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Structural properties of zero-one-inflated negative-binomial crack distribution

B.P. Tlhaloganyang* and S.L. Nokwane
University of Botswana, Gaborone, Botswana
tlhaloganyangs@gmail.com

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Abstract

In modeling of discrete data, common discrete distributions sometimes fail to fit the observed data due to over dispersion resulting from excessive zero counts. In those situations, Zero Inflated models act as best alternative models to handle that kind of data. When the data contains both high frequencies of zero and one counts, Zero-Inflated models can also perform poorly. As a result, this paper aimed at developing a new alternative model titled the Zero-One Inflated Negative Binomial Crack (ZOINBCR) distribution that would account for excessive zeroes and ones in dataset. Properties and generalizations of ZOINBCR distribution were provided. Its parameters were also obtained based on the Method of Moments procedure and the Maximum Likelihood estimation procedure.

Keywords: Zero, One, Count, Negative Binomial distribution and Crack distribution.

Introduction

Count data is found in many fields such as manufacturing, health, transport and many more. Analysis of this type of data is usually carried out using Poisson and Negative Binomial distributions. These discrete distributions at times poorly fit the observed count data due to over dispersion. Over dispersion arises when the data is characterized lager variability that cannot be handled by the assumed distribution. In discussion about the consequences of over-dispersion Hinde[1], Cameron[2] and Cox[3] states that the standard errors may be underestimated and, consequently, the significance of individual parameters may be incorrectly assessed.

From previous studies, Negative Binomial frequently performs better than Poisson distribution in dispersion cases but it can also at times perform poorly in over-dispersion cases. Moreover, Negative Binomial Crack (NBCR) distribution[4] is another alternative distribution that can also provide a better fit when modelling over-dispersed data. NBCR distribution was built through compounding Negative Binomial distribution and Crack distribution. It is a general class that constitutes three special cases distributions being a mixture of Negative binomial with Inverse Gaussian (NBIG), Birnbaum Saunders (NBBS) and Length Biased Inverse Gaussian (NBLBIG).

In their study, Perumean-Chaney[5] recommended that when there is evidence of over-dispersion resulting from excess number of zeroes, standard models should be replaced with zero-inflated models. Excess number of zeroes increases the sample size on the other side decreasing the total sum of observed data. This results in reduction in the mean value and an increase in dispersion index (variance to mean ratio). This violates the Poisson theoretical assumption of mean to variance equality. Excess number of zeroes in count data is experienced in many fields. Production defects in manufacturing are usually modelled using Poisson, which sometimes fails due to excess number of zeroes[6]. Moreover, traffic and motor vehicle crashes studies, entomology studies, agricultural studies and many more studies usually exhibit excess number of zero counts[7].

In literature, researchers are striving to seek for alternative distributions to handle zero-inflated data. Modification of standard distributions seems to be providing a better fit under the condition over dispersion as a result of zero inflation. This modification included adding an extra parameter that specifically handles observed zero counts under predicted by the assumed standard distribution. As a result of this additional parameter, new distributions such as Zero Inflated Power Series[18] (ZIPS) distributions were developed. Poisson, Binomial, Negative-Binomial, Geometric and Logarithmic Series distributions are members of this class. ZIPS distributions are frequently applied in many fields that work with count data containing excess number of zeroes. The importance of zero inflated models became significant when Lambert[6] applied Zero Inflated Poisson (ZIP) distribution in modelling number of production defects in manufacturing process. Moreover, Hall[10] and Hall and Berenhaut[11] further applied Zero Inflated Poisson/ Binomial distributions to model real life count data and these distributions provided a far much better fit when compared to Poisson and Binomial distributions. For further improvement of fit, Deng and Zhang[9] decided to model the same data using another distribution modified to handle excessive one counts termed as Zero-One-Inflated Binomial (ZOIB) distribution. Their results[9] marked ZOIB distribution as the best fitting model when compared to other competing models that were considered in the study. Recently, other alternative count data distributions were proposed as a way to provide alternatives to count data with excessive zero counts as different datasets adhere to different datasets. Some of those distributions are Zero Inflated Conway Maxwell Poisson (ZICMP)[12], Zero Inflated Inverse Trinomial Distribution (ZIIT)[13], Zero Inflated Negative Binomial[14] Sushila (ZINBS)and Generalized Negative Binomial[15] Shanker. Moreover, Saengthong[16] extended NBCR to Zero Inflated Negative Binomial Crack (ZINBCR) distribution through inclusion of the zero-inflation parameter. In application to the data with high frequency of zero counts, CR provided a superb fit a real life dataset when it was compared to other competing distributions.

