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Optimal solutions to a stochastic knapsack problem with contagious distributional capacity

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Abstract

The stochastic knapsack problem has continued to generate interest in many areas especially in the area of resource allocation. Of the two forms of the stochastic knapsack problem, the static knapsack problem has been studied over the years by considering the distribution of one of the parameters of a knapsack such as weight, capacity, profit, etc. however, optimal solutions for parameters having contagious distributions have not been considered. This study therefore seeks to obtain optimal solutions to a stochastic knapsack problem following a contagion capacity of Poisson and Gamma distribution. The simplex method of Witchakul et al. (2007) was adopted in developing an algorithm as well as a Monte Carlo and Heuristics algorithms. The result shows optimal solutions were gotten for up to 75,000 variables and the Heuristics algorithm performed much better than the main algorithm and Monte Carlo algorithms respectively.

Keywords: Stochastic Knapsack Problem, Contagious distribution, Optimal Solutions, Knapsack problems, Optimal Solutions.

Introduction

Stochastic knapsack problems is an aspect of the knapsack problem which are often used to model real life situations involving problems such as resource allocation, cargo loading, assignment problems, investment planning, etc. where allocation of scarce or limited resources is done to minimize risk or costs and maximize profit or investment returns. The stochastic knapsack problem is a non-deterministic knapsack problem whereby the parameters of a knapsack problem are known with uncertainty. These parameters such as weight, profit, or capacity are therefore treated as random variables with certain distributions.

Over the years, the stochastic knapsack problem has been studied based on two concepts, the static stochastic knapsack problem and the dynamic stochastic knapsack problem as shown in the literature. Studies on the dynamic stochastic knapsack problem includes Stewart[1], Mendelson *et al.*[2], Kelywegt and Papastavrou[3], Dean *et al.*[4] and Levin and Vainer[5]. For the static stochastic knapsack problem, Steinberg and Parks[6], Sniedovich[7], Cohn and Barnhart[8], Witchakul*et al.*[9], Agrali and Geunes[10], Merzifonlouglu*et al.*[11] and Chen and Ross[12] all studied the static stochastic knapsack problem in different areas and concepts especially with respect to the distributions of the parameters of the knapsack problem.

Steinberg and Parks[6], Sniedovich[7], Henig[13] all considered the situation whereby the profits and weights are uncertain. Caraway *et al.*[14] examined the stochastic knapsack problem when the profits are normally and independently distributed random variables, Goel and Indyk[15] considered the case of the weights following Poisson, exponential and Bernoulli distributions respectively while Kosuch and Lisser[16,17] focused on normally distributed weights. Witchakul *et al.*[9,18] proposed methods of solving the stochastic knapsack problem when the capacity follows a certain distribution. Merzifonlouglu *et al.*[11]followed it up by considering normally distributed items and random capacity while Chen and Ross[12] considered an exponentially distributed capacity.

Recently, the distributions of the parameters of a stochastic knapsack problem has been examined using the multiplicative and additive contagious distributions of Mood *et al.*[19]. Etuk *et al.*[20] and Akpan and Etuk[21] presented a stochastic knapsack problem where the capacity is a multiplicative and additive contagious distribution of Poisson and exponential distribution respectively, while Akpan *et al.*[22] considered when the weight is an additive contagion of Gamma and Exponential distributions. These studies were basically focused on the graphical properties of the mixture as well as its first and second moments. However, optimal solutions have not been obtained for a stochastic knapsack problem with contagious distributions.

This study therefore considers a stochastic knapsack problem with the capacity following a multiplicative contagion of Poisson and Gamma distributions. The study adopts the simplex method approach of Witchakul *et al.*[18] in solving the problem and the optimal solutions obtained will be compared the that of Witchakul *et al.*[18].

The rest of the paper is organized as follows. Section 2 deals with the contagious distribution of Poisson and Gamma and the methodology using the simplex method approach. Section 3 will show the results and discussions. Section 4 will then be the conclusion of the study.

Methodology

In this section, the mixture of Poisson and Gamma distribution is shown and then the simplex method approach of solving the stochastic knapsack problem of Witchakul *et al.*[18] is applied to the stochastic knapsack problem with the capacity following a contagious distribution. The materials employed in the course of this analysis are MATLAB 15a software and a laptop, Intel(R) Core(TM) i3 M350 Processor @ 2.27 GHz, 6.00 GB RAM, 297 GB hard drive.

Contagious distribution: The multiplicative form of a contagious distribution or mixture is stated as let $\{f(x;\theta)\}$be a family of density functions parameterized by θ . Let also the totality of values the parameter θ can assume be denoted by Θ . If Θ is an interval and $g(\theta)$ is a pdf which is 0 for all arguments not in Θ , then $\int_{\Theta} f(x;\theta)g(\theta)d\theta$ is a density function called a mixture (Mood *et al*., 1973).

