



Dispersion of longitudinal electro-kinetic waves in Ion-Implanted Quantum Semiconductor Plasmas

Chaudhary S., Yadav Nishchhal and Ghosh S.

School of Studies in Physics, Vikram University, Ujjain-456010, MP, INDIA

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Abstract

Dispersion properties of longitudinal electro-kinetic wave in multi-component (electron, ion and dust (e-i-d)) quantum plasmas are studied. The variation of dispersion characteristics of longitudinal electro-kinetic wave, with and without quantum effect in ion-implanted semiconductor plasma is also explored. It is found that the quantum Bohm potential term modifies the dispersion characteristics of fast and slow electro-kinetic waves in multi-component (e-i-d) plasmas.

Keywords: Bohm potential, electro-kinetic wave, ion-implantation, semiconductor plasma.

Introduction

During the past few years quantum plasma have generated tremendous interest owing to their wide ranging applications in dusty plasmas^{1,2}, in dense astrophysical environment³, (such as white dwarfs and neutron stars), in micro-electronic devices⁴, in intense laser beam produced plasmas⁵, in nonlinear quantum optics^{6,7}, etc. Plasmas, in general, are characterized by high temperature and low density regimes where quantum effects are negligible. However, there are examples in nature where both plasma and quantum effects can coexist. The quantum effects become important in plasmas, when the de-Broglie wavelength associated with particles is comparable to dimension of the system. In such situation plasmas behave like Fermi gas and quantum mechanical effect are expected to play a significant role in the behavior of charged particles⁸⁻¹².

The quantum hydrodynamic model (QHD) for plasmas was enunciated by Manfredi and Hass¹³. Later, the same methodology has been applied to a multitude of problems involving the ion acoustic wave excitations in plasmas using two species quantum plasma and was reported by Haas et al.¹⁴. The stimulated Brillouin scattering of a laser radiation in an unmagnetized piezoelectric semiconductor using QHD model was reported by Uzma et al.¹⁵. The parametric amplification characteristics in piezoelectric semiconductor were reported by Ghosh et al.¹⁶. This status reflects tremendous scope of work in the field of quantum semiconductor plasmas.

The quantum hydrodynamic model (QHD) is an extension of classical fluid model used for plasma media. In QHD model, there are two different quantum effects exist – (1) quantum diffraction, and (2) quantum statistics. Quantum diffraction will be taken into account by the terms proportional to $\hbar^2$ in equations of motion and continuity in the quantum hydrodynamic model. These contributions may be interpreted alternative as quantum pressure terms or as quantum Bohm

potential. The quantum statistics will be included in the model via the equation of state, which takes into account the fermionic character of electrons. The equation of state for electrons shall be found assuming a local zero temperature Fermi distribution, a choice dictated by the spin $\frac{1}{2}$ statistics for these particles. The Bohm potential, on the other hand, will exist even for a pure quantum mechanical state and has nothing to do with the statistical properties of the system. In a broad sense, we will refer to these particularities arising from the wave like nature of the charge carriers as quantum diffraction effects. The Bohm potential term appropriately describes the negative differential resistance in resonance tunneling diodes. Negative resistance is based on resonant tunneling which is a quantum phenomenon and it does not occur in classical transport model.

In quantum dusty plasmas, the dispersion caused by the strong density correlations due to quantum fluctuations play an important role in the investigation of collective effects. It is known that cold quantum plasmas can support new dust modes^{1,8} and the observations suggest that the frequency spectra of these modes should be useful in the study of charged dust impurities in micro-electro-mechanical systems¹⁰. Shukla and Ali¹ studied the propagation of dust acoustic wave in quantum plasma and found that the dispersion characteristics of the wave get significantly modified due to the quantum corrections. The study of linear and nonlinear quantum ion acoustic waves have been investigated in unmagnetized electron-ion plasmas by Hass et al.¹⁴ using the well known QHD model.

In classical frame work, Ghosh and Thakur¹⁷, have studied longitudinal electro-kinetic wave, in ion-implanted semiconductor plasma and reported that the effects of charged colloids on the properties of the host semiconducting medium modify existing modes and also introduce new modes. The studies of electro-kinetic wave have been a subject of many investigations¹⁸⁻²². However; these important study has not been worked out in a multi-component quantum plasma case according to the authors' best information.

