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# Log compound Rayleigh Distribution with Applications to the Reliability Data Sets

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## Abstract

*The main purpose of this research is to familiarize Log compound Rayleigh distribution using log transformation to the random variable of compound Rayleigh distribution. We obtain its basic reliability properties, order statistics and parameter estimation by method of likelihood estimates. Lastly we apply this proposed model to the two reliability data sets and compare its superiority with Gompertz, Nadarajah Haghighi, Weibull, Type II Topp Leone Inverse Rayleigh, Lomax and Log Dagum distributions*.

**Keywords:** Log compound Rayleigh, compound Rayleigh, ML estimates.

## Introduction

Slightly change in random variable cause a new invention of the distribution. Many authors introduced different random variables using log transformation, inverting variable, parametric change etc. Some examples are important to mention here, Domma[1] introduced Log-Dagum distribution by logarithmic transformation. The random variable of Dagum distribution has the pdf as

$$f_{Da}(z,\alpha,\lambda,\theta)=\alpha\lambda\theta z^{-\alpha-1}(1+\lambda z^{-\alpha})^{-\theta-1}$$

and its cdf is

$$F_{Da}(z,\alpha,\lambda,\theta)=(1+\lambda z^{-\alpha})^{-\theta}$$

Logarithmic transformation, $x=\log z$ generates a new variable so called log-Dagum random variable having pdf and cdf as

$$f_{LDa}(e^{x},\alpha,\lambda,\theta)=\alpha\lambda\theta e^{-\alpha x}(1+\lambda e^{-\alpha x})^{-\theta-1}$$

and

$$F_{LDa}(e^{x},\alpha,\lambda,\theta)=(1+\lambda e^{-\alpha x})^{-\theta}$$

Similarly Voda[2] suggested inverse Rayleigh distribution using inverting the random variable of Rayleigh distribution. In the same way Tahir et al[3]. Introduced inverted Nadarajah Haghighi distribution. From above mentioned suggestions we modify compound Rayleigh distribution using logarithmic transformation and obtain Log compound Rayleigh distribution.

## Log compound Rayleigh distribution

A continuous random variable $z$ called compound Rayleigh distribution having probability density function

$$f(z)=2\theta\lambda^{\theta}z(\lambda+z^{2})^{-(\theta+1)}$$

With cumulative distribution function

$$F(z)=1-\lambda^{\theta}(\lambda+z^{2})^{-\theta}$$

Then simple logarithmic transformation $x=\ln z$ we obtain Log compound Rayleigh random variable with pdf

$$f(x)=2\theta\lambda^{\theta}e^{2x}(\lambda+e^{2x})^{-(\theta+1)},\quad 0<x<\infty \tag{1}$$

and its cdf is

$$F(x)=1-\lambda^{\theta}(\lambda+e^{2x})^{-\theta} \tag{2}$$

## Some Reliability Expressions of LCR

With random variable $x$ Log Compound Rayleigh distribution has following Reliability measures

**Reliability function**

$$R(x)=\lambda^{\theta}(\lambda+e^{2x})^{-\theta}$$

**Failure rate function**

$$h(x)=\frac{2\theta\lambda^{\theta}e^{2x}(\lambda+e^{2x})^{-1}}{(\lambda+e^{2x})^{\theta}-\lambda^{\theta}}$$

**Cumulative Failure Rate**

$$H(x)=\ln\left(\lambda^{\theta}(\lambda+e^{2x})^{-\theta}\right)$$

**Reverse Hazard function**

$$\boldsymbol{r(x)}=\frac{2\theta\lambda^{\theta}e^{2x}(\lambda+e^{2x})^{-(\theta+1)}}{1-\lambda^{\theta}(\lambda+e^{2x})^{-\theta}}$$

**Odd function**

$$O(x)=\frac{(\lambda+e^{2x})^{\theta}-\lambda^{\theta}}{\lambda^{\theta}}$$

**Quantile function:** Inverting equation (2) we obtain quantile function of the distribution as

$$x_q = \frac{1}{2}\ln\left\{\lambda(1-u)^{-\frac{1}{\theta}} - \lambda\right\}$$

Where: $u \sim \text{uniform}(0,1)$: From this function we obtain Median ($x_{0.5}$), coefficient of skewness[4] ($\gamma_1$) and Moors kurtosis[5] ($\gamma_2$) by replacing values of $u$ as the table is presented below of suitable values of parameters.