Here, $f(x;\theta)$ follows a Poisson distribution with parameter θ given by $f(x;\theta)=\frac{e^{-\theta}\theta^x}{x!}$ for $x=0,1,...,n$

and $g(\theta)=\frac{\lambda^r}{\Gamma(r)}\theta^{r-1}e^{-\lambda\theta}I_{(0,\infty)}(\theta)$

follows a Gamma density function. Then,

$$\int_0^\infty f(x;\theta)\cdot g(\theta)d\theta=\int_0^\infty \frac{e^{-\theta}\theta^x}{x!}\cdot\frac{\lambda^r}{\Gamma(r)}\theta^{r-1}e^{-\theta\lambda}d\theta \quad (1)$$

$$=\frac{\lambda^r}{x!\Gamma(r)}\int_0^\infty \theta^{r+x-1}\cdot e^{-(\lambda+1)\theta}d\theta$$

Applying integration by parts,

$$=\frac{\lambda^r}{x!\Gamma(r)}\cdot\frac{1}{(\lambda+1)^{r+x}}\cdot\frac{\Gamma(r+x)}{\Gamma(r+x)}\int_0^\infty[(\lambda+1)\theta]^{r+x-1}e^{-(\lambda+1)\theta}d[(\lambda+1)\theta] \quad (2)$$

$$=\frac{\Gamma(r+x)}{(x!)\Gamma(r)}\cdot\left(\frac{\lambda}{\lambda+1}\right)^r\cdot\frac{1}{(\lambda+1)^x} \quad (3)$$

$$=\binom{r+x-1}{x}\cdot\left(\frac{\lambda}{\lambda+1}\right)^r\cdot\left(\frac{1}{\lambda+1}\right)^x \text{ for } x=0,1,...,n \quad (4)$$

which is the density function of the new mixture with parameters r and $\lambda/(\lambda+1)$.

Stochastic Knapsack Problem with mixed capacity (SKPMC): The problem can be formulated as shown below.

Min

$$f=\sum_{i=1}^n p_i x_i+\sum_{j=1}^m\left\{g\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x u_j+h\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x v_j\right\} (5).$$

Subject to $\sum_{i=1}^n w_i x_i+u_j-v_j=c_j$

$0\le x_i\le t_i$ and $x=0,1,...,n$

$\sum_{x=0}\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x=1$, and

$z_x=\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x$, $r=1,2,3,...$

where x_i is the decision variable, $i=1,...,n,$

t_i is the upper bound of x_i, $i=1,...,n,$

u_j is the slack variable and $u_j\ge 0$, $j=1,...m,$

v_j is the surplus variable and $v_j\ge 0$, $j=1,...m,$

w_i is the coefficient weight of item i and $w_i\ge 0, i=1,...,n,$,

p_i is the coefficient cost of item j and $p_i\ge 0$, $i=1,...,n,$,

c_j is the capacity of alternative j and $c_j\ge 0$, $j=1,...m,$

g is the cost per unit of having u_j, $g\ge 0$, and

h is the cost per unit of having v_j, $h\ge 0$.

z_x is the probability distribution mixture of having capacity $c_j, z_x\ge 0$, $j=1,...m,$

We propose an algorithm for solving the problem using the simplex approach of Witchakul et al. The underlying assumptions following these algorithms are $c_j\le c_{j+1}$, for $j=1,...,m-1$, and $p_i/w_i\le p_{i+1}/w_{i+1}$, $i=1,...,n-1$.

Algorithm for SKPMC: Following Witchakul et al. (2007), four optimality conditions exists for this problem and are proven using the principles of simplex algorithm. Each of the conditions are proved by writing basic variables in terms of non-basic variables. Then the basic variables are substituted into the objective function. Finally, when the cost of the non-basic variables has been reduced and are greater than or equal to zero, we obtain the optimal solution.

For the first case, we obtain the optimal solution as shown below.

$$x_i = 0, \forall i \quad (6)$$

$$v_j = 0, \forall j \quad (7)$$

$$u_j = c_j, \forall j \quad (8)$$

Where $p_1 / w_1 \geq g$ and $x = 0,1,..., n$

Proof

$$u_j = c_j - \sum_{i=1}^{n} w_i x_i + v_j, \; j = 1,...,m \quad (9)$$

We then proceed to substitute (9) in (5) above as follows.

$$f = \sum_{i=1}^{n} p_i x_i + g \sum_{j=1}^{m} \left[\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^r \left(\frac{1}{\lambda+1}\right)^x \right] \left[c_j - \sum_{i=1}^{n} w_i x_i + v_j \right] + h \sum_{j=1}^{m} \left[\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^r \left(\frac{1}{\lambda+1}\right)^x \right] v_j \quad (10)$$

$$= g \sum_{j=1}^{m} \left[\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^r \left(\frac{1}{\lambda+1}\right)^x \right] c_j + \sum_{j=1}^{m} (g+h) \left[\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^r \left(\frac{1}{\lambda+1}\right)^x \right] v_j + \sum_{i=1}^{n} \left(p_i - g w_i \sum_{x=0}^{m} \binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^r \left(\frac{1}{\lambda+1}\right)^x \right) x_i$$

$$= g \sum_{j=1}^{m} \left[\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^r \left(\frac{1}{\lambda+1}\right)^x \right] c_j + \sum_{j=1}^{m} (g+h) \left[\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^r \left(\frac{1}{\lambda+1}\right)^x \right] v_j + \sum_{i=1}^{n} (p_i - g w_i) x_i \quad (11)$$

Since, the minimized objective function, *f* is obtained when all the reduced cost of the non-basic variables are greater than or equal to zero, we proceed to find the non-basic reduced variables greater than or equal to zero. For this first case, non-basic variables are $x_i \forall i$ and $v_j \forall j$, hence for the minimum value of *f*, the reduced costs of these two non-basic variables must be greater than or equal to zero that implies, $(g+h)\left[\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^r \left(\frac{1}{\lambda+1}\right)^x \right] \geq 0, \quad \forall j$ and $(p_i - g w_i) \geq 0, \forall i$.