Motivated by the present status and the works of Ghosh et al.¹⁷⁻²², in this paper, authors have focused their attention on the modifications occurred in dispersion characteristics of longitudinal electro-kinetic wave (LEKW) in ion-implanted semiconductor plasma due to quantum effect through Bohm potential.

The paper is organized in the following way: The basic equations describing electro-kinetic wave propagation and derivation of the dispersion relation for electro-kinetic wave in quantum dusty plasmas, using QHD model are given in section 2. In section 3, authors present numerical appreciation of the results obtained followed by physical discussion. Finally, conclusions drawn from the study are listed in section 4.

Methodology

Basic Equations and Dispersion Relation: We are studying the nature of longitudinal electro-kinetic wave (LEKW) in ion-implanted quantum semiconductor plasma, consisting of electrons, holes and negatively charged colloids as carriers. The colloids may collect both electrons and holes, but since the electrons moves more swiftly than the holes, therefore we have considered that colloids tend to acquire net negative charge. These negatively charged colloids act as a third species or foreign particles inside the medium, but have significant effect on the behaviour of the plasma. Hence, now this ion-implanted quantum semiconductor plasma that contains electrons, holes and negatively charged colloids may be treated with 'multi-component plasma model'. In such multi-component plasma system the charge neutrality shall be given by

$$z_h n_{0h} = z_e n_{0e} + z_d n_{0d}, \quad (1)$$

where $n_{0\alpha} (\alpha=e, h, d)$ is the number density, z_α is the charge states of carriers, in which $z_d = \frac{q_d}{e}$ is the ratio of negative charges q_d resided over the colloidal grains to the charge e on electrons. It is assumed that $z_e = -1$ and $z_h = 1$ for further calculations.

If each component has a mass m_α , charge state z_α , density n_α , Fermi velocity V_F , momentum transfer collision frequency ν_α and Fermi pressure P_α , then this multi-component quantum plasma system is described by their continuity and momentum equations as

$$\frac{\partial n_\alpha}{\partial t} + \frac{\partial}{\partial z} (n_\alpha v_\alpha) = 0 \quad (2)$$

$$\frac{\partial v_{z1\alpha}}{\partial t} = \frac{z_\alpha q_\alpha}{m_\alpha} E_{z1} - \nu_\alpha v_{z1} - \frac{1}{m_\alpha n_{0\alpha}} \left[\nabla P_\alpha - \frac{\hbar^2 \nabla (\nabla^2 n_{1\alpha})}{4m_\alpha} \right] \quad (3)$$

where, $p_\alpha = \frac{m_\alpha V_F^2 n^3}{3n_{0\alpha}^2}$ is the Fermi pressure in which Fermi velocity $V_F = \left(\frac{2k_B T_F}{m_\alpha} \right)^{\frac{1}{2}}$,

Here the subscripts 0 and 1 represent zero and first order quantities, respectively. Assuming the first order quantities varying as $\exp[i(\omega t - kz)]$ [where ω and k are frequency and wave number of the propagating mode respectively], and following the procedure adopted by Steele and Vural²², the dispersion relation for longitudinal electro-kinetic wave is obtained with the help of quantum hydrodynamic model in ion-implanted quantum plasma as,

$$\epsilon(\omega, k) = 1 - \frac{\omega_{pe}^2}{(\omega - i\nu_e \omega - k^2 V_{Fe}^2 (1 + \gamma_e))} - \frac{\omega_{ph}^2}{(\omega - i\nu_h \omega - k^2 V_{Fh}^2 (1 + \gamma_h))} - \frac{\omega_{pd}^2}{\omega^2} = 0 \quad (4)$$

where $\omega_{p\alpha}^2 = \frac{(z_\alpha e)^2 n_{0\alpha}}{\epsilon m_\alpha}$, $\gamma_\alpha = \frac{\hbar^2 k^2}{8m_\alpha k_B T_F}$ and $\epsilon = \epsilon_0 \epsilon_L$; ϵ_L being the lattice dielectric constant.