Based on the strategy of inclusion of extra parameter, Alshkaki[17] extended the ZIPS distributions through addition of the one-inflation parameter provide alternative distributions for a count data that is found to be characterized by excessive zero and one counts. These new distributions were then termed as Zero One Inflated Power Series (ZOIPS) distributions. Poisson, Binomial, Negative-Binomial, Geometric and Logarithmic Series distributions were its limiting cases under different conditions. Some structural properties of this general class were also provided along with parameter estimates that were derived based on Maximum Likelihood and Method of Moments procedures. In application, Zero-One Inflated Geometric Distribution (ZOIGD) was considered in analysis of a real life data that was previously studied by Edwin[18]. Results were compared to the one obtained by Edwin[18] and it was found that ZOIGD provided a better fit than Zero Inflated Geometric Distribution (ZIGD). Furthermore, Alshkaki[19-21] also provided mathematical properties of Zero-One Inflated Poisson (ZOIPD), Zero-One Inflated Negative Binomial (ZOINB), Zero-One Inflated Logarithmic Series (ZOILSD) and Zero-One Inflated Binomial (ZOIBD) distributions. Another real life application was carried out using ZOIPD distribution which performed better than the ZIP distribution.

For that reason, this paper also aimed at developing an alternative distribution that is expected to handle variability from excessive zeros and ones in the data. Thus, in this paper, an extra parameter was added in ZINBCR distribution[16] to make it more suitable for a real life count dataset characterized by excessive zeroes and ones. As a result, this extended ZINB-CR distribution was termed as "Zero-One Inflated Negative Binomial- Crack (ZOINBCR)" distribution. This model was expected to perform effectively in count data with a lot of zeroes and ones that cannot be handled by zero inflated distributions. ZOINBCR is made up three distributions mainly depending on whether a count is zero or one in a dataset.

The paper was organized as follows: The next section portrayed properties and generalizations of NBCR and ZINBCR distributions. The section continued with introduction of the new distribution ZOINBCR together with provision of its mathematical properties such as the mean and the variance. Its parameters were estimated using Maximum Likelihood and Method of Moments estimation procedures. Last section deals with conclusion that included future research plans.

Methodology

Negative-Binomial Crack distribution (NBCR): The basic NBCR probability function with a random variable $y$is defined as

$$P(Y=y) = \begin{cases} \binom{r+y-1}{y}\sum_{j=0}^{y}\binom{y}{j}(-1)^j \dfrac{exp\,\{\lambda(1-\sqrt{1+2\theta r+2\theta j})\}}{\sqrt{1+2\theta r+2\theta j}} \\ [1-\gamma(1-\sqrt{1+2\theta r+2\theta j})] & for\ \ y=0,1,2,\ldots, \end{cases}$$

From the p.m.f, the corresponding factorial moment is defined as:

$$\mu_{[k]}(Y) = \frac{\Gamma(r+k)}{\Gamma(r)}\left[\sum_{j=0}^{k}\binom{k}{j}(-1)^j \frac{exp\left[\lambda\left(1-\sqrt{1-2\theta(k-j)}\right)\right]}{\sqrt{1-2\theta(k-j)}} \times \left[1-\gamma\left(1-\sqrt{1-2\theta(k-j)}\right)\right]\right] \quad (1)$$

where $r,\lambda,\theta > 0$ and $0 \le \gamma \le 1$. For proofs and other properties of this p.m.f see Saengthong[4]. NBCR distribution was obtained through mixing Negative Binomial(NB) and Crack distribution assuming that the parameter p of NB is exponentially linked with another parameter that follows Crack distribution.

Special cases of NBCR distribution: From the p.m.f of NBCR distribution, when: i. $\gamma = 1, \lambda = \frac{\psi}{\mu}$ and $\theta = \frac{\mu^2}{\psi}$, Z follows NBIG distribution with parameters r, μ and ψ. ii. $\gamma = \frac{1}{2}$, Z follows NBBS distribution with parameters r, λ and θ. iii. $\gamma = 0$, Z follows NBLBIG distribution with parameters r, λ and θ.