Since $g, h \geq 0$ and $\left[\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^r \left(\frac{1}{\lambda+1}\right)^x \right] \geq 0, \quad \forall j$, $(g+h)\left[\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^r \left(\frac{1}{\lambda+1}\right)^x \right] \geq 0, \quad \forall j$. To minimize *f*, $(p_i - g w_i) \geq 0, \forall i$, must be greater than or equal to zero, that is, equivalent to $p_i / w_i \geq g$, $\forall i$. According to the assumption $p_i / w_i \leq p_{i+1} / w_{i+1}$, for $i = 1,..., n-1$, the condition for this case is $p_1 / w_1 \geq g$. Hence for the first case, the minimum value of *f* is $g \sum_{j}^{m} \left[\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^r \left(\frac{1}{\lambda+1}\right)^x \right] c_j$. (12)

For the second case, we obtain the optimal solution as shown below.

$$x_i = t_i, \forall i \quad (13)$$

$$v_j = \sum_{i=1}^{n} w_i t_i - c_j \text{ and } u_j = 0, \; j \in I_2 \quad (14)$$

$$u_j = c_j - \sum_{i=1}^{n} w_i t_i \text{ and } v_j = 0, \; j \in I_1 \quad (15)$$

Where

$I_1 = \left\{ j = 1:m : c_j \geq \sum_{i=1}^{n} w_i t_i \right\}, I_2 = \left\{ j = 1:m : c_j < \sum_{i=1}^{n} w_i t_i \right\}$,

$p_n / w_n \leq g \sum_{j \in I_1} \binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^r \left(\frac{1}{\lambda+1}\right)^x - h \sum_{j \in I_2} \binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^r \left(\frac{1}{\lambda+1}\right)^x$ and

$x = 0,1,..., n$

Proof

$$x_i = t_i - r_i, \forall i \quad (16)$$

$$r_i \geq 0, \forall i \quad (17)$$

$$u_j = c_j - \sum_{i=1}^{n} w_i t_i + \sum_{i=1}^{n} w_i r_i + v_j, \; j \in I_1 \quad (18)$$

$$v_j = -c_j + \sum_{i=1}^{n} w_i t_i - \sum_{i=1}^{n} w_i r_i + u_j, \; j \in I_2 \quad (19)$$

We then proceed to substitute (16), (17), (18) and (19) into (5) as follows.

$$f = \sum_{i=1}^{n} p_i (t_i - r_i) + g \sum_{j \in I_1} \left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^r \left(\frac{1}{\lambda+1}\right)^x \right) \left(c_j - \sum_{i=1}^{n} w_i t_i + \sum_{i=1}^{n} w_i r_i + v_j \right) + h \sum_{j \in I_1} \left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^r \left(\frac{1}{\lambda+1}\right)^x \right) v_j$$
$$+ h \sum_{j \in I_2} \left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^r \left(\frac{1}{\lambda+1}\right)^x \right) \left(-c_j + \sum_{i=1}^{n} w_i t_i - \sum_{i=1}^{n} w_i r_i + u_j \right) + g \sum_{j \in I_2} \left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^r \left(\frac{1}{\lambda+1}\right)^x \right) u_j \quad (20)$$

$$= \sum_{i=1}^{n} p_i t_i + g \sum_{j \in I_1} \left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^r \left(\frac{1}{\lambda+1}\right)^x \right) \left(c_j - \sum_{i=1}^{n} w_i t_i \right) + h \sum_{j \in I_2} \left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^r \left(\frac{1}{\lambda+1}\right)^x \right) \left(-c_j + \sum_{i=1}^{n} w_i t_i \right)$$
$$+ \sum_{j \in I_1} (g+h) \left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^r \left(\frac{1}{\lambda+1}\right)^x \right) v_j + \sum_{j \in I_2} (g+h) \left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^r \left(\frac{1}{\lambda+1}\right)^x \right) u_j$$
$$+ \sum_{i=1}^{n} \left(-p_i + g w_i \sum_{j \in I_1} \left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^r \left(\frac{1}{\lambda+1}\right)^x \right) - h w_i \sum_{j \in I_2} \binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^r \left(\frac{1}{\lambda+1}\right)^x \right) r_i \quad (21)$$