**Order Statistics:** The pdf of r[th] order statistics with sample size $n$ order values is

$$f_{r,n}(x) = \frac{f(x)}{B(r, n-r+1)}\{F(x)\}^{r-1}[1-F(x)]^{n-r}$$

Using equation we obtain orders statistics for Log compound Rayleigh distribution is

$$f_{r,n}(x) = \frac{2\theta\lambda^{\theta}e^{2x}(\lambda+e^{2x})^{-(\theta+1)}}{B(r,n-r+1)} \times \left\{1-\lambda^{\theta}(\lambda+e^{2x})^{-\theta}\right\}^{r-1}\left\{\lambda^{\theta}(\lambda+e^{2x})^{-\theta}\right\}^{n-r}$$

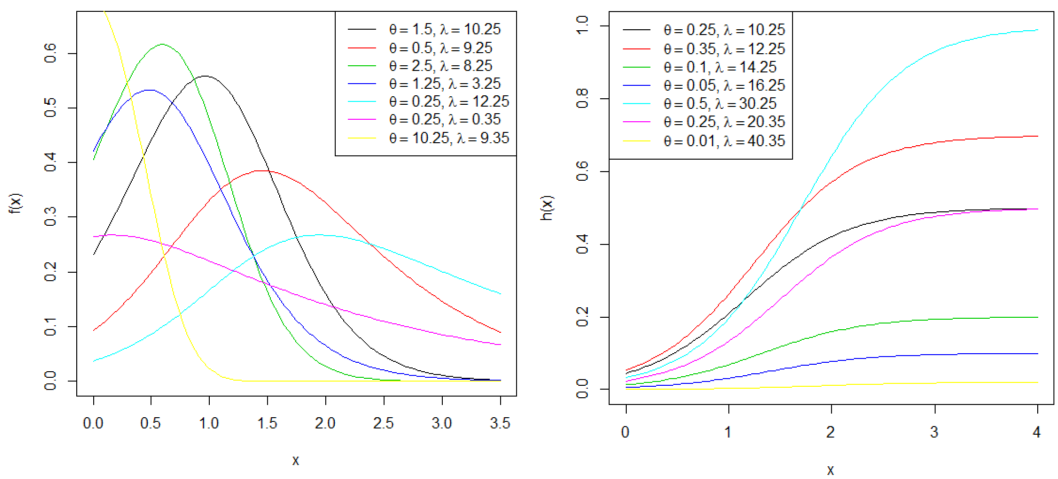


**Figure-1:** Pdf and hazard rate plots.

**Table-1:** Basic Statistical measures using assumed parametric values.

| $\lambda$ | $\theta$ | $x_{0.25}$ | $x_{0.5}$ | $x_{0.75}$ | $D = \frac{x_{0.75} - x_{0.25}}{2}$ | $\gamma_1$ | $\gamma_2$ |
|---|---|---|---|---|---|---|---|
| 1 | 0.1 | 1.256153 | 1.472219 | 1.831781 | 0.28781 | 0.249284 | 1.292289 |
| | 0.4 | 0.423649 | 0.693147 | 1.098612 | 0.33748 | 0.201443 | 1.251483 |
| | 0.7 | -0.05 | 0.3095 | 0.7753 | 0.41267 | 0.128696 | 1.22415 |
| | 0.9 | -0.3654 | 0.10034 | 0.61838 | 0.49191 | 0.0531261 | 1.24494 |
| 3 | 0.1 | 1.80546 | 2.02153 | 2.38109 | 0.28781 | 0.295010 | 1.292289 |
| | 0.4 | 0.97296 | 1.24245 | 1.64792 | 0.33748 | 0.249284 | 1.251483 |
| | 0.7 | 0.49926 | 0.85883 | 1.3246 | 0.41267 | 0.128695 | 1.22415 |
| | 0.9 | 0.18386 | 0.64964 | 1.16769 | 0.49191 | 0.0531261 | 1.24494 |
| 5 | 0.1 | 2.06087 | 2.27694 | 2.6365 | 0.28781 | 0.249284 | 1.292289 |
| | 0.4 | 1.22837 | 1.49787 | 1.90333 | 0.33748 | 0.201443 | 1.251483 |
| | 0.7 | 0.75468 | 1.11424 | 1.58002 | 0.41267 | 0.128695 | 1.22415 |
| | 0.9 | 0.43928 | 0.90505 | 1.4231 | 0.49191 | 0.0531261 | 1.24494 |