Zero-Inflated Negative-Binomial Crack (ZINBCR) distribution: The ZINBCR probability distributionis presented as

$$P(Y=y) = \begin{cases} \alpha + (1-\alpha)\dfrac{exp\,[\lambda(1-\sqrt{1+2\theta r})]}{\sqrt{1+2\theta r}}[1-\gamma(1-\sqrt{1+2\theta r})] & for\ \ y=0 \\ (1-\alpha)\sum_{j=0}^{y}\left[\binom{r+y-1}{y}\binom{y}{j}(-1)^j \dfrac{[1-\gamma(1-\sqrt{1+2\theta r+2\theta j})]exp\,[\lambda(1-\sqrt{1+2\theta r+2\theta j})]}{\sqrt{1+2\theta r+2\theta j}}\right] \\ & for\ \ y=1,2,3,\ldots, \end{cases}$$

where $\alpha(0 < \alpha < 1)$ is the zero inflation parameter that handles zeros when the Negative-Binomial underpredict them. The mean, variance and other properties of this distribution can be seen in a research study by Saengthong[16].

Zero-One-Inflated Negative-Binomial Crack distribution (ZOINBCR): In the data set that contains excess number of zero and one counts, let there be a mixture that assigns a mass of α to the extra zeros and a mass of β to the extra ones. Furthermore, a mass of $(1-\alpha-\beta)$ assigned to the NBCR distribution leads to Zero-One-Inflated Negative-Binomial Crack (ZOINBCR) distribution. If we let a random variable Y to follow ZOINBCR distribution, its corresponding p.m.f is presented as

$$\P(Y=y) = \begin{cases} \alpha + (1-\alpha-\beta)\left[\frac{exp\,[\lambda(1-\sqrt{1+2\theta r})]}{\sqrt{1+2\theta r}}\left[1-\gamma(1-\sqrt{1+2\theta r})\right]\right] & for\ y=0 \\ \beta + (1-\alpha-\beta)\sum_{j=0}^{1}\left[(-1)^j \frac{r\,exp\,[\lambda(1-\sqrt{1+2\theta r+2\theta j})]}{\sqrt{1+2\theta r+2\theta j}}\right] \times\left[1-\gamma(1-\sqrt{1+2\theta(r+j)})\right] & for\ y=1 \\ (1-\alpha-\beta)\sum_{j=0}^{y}\left[\binom{r+y-1}{y}\binom{y}{j}(-1)^j \frac{exp\,[\lambda(1-\sqrt{1+2\theta r+2\theta j})]}{\sqrt{1+2\theta(r+j)}}\right] \times\left[1-\gamma(1-\sqrt{1+2\theta r+2\theta j})\right] & for\ y=2,3,4,\ldots, \end{cases} \quad (2)$$

where $0<\alpha<1$, $0<\beta<1$ and $0<\alpha+\beta<1$. If $\beta\to 0$, equation (2) reduces to ZINB-CR distribution. If Y~ZOINB-CR$(r,\lambda,\theta,\gamma,\lambda,\alpha,\beta)$, some basic properties are:

A **factorial moment** for this distribution is presented as:

$$\mu_{[k]}(Y) = \beta + \sum_{y=1}^{\infty}\sum_{j=0}^{y}\left[(-1)^j y^k\binom{y}{j}\binom{r+y-1}{y}\frac{(1-\alpha-\beta)exp\,[\lambda(1-\sqrt{1+2\theta r+2\theta j})]}{\sqrt{1+2\theta r+2\theta j}} \times\left[1-\gamma(1-\sqrt{1+2\theta r+2\theta j})\right]\right] \quad for\ k=1,2,\ldots,$$

b) From the factorial moment, the **Mean, Variance**and the **Moment generating function** of this distribution follows respectively as:

$E(Y)=\beta+r(1-\alpha-\beta)\left[\frac{(1-\gamma(1-\delta))\,exp\,(\lambda(1-\delta))}{\delta}-1\right]$, where $\delta=\sqrt{1-2\theta}$

$Var(Y)=\beta(1-r)+r-\alpha$

$$M_Y(t) = \alpha+\beta+\frac{(1-\alpha-\beta)exp\,[\lambda(1-\sqrt{1+2\theta r})][1-\gamma(1-\sqrt{1+2\theta r})]}{\sqrt{1+2\theta r}} + \sum_{y=1}^{\infty}\sum_{j=0}^{y}\left[\binom{r+y-1}{y}\binom{y}{j}(-1)^j\frac{[1-\gamma-\gamma\sqrt{1+2\theta r+2\theta j}]exp\,[\lambda t y(1-\sqrt{1+2\theta r+2\theta j})]}{\sqrt{1+2\theta r+2\theta j}}\right]$$

Parameter Estimation: This part entails the parameter estimation of $ZOINB-CR(r,\lambda,\theta,\gamma,\lambda,\alpha,\beta)$ distribution based on two methods; Moments procedure and Maximum Likelihood procedure.