The non-basic variables for this case are v_j, $j \in I_1$, u_j, $j \in I_2$ and $r_i \forall i$. so in order to obtain the minimum value of *f*, the reduced costs of these non-basic variables must be greater than or equal to zero and that means,

$$(g+h)\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)\geq 0 \;\; j\in I_1,$$

$$(g+h)\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)\geq 0 \;\; j\in I_2 \text{ and}$$

$$\left(-p_i+gw_i\sum_{j\in I_1}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)-hw_i\sum_{j\in I_2}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)\right)\geq 0 \quad \forall i$$

Since $g, h \geq 0$ and $\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)\geq 0\,\forall x$, it implies that condition 1and 2 have been satisfied. To minimize *f*, the reduced cost of $\left(-p_i+gw_i\sum_{j\in I_1}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)-hw_i\sum_{j\in I_2}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)\right)\geq 0$ $\forall i$ must be greater than or equal to zero that is equivalent to $p_i/w_i \leq g\sum_{j\in I_1}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)-h\sum_{j\in I_2}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)$ $\forall i$.

According to the assumption $p_i/w_i \leq p_{i+1}/w_{i+1}$, $i=1,2,3,...,n-1$, the condition for this case is $p_n/w_n \leq g\sum_{j\in I_1}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)-h\sum_{j\in I_2}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)$. Hence for this case, the minimum value of *f* is

$$\sum_{i=1}^{n}p_it_i+g\sum_{j\in I_1}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)\left(c_j-\sum_{i=1}^{n}w_it_i\right)+h\sum_{j\in I_2}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)\left(-c_j+\sum_{i=1}^{n}w_it_i\right) \quad (22)$$

For the third case, we obtain the optimal solution as shown below.

$$x_i=t_i, i=1,\ldots,k-1 \quad (23)$$

$$x_k=\frac{c_q-\sum_{i=1}^{n}w_it_i}{w_i} \quad (24)$$

$$x_i=0, i=k+1,\ldots,n \quad (25)$$

$$v_j=c_q-c_j \text{ and } u_j=0, j=1,\ldots q-1 \quad (26)$$

$$u_q=0 \text{ and } v_q=0 \quad (27)$$

$$u_j=c_j-c_q \text{ and } v_j=0, j=q+1,q+2,\ldots,m \quad (28)$$

Where $q\in\{1,2,...,m\}$ and $k\in\{1,2,...,n\}$ such that

$$\left(-p_k/w_k+g\sum_{j=q}^{m}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)-h\sum_{j=1}^{q-1}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)\right)\geq 0,$$

$$\left(p_k/w_k+h\sum_{i=1}^{q}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)-g\sum_{i=q+1}^{m}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)\right)\geq 0, \sum_{i=1}^{k-1}w_it_i\leq c_q \text{ and}$$

$\sum_{i=1}^{k}w_it_i\geq c_q$. If $\sum_{i=1}^{n}w_it_i\leq c_q$, then $k=n$.

Proof

Supposing *k* and *q* is specified

$$f(q)=\sum_{i=1}^{k-1}p_ix_i+p_kx_k+\sum_{i=k+1}^{n}p_ix_i+g\sum_{j=1}^{q-1}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)u_j+h\sum_{j=1}^{q-1}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)v_j$$
$$+g\sum_{j=q+1}^{m}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)u_j+h\sum_{j=q+1}^{m}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)v_j+g\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)u_q$$
$$+h\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)v_q \quad (29)$$

Let,

$$x_i = t_i, i = 1,2,\ldots,k-1, \tag{30}$$

$$x_k = \left(c_q - u_q + v_q - \sum_{i=1}^{k=1} w_i x_i - \sum_{i=k+1}^{n} w_i x_i \right) \Big/ w_k \tag{31}$$

$$x_k = \left(c_q - u_q + v_q - \sum_{i=1}^{k=1} w_i t_i - \sum_{i=k+1}^{n} w_i x_i \right) \Big/ w_k \tag{32}$$

$$v_j = \sum_{i=1}^{k-1} w_i x_i + w_k x_k + \sum_{i=k+1}^{n} w_i x_i + u_k - c_j \tag{33}$$

$$= \sum_{i=1}^{k-1} w_i x_i + w_k \left(\left(c_q - u_q + v_q - \sum_{i=1}^{k+1} w_i x_i - \sum_{i=k+1}^{n} w_i x_i \right) \Big/ w_k \right) + \sum_{\substack{i=k+1 \\ 1}}^{n} w_i x_i + u_j - c_j \tag{34}$$

$= c_q - c_j + u_j - u_q + v_q, j = 1,2,\ldots,q-1$, and

$$u_j = c_j + v_j - \sum_{i=1}^{k-1} w_i x_i - w_k x_k - \sum_{i=k+1}^{n} w_i x_i$$

$$= c_j + v_j - \sum_{i=1}^{k-1} w_i x_i - w_k \left(\left(c_q - u_q + v_q - \sum_{i=1}^{k-1} w_i x_i - \sum_{i=k+1}^{n} w_i x_i \right) \Big/ w_k \right) - \sum_{i=k+1}^{n} w_i x_i$$

$$= c_j - c_q + v_j + u_q - v_q, j = q+1, q+2, \ldots, m.$$

Next, we substitute the basic variables above into (29). Thus (29) becomes as follows.