From above equation consider the factor

$$\{1-\lambda^{\theta}(\lambda+e^{2x})^{-\theta}\}^{r-1}=\sum_{k=0}^{\infty}\binom{r-1}{j}(-1)^{j}\{\lambda^{\theta}(\lambda+e^{2x})^{-\theta}\}^{j}$$

After some basic simplification

$$f_{r,n}(x)=\frac{2\theta\lambda^{\theta}e^{2x}(\lambda+e^{2x})^{-(\theta+1)}\sum_{k=0}^{\infty}\binom{r-1}{j}(-1)^{j}\{\lambda^{\theta}(\lambda+e^{2x})^{-\theta}\}^{j+n-r}}{B(r,n-r+1)}$$

## Parameter Estimation by Maximum likelihood

Suppose that we have n values of random variable$x$ whose values are $x_1,x_2,\dots,x_n$ obtained from the Log compound Rayleigh distribution then by definition of Log likelihood function defined as

$$l=\prod_{i=0}^{n}f(x_i)$$

$$l=2^{n}\theta^{n}\lambda^{n\theta}e^{2\sum_{i=1}^{n}x_i}\prod_{i=1}^{n}(\lambda+e^{2x})^{-(\theta+1)}$$

Further log likelihood function for the distribution is

$$L(\phi)=n\ln(2)+n\ln(\theta)+n\theta\ln(\lambda)+2\sum_{i=1}^{n}x_i-(\theta+1)\sum_{i=1}^{n}\ln(\lambda+e^{2x})$$

Differentiating the above equation with parameter space $\phi(\lambda,\theta)$ and we obtain estimate

$$\frac{\partial L(\phi)}{\partial\theta}=\frac{n}{\theta}+n\ln(\lambda)-\sum_{i=1}^{n}\ln(\lambda+e^{2x})=0$$

$$\frac{\partial L(\phi)}{\partial\lambda}=\frac{n\theta}{\lambda}-(\theta+1)\sum_{i=1}^{n}(\lambda+e^{2x})^{-1}=0$$

Further these equations are solved and there are no of software are available for this purpose. To obtain $100(1-\alpha)\%$ confidence interval we need Fisher information matrix and this matrix is easily obtained from the further derivatives

$$\frac{\partial^2 L(\phi)}{\partial\lambda^2}=\frac{-n\theta}{\lambda^2}-(\theta+1)\sum_{i=1}^{n}(\lambda+e^{2x})^{-1},\qquad \frac{\partial^2 L(\phi)}{\partial\theta^2}=\frac{-n}{\theta}$$

$$,\frac{\partial^2 L(\phi)}{\partial\lambda\partial\theta}=n\lambda^{-1}-\sum_{i=1}^{n}(\lambda+e^{2x})^{-1}$$

## Applications

In applied Statistics the application of proposed model is necessary and for this purpose we present two application of the Log compound Rayleigh distribution to the practical data sets. The analysis carried out with the help of R-Language, a useful language used by no of authors. In this analysis we use Adequacy model package of the language with "L-BFGS-B"[6]. Here we use $\tau$ (Cramer-von Misses statistic), $\varphi$ (Anderson Darlling statistic), $\omega$ (Kolmogorov Smirnov statistic), log-likelihood (Value) and other measures. We compare superiority of the proposed model with other models in which Gompertz (go), Lomax (lom), Type II Topp Leone Inverse Rayleigh[7] (TLIR), Nadarajah Haghighi (NH), Log Dagum (LDa) and Weibull (We) distribution are include having pdfs

$$f_{Gom}(x)=\alpha\theta e^{\alpha}e^{\theta x}e^{-\alpha e^{\theta x}},f_{Lom}(x)=\frac{\alpha}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)}$$

$$f_{NH}(x)=\alpha\theta(1+\theta x)^{\alpha-1}e^{(1-(1+\theta x)^{\alpha})},$$

$$f_{We}(x)=\frac{\alpha}{\theta}\left(\frac{x}{\theta}\right)^{\alpha-1}e^{\left(-\left(\frac{x}{\theta}\right)^{\alpha}\right)}$$

$$f_{TLIR}(x)=4\lambda\alpha^{2}x^{-3}e^{-2\left(\frac{\alpha}{x}\right)^{2}}\left\{1-e^{-2\left(\frac{\alpha}{x}\right)^{2}}\right\}^{\lambda-1}$$

Data sets used in this anylisis are presented Table-2.
Summary statistics of data set 1 and 2 are presented with the help of Tables

