree. The highest $\Delta\alpha$ obtained during 2010-2019 period suggests that compared to others time periods, in this sub period, the runoff time series present the greatest irregularity, heterogeneity, intermittencies and multifractality. Thus, changes in the runoff time series during 2010-2019 period are more extreme and the prediction will be the most difficult in this sub period. From Figure-6b, it is observed that SAI values are positive. These results suggest that the multifractal spectrum of each sub period is characterized by a left-hand deviation, indicating some degree of local high fluctuations. Thus, in Diani River high fluctuation is responsible for runoff time serie dynamics. Qualitatively, our findings regarding the persistency and multifractality characteristics of runoff align with results reported in other regions of the world, such as China[7,25], Canada[10], New Zealand [26] and Brazil[27].

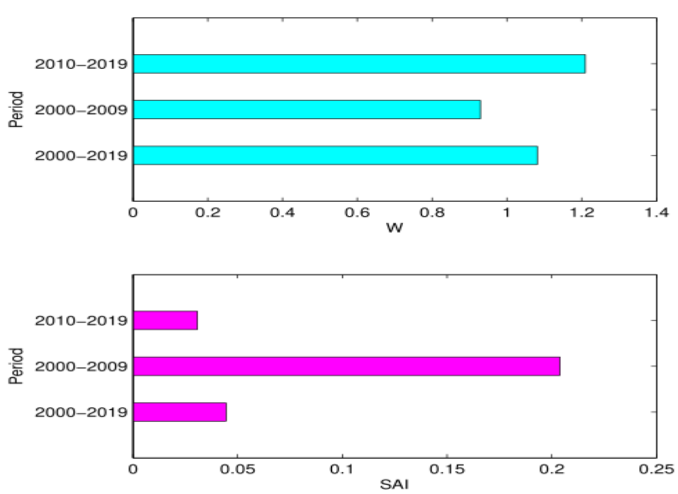

Figure-6: Temporal variation of (a) multifractal spectrum ($\Delta\alpha$), (b) spectrum symmetry index (SAI) of runoff time series.

Conclusion

This research work aimed to investigate for the first time the intrinsic multifractal and chaotic characteristics involved in runoff time series dynamics recorded from 2000 to 2019 in Diani River by using Fractal Theory. For this end the 2000-2019 period is divided in two sub periods: 2000-2009 and 2010-2019. From this work, the following conclusions can be drawn: 1. Independently of the time period, runoff series are persistent and multifractal in Diani River. However, the persistency level and the multifractality severity depend on the time period considered. ii. predictive instability is identified in the mechanism governing runoff series. iii. It seems that the predictive scheme based on multifractality and the trend of the previous elements is adapted for Dianireiverrun off prediction. This study is a preliminary work on the modeling of runoff time serie in Guinea. Furthermore, the limitation of the present study is related to the using of only one hydrological station. However, this situation can be explained by the unavailability of hydrological variables over a long period in Guinea, the presence of a large number of missing values in the hydrological variables time series in most of the country' rivers, and by the fact that the measurement networks are less densely populated.

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