$$f(q) = \sum_{i=1}^{k-1} p_i t_i + p_k \left(\left(c_q - u_q + v_q - \sum_{i=1}^{k-1} w_i t_i - \sum_{i=k+1}^{n} w_i x_i \right) \Big/ w_k \right)$$
$$+ \sum_{i=k+1}^{n} p_i x_i + g \sum_{j=1}^{q-1} \left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^{r} \left(\frac{1}{\lambda+1}\right)^{x} \right) u_j$$
$$+ h \sum_{j=1}^{q-1} \left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^{r} \left(\frac{1}{\lambda+1}\right)^{x} \right) \left(c_q - c_j + u_j - u_q + v_q \right)$$
$$+ g \sum_{j=q+1}^{m} \left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^{r} \left(\frac{1}{\lambda+1}\right)^{x} \right) \left(c_j - c_q + v_j + u_q - v_q \right)$$
$$+ h \sum_{j=q+1}^{m} \left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^{r} \left(\frac{1}{\lambda+1}\right)^{x} \right) v_j + g \left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^{r} \left(\frac{1}{\lambda+1}\right)^{x} \right)$$
$$u_q + h \left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^{r} \left(\frac{1}{\lambda+1}\right)^{x} \right) v_q \tag{35}$$

$$= \sum_{i=1}^{k-1} p_i t_i + p_k \left(\left(c_q - \sum_{i=1}^{k-1} w_i t_i \right) \Big/ w_k \right) + h \sum_{j=1}^{q-1} \left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^{r} \left(\frac{1}{\lambda+1}\right)^{x} \right) \left(c_q - c_j \right)$$
$$+ g \sum_{j=q+1}^{m} \left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^{r} \left(\frac{1}{\lambda+1}\right)^{x} \right) \left(c_j - c_q \right)$$
$$+ \sum_{i=k+1}^{n} (p_i - p_k w_i / w_k) x_i + \sum_{j=1}^{q-1} (g+h) \left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^{r} \left(\frac{1}{\lambda+1}\right)^{x} \right) u_j + \sum_{j=q+1}^{m} (g+h) \left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^{r} \left(\frac{1}{\lambda+1}\right)^{x} \right) v_j$$
$$+ \left(-p_k \Big/ w_k + g \sum_{j=q}^{m} \left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^{r} \left(\frac{1}{\lambda+1}\right)^{x} \right) - h \sum_{j=1}^{q-1} \left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^{r} \left(\frac{1}{\lambda+1}\right)^{x} \right) \right) u_q$$
$$+ \left(p_k \Big/ w_k + h \sum_{j=1}^{q} \left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^{r} \left(\frac{1}{\lambda+1}\right)^{x} \right) - g \sum_{j=q+1}^{m} \left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^{r} \left(\frac{1}{\lambda+1}\right)^{x} \right) \right) v_q \tag{36}$$

Here, the non-basic variables are x_i, $i = k+1, k+2, \ldots, n$, v_j, $j = q+1, q+2, \ldots, m$, u_j, $j = 1,2,\ldots, q-1$, u_q and v_q. Hence the reduced costs of these non-basic variables must be greater than or equal to zero in order to obtain the minimum value of *f*, that means $p_i - p_k w_i / w_k \geq 0$, $i = k+1, k+2, \ldots, n$, $(g+h)\left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^{r} \left(\frac{1}{\lambda+1}\right)^{x} \right) \geq 0$, $j = 1,2,\ldots, q-1$, $(g+h)\left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^{r} \left(\frac{1}{\lambda+1}\right)^{x} \right) \geq 0$, $j = q+1, q+2, \ldots, m$,

$$\left(-p_k \Big/ w_k + g \sum_{j=q}^{m} \left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^{r} \left(\frac{1}{\lambda+1}\right)^{x} \right) - h \sum_{j=1}^{q-1} \left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^{r} \left(\frac{1}{\lambda+1}\right)^{x} \right) \right) \geq 0$$

and

$$\left(p_k \Big/ w_k + h \sum_{j=1}^{q} \left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^{r} \left(\frac{1}{\lambda+1}\right)^{x} \right) - g \sum_{j=q+1}^{m} \left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^{r} \left(\frac{1}{\lambda+1}\right)^{x} \right) \right) \geq 0$$

From assumption, $p_i / w_i \leq p_{i+1} / w_{i+1}$, $i = 1,2,\ldots, n-1$, then $p_i - p_k w_i / w_k \geq 0$, $i = k+1, k+2, \ldots, n$. since $g, h \geq 0$ and $\left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^{r} \left(\frac{1}{\lambda+1}\right)^{x} \right) \geq 0$, $\forall$ j, $(g+h)\left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^{r} \left(\frac{1}{\lambda+1}\right)^{x} \right) \geq 0$, $j = 1,2,\ldots, q-1$, and $(g+h)\left(\binom{r+x-1}{x} \left(\frac{\lambda}{\lambda+1}\right)^{r} \left(\frac{1}{\lambda+1}\right)^{x} \right) \geq 0$, $j = q+1, q+2, \ldots, m$ have been satisfied.