**Table-2:** Data sets and their descriptions.

| | |
|---|---|
| Data 1 referese to carbon fibers data[8] | 3.7,3.11,4.42,3.28,3.75,2.96,3.39,3.31,3.15,2.81,1.41,2.76,3.19,1.59,2.17,3.51,1.84,1.61, 1.57,1.89,2.74,3.27,2.41,3.09,2.43,2.53,2.81,3.31,2.35,2.77,2.6,4.91,1.57,2.00,1.17,2.17, 0.39,2.79,1.08,2.88,2.73,2.87,3.19,1.87,2.95,2.67,4.20,2.85,2.55,2.17,2.97,3.68,0.81,1.2 2,5.08,1.69,3.68,4.70,2.03,2.82,2.50,1.47,3.22,3.15,2.97,2.93,3.33,2.56,2.59,2.83,1.36,1. 84,5.56,1.12,2.48,1.25,2.48,2.03,1.61,2.05,3.60,3.11,1.69,4.90,3.39,3.22,2.55,3.56,2.38, 1.92,0.98,1.59,1.73,1.71,1.18,4.38,0.85,1.80,2.12,3.6. |
| Data 2 consits of 66 observations referes to carbon fibers dat[9] | 0.39,0.85,1.08,1.25,1.47,1.57,1.61,1.61,1.69,1.80,1.84,1.87,1.89,2.03,2.03,2.05,2.12, 2.35,2.41,2.43,2.48,2.50,2.53,2.55,2.55,2.56,2.59,2.67,2.73,2.74,2.79,2.81,2.82,2.85, 2.87,2.88,2.93,2.95,2.96,2.97,3.09,3.11,3.11,3.15,3.15,3.19,3.22,3.22,3.27,3.28,3.31, 3.31,3.33,3.39,3.39,3.56, 3.60,3.65,3.68,3.70,3.75,4.20,4.38,4.42,4.70,4.90. |

**Table-3:** Sumarry statistics of data set 1 and 2.

| Data | $n$ | Min | Max | 1st Q. | Median | 3rd Q | Mean | Skew | Kurt |
|---|---|---|---|---|---|---|---|---|---|
| Data 1 | 66 | 0.390 | 5.560 | 1.840 | 2.675 | 3.198 | 2.611 | 0.3985 | 0.24923 |
| Data 2 | 66 | 0.390 | 4.900 | 2.178 | 2.835 | 3.278 | 2.760 | -0.1345 | 0.337643 |

**Table-4:** Analysis resutl of data set 1.

| Distr | AIC | BIC | CAIC | HQIC | $\omega$(PV) |
|---|---|---|---|---|---|
| LCR | 288.6297 | 293.8401 | 288.7535 | 290.7385 | 0.0653 (**0.7872**) |
| Gom | 301.868 | 307.0783 | 301.9917 | 303.9767 | 0.1026 (0.2436) |
| Lom | 395.9796 | 401.19 | 396.1033 | 398.0884 | 0.3219 (0.000 ) |
| NH | 346.0316 | 351.2419 | 346.1553 | 348.1403 | 0.2914 (0.00000 ) |
| LDa | 289.3847 | 297.2002 | 289.6347 | 292.5478 | 0.071 (0.6939) |
| TLIR | 352.0966 | 357.3069 | 352.2203 | 354.2053 | 0.2013 (0.0006064) |

**Table-5:** Analysis resutl of data set 2**.**

| Distr | AIC | BIC | CAIC | HQIC | Value | $\tau$ | $\varphi$ | $\omega$<br>PV |
|---|---|---|---|---|---|---|---|---|
| LCR | 174.6628 | 179.0421 | 174.8532 | 176.3932 | 85.33138 | 0.0472 | 0.2870 | 0.0605<br>**0.969** |
| Gom | 180.1768 | 184.5561 | 180.3672 | 181.9072 | 88.08839 | 0.1099 | 0.7840 | 0.1118<br>0.3813 |
| Lom | 269.9898 | 274.3691 | 270.1803 | 271.7203 | 132.9949 | 0.2462 | 1.3335 | 0.3583<br>0.0000 |
| NH | 233.21 | 237.5893 | 233.4005 | 234.9405 | 114.605 | 0.1309 | 0.7064 | 0.3372<br>0.0000 |
| We | 176.1352 | 180.5145 | 176.3256 | 177.8656 | 86.06759 | 0.0929 | 0.5261 | 0.0823<br>0.7625 |
| TLIR | 254.4565 | 258.8358 | 254.647 | 256.187 | 125.2282 | 1.1269 | 6.2835 | 0.2805<br>0.0000 |

One can easily see that from above tables all comparison mesures of the LCR model are smaller than of other models. It is an indication that LCR is best fitted model to the above mentioned data sets. Now we provide maximum likelihood estimates of the distribution parameters with their standard errors.