Method of Moments procedure: Parameter estimates were obtained through equating the moments computed from the sample about zero to corresponding population (distribution) moment. The first six moments of the ZOINB-CR distribution corresponding to the six parameters are given as:

$$E(Y)=\beta+r[1-\alpha-\beta]\left[\frac{(1-\gamma(1-\delta))\,exp\,(\lambda(1-\delta))}{\delta}-1\right]$$

$$E(Y^2) = \beta+r(1-\alpha-\beta)(r+1)\left[\frac{(1-\gamma(1-\zeta))\,exp\,(\lambda(1-\zeta))}{\zeta}-\frac{2(1-\gamma(1-\delta))\,exp\,(\lambda(1-\delta))}{\delta}+1\right]$$

$$E(Y^3) = \beta+(1-\alpha-\beta)\left[\frac{(r^3+3r^2+2r)(1-\gamma(1-\vartheta))\,exp\,(-\lambda\vartheta)}{\vartheta} - \frac{(3r^3+6r^2+3r)(1-\gamma(1-\zeta))\,exp\,(-\lambda\zeta)}{\zeta}+\frac{(3r^3+3r^2+r)(1-\gamma(1-\delta))\,exp\,(-\lambda\delta)}{\delta}-r^3\right]$$

$$E(Y^4)=\beta+(1-\alpha-\beta)\left[\frac{(4r^4+18r^3+26r^2+12r)(1-\gamma(1-\vartheta))\,exp\,(\lambda(1-\vartheta))}{\vartheta} + \frac{(6r^4+18r^3+19r^2+7r)(1-\gamma(1-\zeta))\,exp\,(\lambda(1-\zeta))}{\zeta} - \frac{(4r^4+6r^3+4r^2+r)(1-\gamma(1-\delta))\,exp\,(\lambda(1-\delta))}{\delta}+r^4\right]$$

$$E(Y^5)=\beta+(1-\alpha-\beta)\left[\frac{(r^5+10r^4+35r^3+50r^2+24r)(1-\gamma(1-\upsilon))\,exp\,(\lambda(1-\upsilon))}{\upsilon} - \frac{(5r^5+40r^4+115r^3+140r^2+60r)(1-\gamma(1-\kappa))\,exp\,(\lambda(1-\kappa))}{\kappa} + \frac{(10r^5+60r^4+135r^3+135r^2+50r)(1-\gamma(1-\vartheta))\,exp\,(\lambda(1-\vartheta))}{\vartheta} - \frac{(10r^5+40r^4+65r^3+50r^2+15r)(1-\gamma(1-\zeta))\,exp\,(\lambda(1-\zeta))}{\zeta} + \frac{(5r^5+10r^4+10r^3+5r^2+r)(1-\gamma(1-\delta))\,exp\,(\lambda(1-\delta))}{\delta}-r^5\right]$$

$$E(Y^6)=\beta+(1-\alpha-\beta)\frac{\Gamma(r+6)}{\Gamma(r)}\left[\frac{(1-\gamma(1-\rho))\,exp\,(\lambda(1-\rho))}{\rho}-\frac{6(1-\gamma(1-\upsilon))\,exp\,(\lambda(1-\upsilon))}{\upsilon} + \frac{15(1-\gamma(1-\kappa))\,exp\,(\lambda(1-\kappa))}{\kappa}-\frac{20(1-\gamma(1-\vartheta))\,exp\,(\lambda(1-\vartheta))}{\vartheta} + \frac{15(1-\gamma(1-\zeta))\,exp\,(\lambda(1-\zeta))}{\zeta}-\frac{6(1-\gamma(1-\delta))\,exp\,(\lambda(1-\delta))}{\delta}+1\right]$$

where $\delta=\sqrt{1-2\theta}$,$\zeta=\sqrt{1-4\theta}$,$\vartheta=\sqrt{1-6\theta}$,$\kappa=\sqrt{1-8\theta}$, $\upsilon=\sqrt{1-10\theta}$ and $\rho=\sqrt{1-12\theta}$. Let y_i(i = 1,2, ..., n) be a sample from ZOINB-CR and let

$$m_k^{'}=\frac{1}{n}\sum_{i=1}^{n}y_i^k,\quad k=1,2,\ldots,6$$

be their sample moments about the origin. From Central Limit Theorem, we know that moments computed from the sample provides reliable estimates of population moments. We obtain moment estimators by solving equations $m_1=E(Y)$, $m_2=E(Y^2)$, $m_3=E(Y^3)$, $m_4=E(Y^4)$, $m_5=E(Y^5)$ and $m_6=E(Y^6)$.