On neglecting the quantum correction ($\gamma_\alpha = 0$) term the dispersion relation given by equation (4) reduces to dispersion relation obtained by Ghosh and Thakur¹⁷. In addition if we ignore the presence of colloids also, the derived dispersion relation reduces to equation (4-3b) of Steele and Vural²². Hence the derived dispersion relation for LEKW shows the modifications due to quantum effect in ion-implanted semiconductor plasmas.

Now, authors will focus their attention towards the principal point of the paper i.e. modification in dispersion characteristics due to quantum effects in electro-kinetic mode propagating through ion-implanted semiconductor plasma. For the same we shall study this dispersion relation under two different frequency regimes.

Case I- Slow electro-kinetic mode ($\omega \ll kV_{Fe}, kV_{Fh}$): If the phase velocity of the wave is less than the Fermi velocities of electrons and holes both, the mode may be termed as slow electro-kinetic mode. Therefore, for slow electro-kinetic mode, under collision dominated or low frequency regime ($\omega \ll V_{Fe}, V_{Fh}$), dispersion relation (4) reduces to

$$1 + \frac{\omega_{pe}^2}{(i\omega \nu_e + k^2 V_{Fe}^2 (1 + \gamma_e))} + \frac{\omega_{ph}^2}{(i\omega \nu_h + k^2 V_{Fh}^2 (1 + \gamma_h))} - \frac{\omega_{pd}^2}{\omega^2} = 0. \quad (5)$$

Equation (5) may be written in the form of polynomial in ' ω ' as,

$$\begin{aligned} & -\omega^4 + i\omega^3 [\omega_{Re} \{1 + k^2 \mathcal{L}_{Fe}^2 (1 + \gamma_e)\} + \omega_{Rh} \{1 + k^2 \mathcal{L}_{Fh}^2 (1 + \gamma_h)\}] \\ & + \omega^2 [k^2 \{\mathcal{L}_{Fe}^2 \mathcal{L}_{Fh}^2 (1 + \gamma_e)(1 + \gamma_h) + \mathcal{L}_{Fe}^2 (1 + \gamma_e) + \mathcal{L}_{Fh}^2 (1 + \gamma_h)\} \omega_{Re} \omega_{Rh} + \omega_{pd}^2] \\ & - i\omega [k^2 \omega_{pd}^2 \{\mathcal{L}_{Fe}^2 \omega_{Re} (1 + \gamma_e) + \mathcal{L}_{Fh}^2 \omega_{Rh} (1 + \gamma_h)\}] - k^4 \mathcal{L}_{Fe}^2 \mathcal{L}_{Fh}^2 \omega_{pd}^2 \omega_{Re} \omega_{Rh} (1 + \gamma_e)(1 + \gamma_h) = 0. \end{aligned} \quad (6)$$

where, $\omega_{Re, h} = \frac{\omega_{pe, h}^2}{V_{e, h}}$, are the dielectric relaxation frequencies of electrons and holes, respectively.

Case II- Fast electro-kinetic mode ($kV_{Fh} \ll \omega \ll kV_{Fe}$): If the phase velocity of the mode is less than electron Fermi velocity but more than the hole Fermi velocity, the mode may be termed as fast electro-kinetic mode. Thus for fast electro-kinetic mode the dispersion relation (4) reduces to,

$$1 + \frac{\omega_{pe}^2}{(i\nu_e\omega + k^2V_{Fe}^2(1+\gamma_e))} - \frac{\omega_{ph}^2}{(\omega^2 - i\nu_h\omega)} - \frac{\omega_{pd}^2}{\omega^2} = 0. \quad (7)$$

Equation (7) may be rewritten in the term of polynomial in ' ω ' as,

$$\begin{aligned} & -\omega^4\omega_{rh} - i\omega^3[\omega_{ph}^2 + \{1+k^2\chi_{re}^2(1+\gamma_e)\}\omega_{re}\omega_{rh}] \\ & + \omega^2[\omega_{rh}(\omega_{ph}^2 + \omega_{pd}^2) + \{1+k^2\chi_{re}^2(1+\gamma_e)\}\omega_{re}\omega_{ph}^2] \\ & - i\omega[\omega_{ph}^2\omega_{pd}^2 + k^2\chi_{re}^2\omega_{re}\omega_{rh}(1+\gamma_e)(\omega_{ph}^2 + \omega_{pd}^2)] - k^2\chi_{re}^2\omega_{ph}^2\omega_{pd}^2\omega_{re}(1+\gamma_e) = 0. \end{aligned} \quad (8)$$

Quantum effects are contained in the $\hbar$ dependent terms in equations (6) and (8) (sometimes called the Bohm potential).