To minimize

$$f,\left(-p_k\Big/w_k+g\sum_{j=q}^{m}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)-h\sum_{j=1}^{q-1}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)\right)$$

and

$$\left(p_k\Big/w_k+h\sum_{j=1}^{q}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)-g\sum_{j=q+1}^{m}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)\right)$$

must be greater than or equal to zero. Since $0\le x_k\le t_k$, it is equivalent to $0\le\frac{c_q-\sum_{i=1}^{k-1}w_it_i}{w_k}\le t_k$. It is also equivalent to $\sum_{i=1}^{k-1}w_it_i\le c_q$ and $\sum_{i=1}^{k}w_it_i\ge c_q$. If $\sum_{i=1}^{n}w_it_i<c_q$, then $k=n$.

Therefore, k and q are selected such that

$$\left(-p_k\Big/w_k+g\sum_{j=q}^{m}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)-h\sum_{j=1}^{q-1}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)\right)\ge 0$$

and

$$\left(p_k\Big/w_k+h\sum_{j=1}^{q}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)-g\sum_{j=q+1}^{m}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)\right)\ge 0$$

and $\sum_{i=1}^{k-1}w_it_i\le c_q$ and $\sum_{i=1}^{k}w_it_i\ge c_q$. If $\sum_{i=1}^{n}w_it_i<c_q$, then $k=n$.

Therefore, for the third case, the minimum value of f is

$$\sum_{i=1}^{k-1}w_it_i+c_k\left(\left(c_q-\sum_{i=1}^{k-1}w_it_i\right)\Big/w_k\right)+$$
$$h\sum_{j=1}^{q-1}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)(c_q-c_j)+g\sum_{j=q+1}^{m}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)(c_q-c_j)$$

For the fourth case, we obtain the optimal solution as shown below.

$$x_i=t_i, i=1,\dots,k \tag{37}$$

$$x_i=0, i=k+1,\dots,n \tag{38}$$

$$v_j=c^*-c_j \text{ and } u_j=0, j=1,\dots,q \tag{39}$$

$$u_j=c_j-c^* \text{ and } v_j=0, j=q+1,q+2,\dots,m \tag{40}$$

Where $q\in\{1,2,\dots,m\}$ and $k\in\{1,2,\dots,n\}$ such that

$$\left(p_{k+1}/w_{k+1}+h\sum_{j=1}^{q}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)-g\sum_{j=q+1}^{m}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)\right)\ge 0,$$

$$\sum_{i=1}^{k}w_it_i>c_q, \sum_{i=1}^{k}w_it_i<c_{q+1} \text{ and } c^*=\sum_{i=1}^{k}w_it_i$$

Proof

Suppose k and q have been specified

$$f(q)=\sum_{i=1}^{k}p_ix_i+\sum_{i=k+1}^{n}p_ix_i+g\sum_{j=1}^{q}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)u_j+h\sum_{j=1}^{q}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)v_j$$
$$+h\sum_{j=q+1}^{m}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)v_j \tag{41}$$

Let

$$x_i=t_i, i=1,2,\dots,k, \tag{42}$$

$$v_j=\sum_{i=1}^{k}w_ix_i+\sum_{i=k+1}^{n}w_ix_i+u_j-c_j \tag{43}$$

$$=\sum_{i=1}^{k}w_it_i+\sum_{i=k+1}^{n}w_ix_i+u_j-c_j, j=1,2,\dots,q, and \tag{44}$$

$$u_j=c_j+v_j-\sum_{i=1}^{k}w_ix_i-\sum_{i=k+1}^{n}w_ix_i$$
$$=c_j+v_j-\sum_{i=1}^{k}w_it_i-\sum_{i=k+1}^{n}w_ix_i, j=q+1,q+2,\dots,m.$$

The basic variables above will subsequently be substituted into equation (41). Thus equation (41) becomes as follows.

$$f(q)=\sum_{i=1}^{k}p_it_i+\sum_{i=k+1}^{n}p_ix_i+g\sum_{j=1}^{q}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)u_j$$

$$+h\sum_{j=1}^{q}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)\left(\sum_{i=1}^{k}w_it_i-c_j+u_j+\sum_{i=k+1}^{n}w_it_i\right)$$

$$+g\sum_{j=q+1}^{m}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)\left(c_j-\sum_{i=1}^{k}w_it_i+v_j-\sum_{i=k+1}^{n}w_it_i\right)+h\sum_{j=q+1}^{m}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)v_j \tag{45}$$

$$f(q)=\sum_{i=1}^{k}p_it_i+h\sum_{j=1}^{q}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)\left(\sum_{i=1}^{k}w_it_i-c_j\right)+g\sum_{j=q+1}^{m}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)\left(c_j-\sum_{i=1}^{k}w_it_i\right)$$

$$\sum_{j=1}^{q}(g+h)\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)u_j+\sum_{j=q+1}^{m}(g+h)\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)v_j$$

$$+\sum_{i=k+1}^{n}\left(p_i + hw_i\sum_{j=1}^{q}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right) - gw_i\sum_{j=1}^{q-1}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)\right)x_i \qquad (46)$$

The non-basic variables here are $x_i, i = k+1, k+2, ..., n$, v_j, $j = q+1, q+2, q+3, ..., m$, u_j, $j = 1,2,3,..., q$. Therefore to obtain the minimum value of *f*, the reduced costs of these non-basic variables must be greater than or equal to zero that means

$$(g+h)\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right) \ge 0, \; j = 1,2,3,..., q$$

$$(g+h)\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right) \ge 0,$$

$j = q+1, q+2, q+3, ..., m$, and

$$\left(p_i + hw_i\sum_{j=1}^{q}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right) - gw_i\sum_{j=q+1}^{m}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)\right) \ge 0$$

$, i = k+1, k+2, k+3, ..., n$.