Maximum Likelihood procedure: Let y_i(i = 1,2, ..., n), represent a known sample of size n and moreover, let

$$C_j=\sqrt{1+2\theta r+2\theta j},\quad j=0,1,2,\ldots,y_i$$

$$P=\alpha+\frac{\left[(1-\alpha-\beta)\left(1-\gamma(1-C_j)\right)\right]exp\,[\lambda(1-C_j)]}{C_j} \quad \text{for } y_i=0$$

$$Q=\beta+(r-\alpha r-\beta r)\sum_{j=0}^{1}\left[(-1)^j\frac{[1-\gamma(1-C_j)]exp\,[\lambda(1-C_j)]}{C_j}\right] \quad \text{for } y_i=1$$

$$R=(1-\alpha-\beta)\binom{r+y_i-1}{y_i}\sum_{j=0}^{y}\left[\binom{y_i}{j}(-1)^j\frac{[1-\gamma(1-C_j)]exp\,[\lambda(1-C_j)]}{C_j}\right] \quad \text{for } y_i=2,3,4,\ldots,$$

$$I_0=\begin{cases}1 \text{ if } y_i=0\\0 \text{ otherwise}\end{cases},\quad I_1=\begin{cases}1 \text{ if } y_i=1\\0 \text{ otherwise}\end{cases} \text{ and } I_2=\begin{cases}1 \text{ if } y_i\ge 2\\0 \text{ otherwise}\end{cases}$$

From the above, Log-likelihood function of $ZOINB-CR(r,\lambda,\theta,\gamma,\lambda,\alpha,\beta)$ distribution is defined as:

$$L=\prod_{i=1}^{n}\{I_0P+I_1Q+I_2R\}$$

$$log\ L=\sum_{i=1}^{n}\ log\ \{I_0P+I_1Q+I_2R\}$$

Differentiating log L w.r.t $r,\lambda,\theta,\gamma,\lambda,\alpha,\beta$; we get:

$$\frac{\partial\ log\ L}{\partial r}=\sum_{i=1}^{n}\frac{(1-\alpha-\beta)}{I_0P+I_1Q+I_2R}\left\{I_0\left[\frac{\theta\ \left[\gamma C_j-[1-\gamma(1-C_j)][\lambda C_j+1]\right]exp\left(\lambda(1-C_j)\right)}{C_j^3}\right]\right.$$

$$+I_1\left[\sum_{j=0}^{y_i}(-1)^j\frac{\left[[1-\gamma(1-C_j)][C_j{}^2-\theta r]+r\theta C_j\left[\gamma-\lambda[1-\gamma(1-C_j)]\right]\right]\ exp\ \left(\lambda(1-C_j)\right)}{C_j^3}\right]$$

$$\left.+I_2\left[\binom{r+y_i-1}{y_i}\sum_{j=0}^{y_i}\binom{y_i}{j}(-1)^j\frac{\left[[1-\gamma(1-C_j)][C_j^2(\psi(r+z_i)-\psi(r))-\theta(\lambda C_j+1]+\gamma\theta C_j\right]exp\ \left(\lambda(1-C_j)\right)}{C_j^3}\right]\right\}$$

where $\psi(k)=\frac{\partial\ log\ \Gamma(k)}{\partial k}=\frac{\Gamma^{'}(k)}{\Gamma(k)}$ is the digamma function which can easily be computed by R software.