For both the frequency regimes, the obtained dispersion relations are polynomials of fourth degree in terms of angular frequency ' ω '. Hence we shall have to solve these numerically to study the propagation characteristics of LEKW in ion-implanted quantum semiconductor plasmas.

Results and Discussion

Equations (6) and (8) being of fourth degree in complex wave frequency ($\omega = \omega_r + i\omega_i$) with complex coefficients is not easy to solve analytically and so we have solved it numerically. In numerical calculations the following representative parameters have been used: $m_e = 0.0815 m_0$, m_0 being the free electron mass, $m_h = 4m_e$, $m_d = 10^{-27}$ kg, $\epsilon_L = 15.8$, $n_{0e} = 10^{21} \text{ m}^{-3}$, $n_{0h} = 5 \times 10^{21} \text{ m}^{-3}$, $n_{0d} = 10^{14} \text{ m}^{-3}$, $\nu_e = 3.463 \times 10^{11} \text{ sec}^{-1}$, $\nu_h = 1.194 \times 10^{11} \text{ sec}^{-1}$ and $\nu_d = 3.422 \times 10^8 \text{ sec}^{-1}$, $k_B = 1.3805 \times 10^{-23}$, $h = 6.6 \times 10^{-34}$, $\hbar = \frac{h}{2\pi}$ at 77 K.

The results of our calculations are displayed in figures 1 to 7.

Figures 1-4 displays the variation of real frequency ω_r with positive real value of k for four possible modes, without and with quantum effect under slow wave regime. One may infer from figure 1 that frequency of this mode is found negative reflects that the phase velocity of this mode will be along negative z-axis; hence it is a contra-propagating mode. The magnitude of phase constant increases with increase in value of k , and at higher value of k this mode becomes nearly independent of k in both the cases. Phase constant for this mode

is higher in absence of quantum effect compared to that obtained when quantum effect is present.

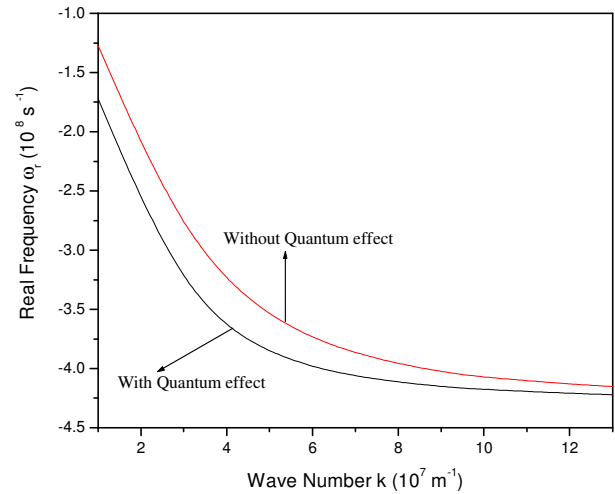


Figure-1
Variation of real frequency of first mode with wave number for slow electro-kinetic wave

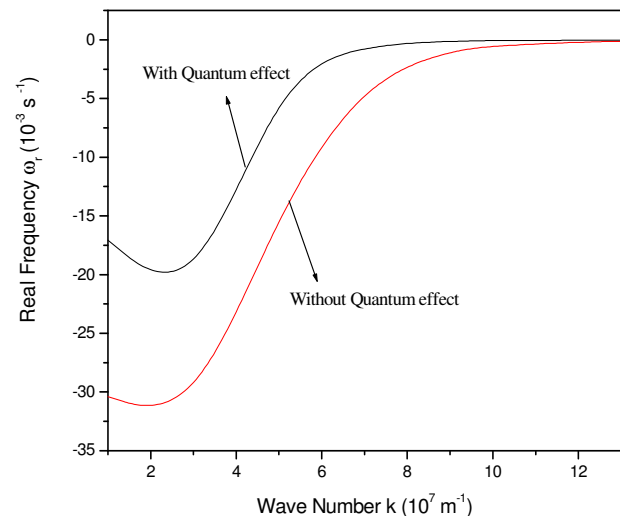


Figure-2
Variation of real frequency of second node with wave number for slow electro-kinetic wave

One may infer from figure 2 that the second mode in slow wave limit is also a contra-propagating mode. The magnitude of phase constant of this mode first increases with increase in k , achieve maximum value at $k \approx 2.5 \times 10^7 \text{ m}^{-1}$ and then saturates. It may be inferred from this figure that the presence of quantum effect enhances the magnitude of phase constant that is opposite to that found for first mode.