**Table-6:** Parameter estimates of data set 1 and 2.

| Estimates of data set 1 | | | | | | |
|---|---|---|---|---|---|---|
| Distr | $\hat{\alpha}$ | $S.E(\hat{\alpha})$ | $\hat{\theta}$ | $S.E(\hat{\theta})$ | $\hat{\lambda}$ | $S.E(\hat{\lambda})$ |
| LCR | - | - | 0.7327408 | 0.1322708 | 105.9950 | 37.6693145 |
| Gom | 0.09910398 | 0.03133496 | 0.78870946 | 0.07733783 | - | - |
| Lom | 36459.47 | 3578.7009 | - | - | 95206.35 | 248.7369 |
| NH | 56.870690 | 32.7190567 | 0.0049062 | 0.0027849 | - | - |
| LDa | 1.445026 | 0.244130 | 2.090543 | 1.471212 | 15.323744 | 22.415384 |
| TLIR | 1.341494 | 0.09165265 | 1.150141 | 0.16745874 | - | - |
| Estimates of data set 2 | | | | | | |
| LCR | - | - | 1.123297 | 0.2978921 | 308.3292 | 136.90181 |
| Gom | 0.034941 | 0.0151527 | 1.06981390 | 0.11275259 | - | - |
| Lom | 53797.44 | 5932.083 | - | - | 148386.93 | 578.912 |
| NH | 56.14234 | 34.797333 | 0.00486291 | 0.00297228 | - | - |
| We | 3.441198 | 0.3309361 | 3.062256 | 0.1149394 | - | - |
| TLIR | 1.340201 | 0.1196565 | 1.041221 | 0.1894495 | - | - |