$$\frac{\partial\ log\ L}{\partial\lambda}=\sum_{i=1}^{n}\frac{(1-\alpha-\beta)}{I_0P+I_1Q+I_2R}\left\{I_0\left[\frac{[1-\gamma(1-C_j)]\ exp\ \left(\lambda(1-C_j)\right)(1-C_j)}{C_j}\right]\right.$$

$$+I_1\left[\sum_{j=0}^{y_i}(-1)^j\frac{r\ exp\ \left(\lambda(1-C_j)\right)(1-C_j)}{C_j}\right]$$

$$\left.+I_2\left[\binom{r+y_i-1}{y_i}\sum_{j=0}^{y_i}\binom{y_i}{j}(-1)^j\frac{[1-\gamma(1-C_j)]\ exp\ \left(\lambda(1-C_j)\right)(1-C_j)}{C_j}\right]\right\}$$

$$\frac{\partial\ log\ L}{\partial\theta}=\sum_{i=1}^{n}\frac{(1-\alpha-\beta)}{I_0P+I_1Q+I_2R}\left\{I_0\left[\frac{r\ exp\ \left(\lambda(1-C_j)\right)}{C_j^2}\left(\gamma-\frac{[1-\gamma(1-C_j)][\lambda C_j-1]}{C_j}\right)\right]\right.$$

$$+I_1\left[\sum_{j=0}^{y_i}(-1)^j\frac{\mathrm{r}(r+j)\ exp\ \left(\lambda(1-C_j)\right)}{C_j^2}\left(\gamma-\frac{[1-\gamma(1-C_j)][\lambda C_j-1]}{C_j}\right)\right]$$

$$\left.+I_2\left[\binom{r+y_i-1}{y_i}\sum_{j=0}^{y_i}\binom{y_i}{j}(-1)^j\frac{(r+j)\ exp\ \left(\lambda(1-C_j)\right)}{C_j^2}\left(\gamma-\frac{[1-\gamma(1-C_j)][\lambda C_j-1]}{C_j}\right)\right]\right\}$$

$$\frac{\partial\ log\ L}{\partial\gamma}=\sum_{i=1}^{n}\frac{(1-\alpha-\beta)}{I_0P+I_1Q+I_2R}\left\{I_0\left[\frac{exp\ \left(\lambda(1-C_j)\right)(C_j-1)}{C_j}\right]\right.$$

$$+I_1\left[\sum_{j=0}^{y_i}(-1)^{j+1}\frac{r\ (1-C_j)exp\left(\lambda(1-C_j)\right)}{C_j}\right]$$

$$\left.+I_2\left[\binom{r+y_i-1}{y_i}\sum_{j=0}^{y_i}\binom{y_i}{j}(-1)^{j+1}\frac{exp\ \left(\lambda(1-C_j)\right)(1-C_j)}{C_j}\right]\right\}$$

$$\frac{\partial\ log\ L}{\partial\alpha}=\sum_{i=1}^{n}\frac{1}{I_0P+I_1Q+I_2R}\left\{I_0\left[1-\frac{[1-\gamma(1-C_j)]exp\ \left(\lambda(1-C_j)\right)}{C_j}\right]\right.$$

$$+I_1\left[\sum_{j=0}^{y_i}(-1)^{j+1}\frac{r[1-\gamma(1-C_j)]exp\ \left(\lambda(1-C_j)\right)}{C_j}\right]$$

$$\left.+I_2\left[\binom{r+y_i-1}{y_i}\sum_{j=0}^{y_i}\binom{y_i}{j}(-1)^{j+1}\frac{[1-\gamma(1-C_j)]exp\ \left(\lambda(1-C_j)\right)(1-C_j)}{C_j}\right]\right\}$$

$$\frac{\partial\ log\ L}{\partial\beta}=\sum_{i=1}^{n}\frac{1}{I_0P+I_1Q+I_2R}\left\{-I_0\left[\frac{[1-\gamma(1-C_j)]\ exp\ \left(\lambda(1-C_j)\right)}{C_j}\right]\right.$$

$$+I_1\left[1+\sum_{j=0}^{y_i}(-1)^{j+1}\frac{r\ exp\ \left(\lambda(1-C_j)\right)}{C_j}\right]$$

$$\left.+I_2\left[\binom{r+y_i-1}{y_i}\sum_{j=0}^{y_i}\binom{y_i}{j}(-1)^{j+1}\frac{[1-\gamma(1-C_j)]exp\ \left(\lambda(1-C_j)\right)(1-C_j)}{C_j}\right]\right\}$$

Conclusion

We introduced a new distribution called ZOINB-CR distribution which was obtained through additional parameter which accounts for excess number of ones in count data. The mean, variance, the factorial moment and the mgf of this distribution was provided. Parameter estimation was also implanted using the Moments procedure and the maximum likelihood procedure. For future work, usefulness of this distribution would be investigated using simulated and real data set with a higher number of zeros and ones.

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