Since $g, h \ge 0$, and $\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right) \ge 0 \,\forall$

$$j, (g+h)\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right) \ge 0,$$

$j = 1,2,3,..., q$

and $(g+h)\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right) \ge 0,$

$j = q+1, q+2, q+3, ..., m$

have been satisfied. To minimize *f*,

$$\left(p_i + hw_i\sum_{j=1}^{q}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right) - gw_i\right.$$

$$\left.\sum_{j=q+1}^{m}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)\right)$$

must be greater than or equal to zero, $i = k+1, k+2, k+3, ..., n$, that is equivalent to

$$\left(p_i + hw_i\sum_{j=1}^{q}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right) - gw_i\sum_{j=q+1}^{m}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)\right) \ge 0,$$

$i = k+1, k+2, k+3, ..., n$. According to the assumption $p_i / w_i \le p_{i+1} / w_{i+1}$, $i = 1,2,3,..., n-1$, the condition here is

$$\left(p_{k+1}/w_{k+1} + h\sum_{j=1}^{q}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right) - g\sum_{j=q+1}^{m}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)\right) \ge 0.$$

Since $c_q < c^* < c_{q+1}$ and $c^* = \sum_{i=1}^{k} w_i t_i$, it is equivalent to

$$\sum_{i=1}^{k} w_i t_i > c_q \text{ and } \sum_{i=1}^{k} w_i t_i < c_{q+1}.$$

Therefore, *k* and *q* will be selected such that

$$\left(p_{k+1}/w_{k+1} + h\sum_{j=1}^{q}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right) - g\sum_{j=q+1}^{m}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)\right) \ge 0$$

$$\sum_{i=1}^{k} w_i t_i > c_q \text{ and } \sum_{i=1}^{k} w_i t_i < c_{q+1}.$$

Finally, the minimum value of *f* here is

$$\sum_{i=1}^{k} p_i t_i + h\sum_{j=1}^{q}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)\left(\sum_{i=1}^{k} w_i t_i - c_j\right) + g\sum_{j=q+1}^{m}\left(\binom{r+x-1}{x}\left(\frac{\lambda}{\lambda+1}\right)^r\left(\frac{1}{\lambda+1}\right)^x\right)\left(c_j - \sum_{i=1}^{k} w_i t_i\right) \qquad (47)$$

Results and discussion

From the Table-1, 2 and 3 in the appendix section, the computing time for the SKPMC algorithm was able to run for up to 75,000 random variables more than the algorithm of Witchakul *et al.*[18] The SKPMC algorithm of Table-1 was more efficient than that of the Monte Carlo algorithm of Table-2. However, the Heuristics algorithm of Table-3 was more efficient than the SKPMC and Monte Carlo algorithms.

Conclusion

This study has shown that optimal solutions can be obtained for stochastic knapsack problem with a contagious distributional capacity. An algorithm for a stochastic knapsack problem (SKP) with a contagious distributional capacity was developed using the simplex methods of Witchakul *et al.*[18] also, a Monte Carlo algorithm and a Heuristic method was developed and ran using a MATLAB software. The results show that the while optimal solutions can be obtained for as large as 75,000 random variables, the optimal solutions from the Heuristics methods was the most efficient of the three methods while the Monte Carlo methods was the least efficient of them all.

Table-1: Computing time (sec) of the SKPMC algorithm

M												
n	100	250	500	750	1,000	2,500	5,000	7,500	10,000	25,000	50,000	75,000
100	0.6206	0.0276	0.0601	0.0972	0.1289	0.3884	0.8304	1.7248	2.2288	8.7315	29.0916	62.3889
250	0.0133	0.0526	0.0565	0.0868	0.1887	0.3319	0.7990	1.3764	1.8550	9.4073	9.4242	62.1279
500	0.0295	0.0442	0.0651	0.0908	0.1442	0.3757	0.9834	1.4382	2.2922	10.3593	30.8231	66.9216
750	0.0180	0.0340	0.0826	0.1714	0.1686	0.4177	0.8654	1.4844	1.8795	9.4538	30.9397	15.0443
1,000	0.0201	0.0625	0.0885	0.1241	0.1841	0.6872	1.2544	1.7528	2.2927	9.3567	32.2625	68.8349
2,500	0.0253	0.0438	0.0948	0.1431	0.1663	0.4923	0.9354	1.7291	2.5449	9.9266	32.3288	67.5332
5,000	0.0251	0.0356	0.0957	0.1432	0.2186	0.5862	1.2635	1.3921	3.0309	4.6391	35.3935	72.6592
7,500	0.0288	0.0709	0.1633	0.2685	0.3240	0.7422	0.9288	1.3937	3.5669	12.4323	38.5549	73.8670
10,000	0.0339	0.0813	0.0964	0.2026	0.2836	0.8016	1.7404	2.8600	1.8547	14.2160	41.4695	82.4798
25,000	0.0354	0.0831	0.3644	0.4266	0.2606	1.7391	3.3058	5.3630	7.0014	25.0308	58.7919	16.1122
50,000	0.1266	0.3167	0.4252	0.6952	1.1446	2.7250	0.9530	9.9642	12.5514	37.4237	83.3753	151.5088
70,000	0.0277	0.1599	0.6032	0.9504	1.3643	3.8470	8.5608	1.4969	17.1314	49.1095	118.4811	193.7150