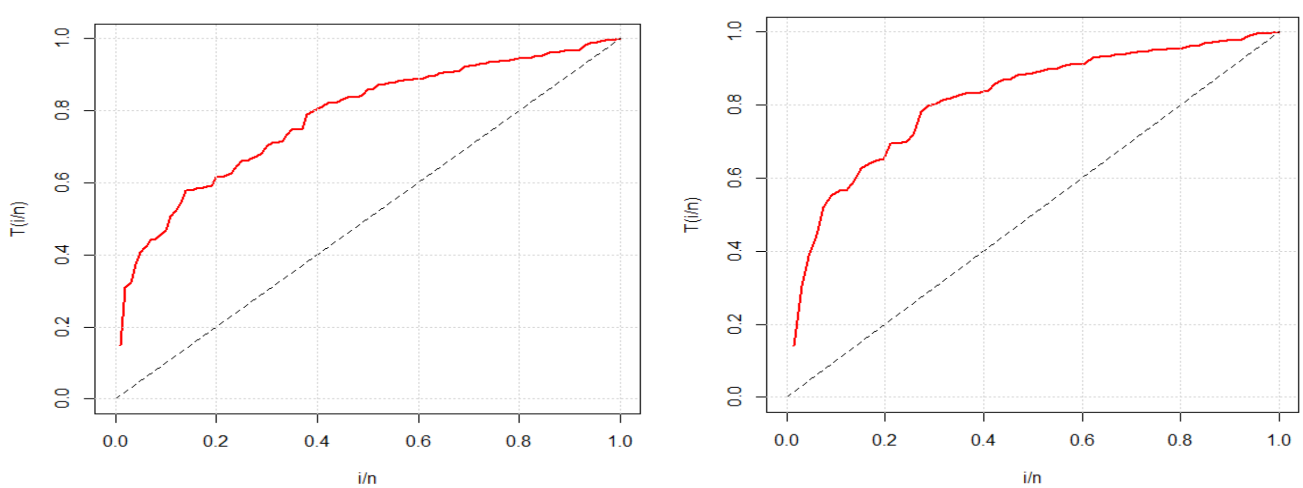


**Figure-2:** TTT plots of data set 1 and 2.

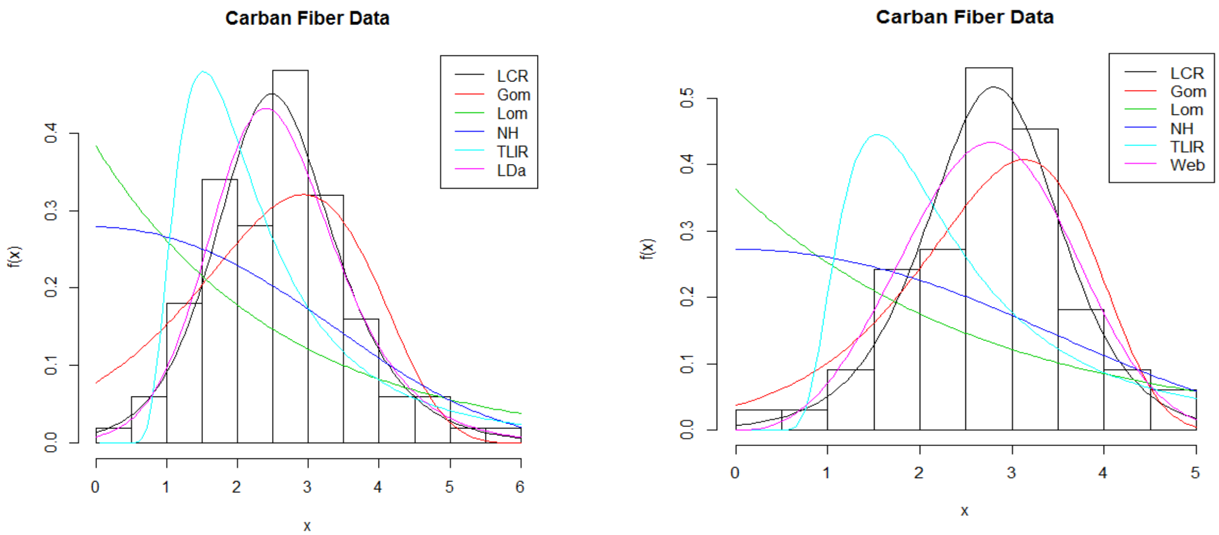


**Figure-3:** Fitted densities of data set 1 and 2.

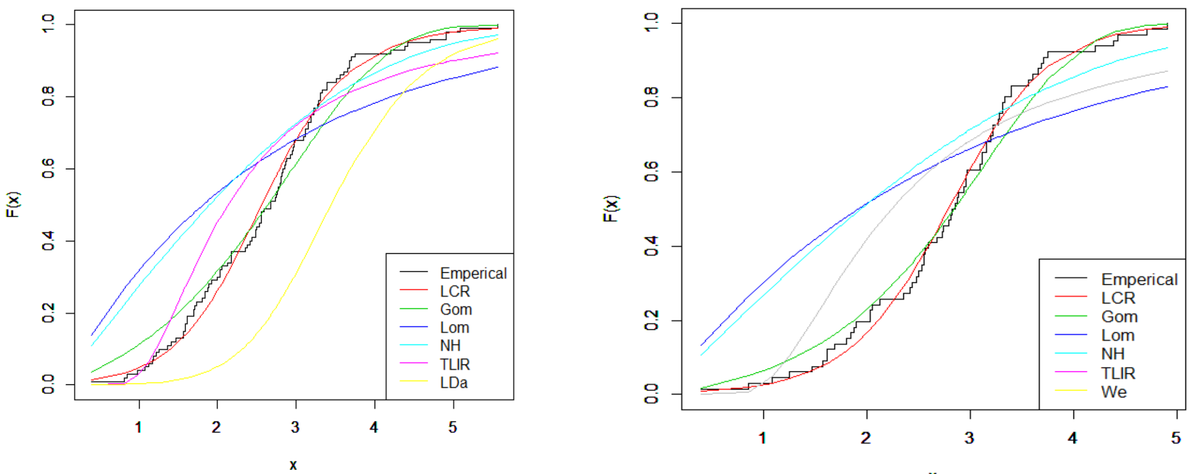


**Figure-4:** Estimated cdfs of data set 1 and 2.

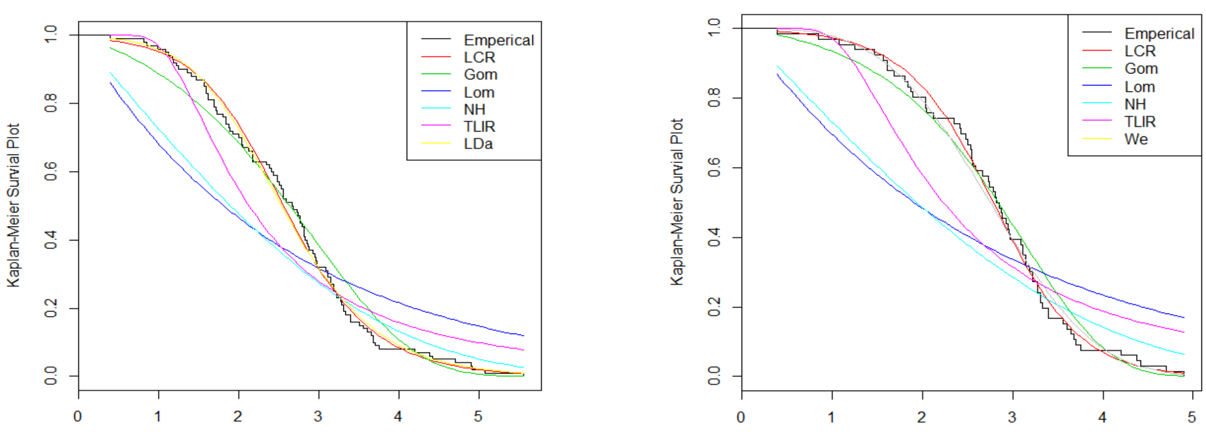


**Figure-5:** Survival plots of data set 1 and 2.

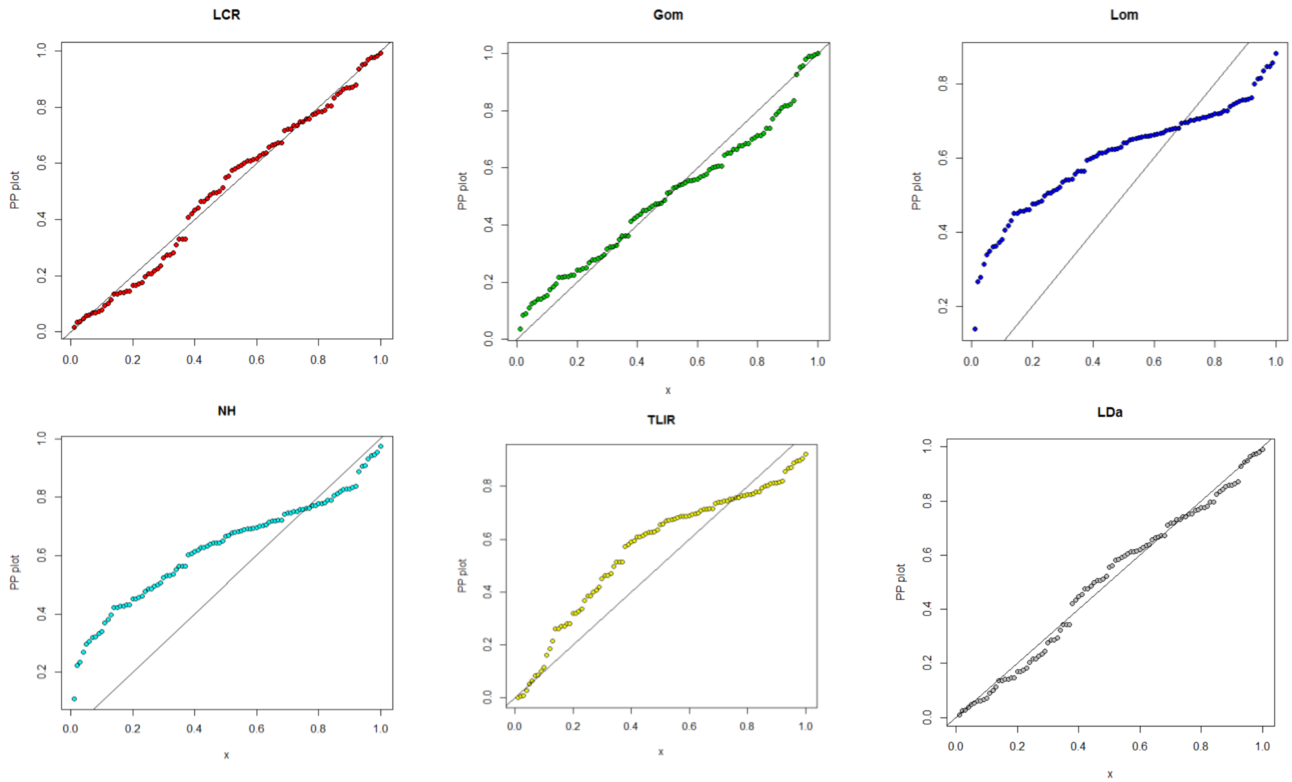


**Figure-6:** PP Plots of data set 1.

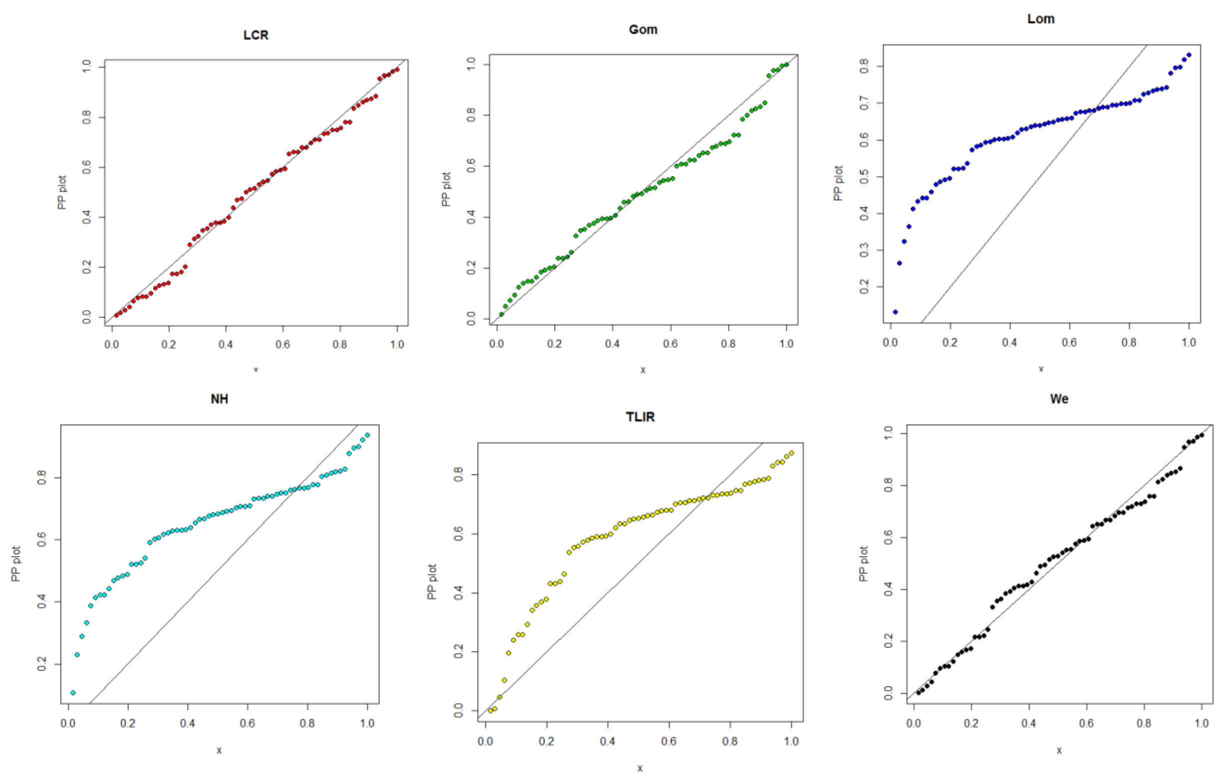


**Figure-7:** PP Plots of data set 2.

**Table-7:** Using Parametric values of ML estimate following measures of LCR.

| Data | $x_{0.25}$ | $x_{0.5}$ | $x_{0.75}$ | D | $\gamma_1$ | $\gamma_2$ |
|---|---|---|---|---|---|---|
| Data set 1 | 2.23226 | 2.60561 | 3.07915 | 0.42345 | 0.118314545 | 1.22372 |
| Data set 2 | 2.02721 | 2.74166 | 3.33577 | 0.65428 | -0.091954492 | 1.791118 |

## Conclusion

In this work we presented a new distribution so called Log compound Rayleigh distributon using basic log transformation to the compound Rayleigh random variable, after introducing this new model we obtained its neccessery reliablity properties. We provided its plots of pdf and failure rates with some parmaetric values of the proposed model. We obtained some basic statstical measures with assumed values of paraeters as well. We also obtained order statistics, ML estimates of the parameters, Lastly we apply this model to the two reliability data sets and compaere its superiority with Gompertz, Nadarajah Hagighi, Weibull, Type II Topp Leone Inverse Rayleigh, Lomax and Log Dagum distributions, we provided its necessary plots as well.

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