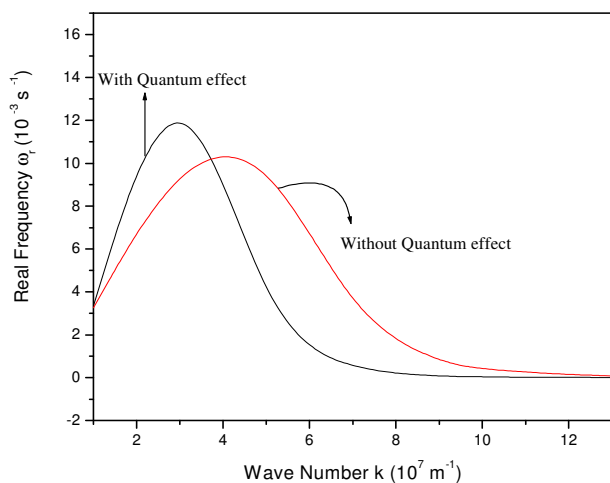


Figure-3
Variation of real frequency of third mode with wave number for slow electro-kinetic wave

Figure 3 illustrates the dispersion curves for third mode with and without quantum effect. It may be inferred from this figure that the mode is propagating along z-direction (co-propagating) in both the cases. Their phase speeds first increase with increase in k and touch maximum at $k = 3 \times 10^7 \text{ m}^{-1}$ in presence of quantum effect, and at $k = 4.5 \times 10^7 \text{ m}^{-1}$ in absence of quantum effect. The maximum value of phase speed is always higher in presence of quantum effect and attains this value towards higher wavelength regime. Beyond these points they decrease sharply and then saturate. Phase speed of this mode is higher in presence of quantum effects towards long wavelength where as lower towards short wavelength regimes.

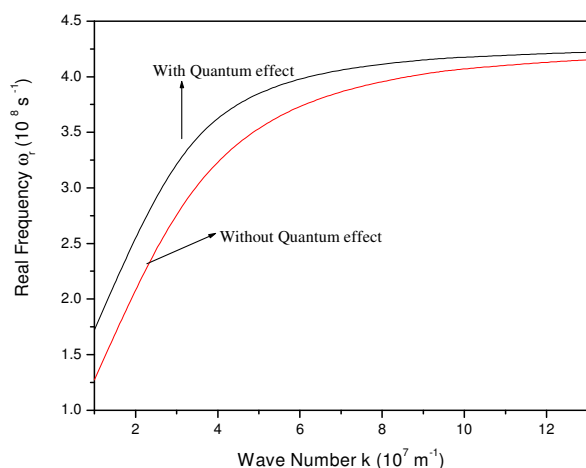


Figure-4
Variation of real frequency of fourth mode with wave number for slow electro-kinetic wave

Figure 4 infers that the phase speed of co-propagating fourth mode increases parabolically with increase in k in both the cases. Figure also depicts that quantum correction enhances the phase speed of the mode in the wavelength regime under study.

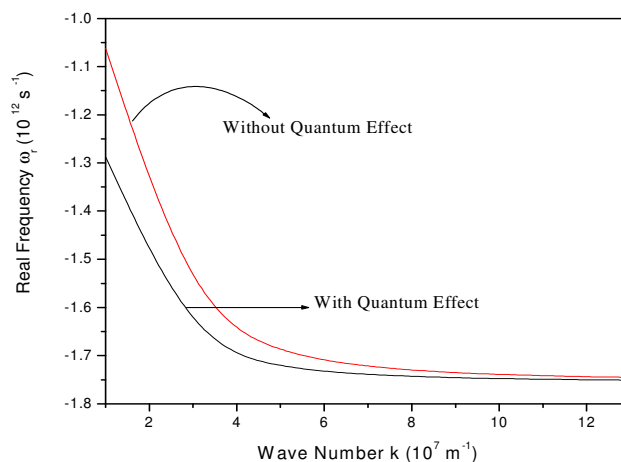


Figure-5
Variation of real frequency of first mode with wave number for fast electro-kinetic wave