Table 2 Computing time (sec) for the Monte Carlo Simulation of SKPMC algorithm.

m												
N	100	250	500	750	1,000	2,500	5,000	7,500	10,000	25,000	50,000	75,000
100	0.4675	0.4731	0.4705	0.5322	0.4631	0.4646	0.4878	0.4983	0.4592	0.4681	0.5070	0.5675
250	0.4553	0.4506	0.4455	0.4431	0.4712	0.4553	0.4799	0.4691	0.4706	0.5067	0.5656	0.6158
500	0.5458	0.5572	0.5604	0.5624	0.5708	0.5664	0.5601	0.5687	0.5844	0.6057	0.6566	0.7235
750	0.6777	0.6865	0.6759	0.6744	0.7530	0.7678	0.8308	0.7819	0.7995	0.7776	0.7686	0.8055
1,000	0.7704	0.7520	0.7663	0.7579	0.7725	0.7809	0.7968	0.7837	0.7482	0.8122	0.8782	0.9048
2,500	1.4807	1.4525	1.4760	1.4673	1.6435	1.6913	1.6364	1.4425	1.4245	1.4721	1.5277	1.5745
5,000	2.6853	2.6777	2.6605	2.7342	3.1677	2.7074	2.6314	2.6698	2.6409	2.6685	2.6714	2.7942
7,500	4.4204	4.4977	4.3750	4.5865	3.9872	3.8075	3.7959	3.8115	3.7924	4.3863	3.8918	3.9309
10,000	5.0404	5.0426	5.5664	5.0092	5.0285	5.0183	5.5920	5.0294	5.0216	5.0391	5.5541	5.9678
25,000	9.4139	8.8606	9.6715	8.7369	9.9461	10.5469	8.8625	9.1812	9.8600	9.6090	10.1020	9.6268
50,000	20.6698	21.1642	21.5789	20.0100	20.8267	21.9285	19.9986	18.9871	20.2151	20.4711	20.0589	20.1342
75,000	30.3224	31.4505	30.7404	29.9412	30.9995	31.1786	30.5339	29.3536	32.6074	31.6547	29.5590	32.3347

Table-3: Computing time (sec) for the Heuristics of SKPMC algorithm.

M											
n	100	250	500	750	1,000	2,500	5,000	7,500	10,000	25,000	50,000	75,000
100	0.0839	0.0925	0.0983	0.0277	0.0380	0.0678	0.0531	0.0109	0.0261	0.0209	0.0117	0.0765
250	0.0368	0.0235	0.0476	0.0251	0.0495	0.0238	0.1164	0.2088	0.0243	0.0224	0.0254	0.0281
500	0.0633	0.0514	0.0466	0.1117	0.1131	0.4245	0.4087	0.1479	0.0749	0.0435	0.2590	0.0530
750	0.0765	0.0949	1.8685	0.0759	0.4833	0.1132	0.4560	0.0665	0.0635	0.0641	0.0634	0.0649
1,000	0.3380	0.9272	0.5273	0.0842	0.0841	0.1669	0.1684	0.0857	0.7599	0.0862	0.0852	0.0898
2,500	0.8289	1.2449	2.7060	2.4827	0.2091	9.3876	4.2491	0.7772	0.3934	3.0318	0.2733	0.7600
5,000	1.2368	0.4134	0.4201	2.4797	0.4234	3.6805	5.7685	0.4659	15.6462	0.8734	10.8135	0.4168
7,500	17.1142	0.6347	2.4925	0.6206	21.3803	1.8792	15.9319	1.2385	0.6164	0.6165	0.6195	0.6295
10,000	3.2820	0.8278	8.1871	12.4181	0.8313	1.6508	12.7358	0.8247	0.8199	2.0209	0.8762	2.0269
25,000	5.0578	4.0503	2.6932	2.3432	2.3130	2.3269	2.3202	2.5172	49.9651	2.0662	2.6333	2.0554
50,000	90.6313	131.1575	38.9214	68.5549	88.9574	4.2267	5.0157	4.4059	4.4023	192.7762	8.1887	6.8908
75,000	10.1798	10.9619	13.5523	12.6031	11.2745	510.5032	5.9721	5.9880	324.8384	600.2312	158.4232	7.1152

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