The dispersion curves for all the possible modes in fast electro-kinetic wave limit in absence and presence of quantum effect via Fermi temperature and Bohm potential are depicted in figures 5-7. Figure 5 shows that the nature of variation of the first mode is identical in both the cases. The magnitude of phase constant first increase with increase in the value of k and then becomes independent of it towards higher value of k . This mode is found to be contra propagating. Phase constant of this mode is found higher in absence of quantum effect compared to that obtained when quantum effect is present.

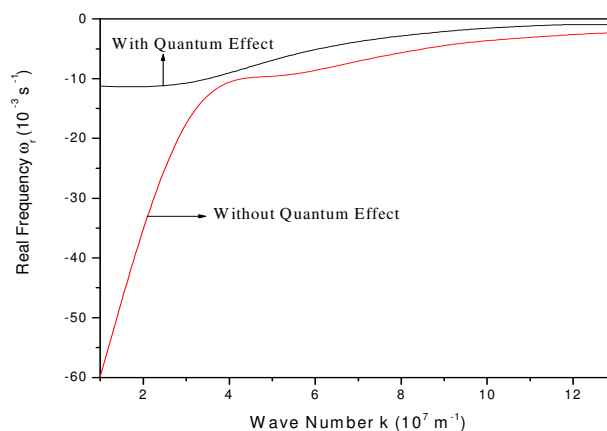


Figure-6
Variation of real frequency of second mode with wave number for fast electro-kinetic wave

Figure 6 displays the variation of real frequency ω_r of second mode with wave number k in absence and presence of quantum effect under fast electro-kinetic wave limit. The phase constant of this mode increases very slowly with k in presence of quantum effect and sharply when quantum effect is not present. In absence of quantum effect, phase constant becomes nearly independent towards higher wave number. Phase constant for this mode is higher in presence of quantum effect compared to that obtained in case when quantum effect is absence.

The third mode under fast electro-kinetic wave limit is found to be aperiodic ($\omega_r = 0$) in nature. Hence it is a dispersionless mode.

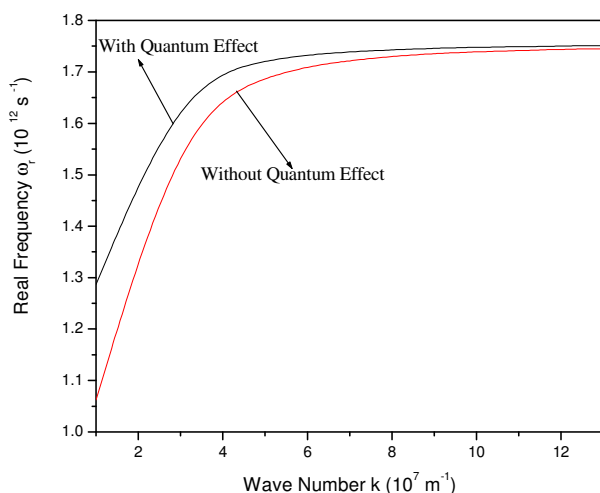


Figure-7

Variation of real frequency of fourth mode with wave number for fast electro-kinetic wave

Figure 7 illustrates the variation of real frequency ω_r of fourth mode with k in absence and presence of quantum effect for fast electro-kinetic mode. The fourth mode is found to be co-propagating in nature whose phase speed increases parabolically with k . The presence of quantum effect enhances the magnitude of phase speed.

Conclusion

On the basis of above study following conclusions may be drawn: In presence of quantum effect all modes get modified. The nature of variation of the phase constant of first mode is identical under slow as well as fast electro-kinetic wave regimes. The presence of quantum effect enhances the interaction. The phase speed is found to be enhanced in the presence of quantum effect under longer wavelength regime. Quantum correction enhances the phase constants of second and fourth modes under both the speed regimes. It is hoped that the present study may add substantially to the present knowledge of

wave interaction and may become useful in designing the semiconducting devices.

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