Research Journal of Mathematical and Statistical Sciences ______________________________ISSN 2320-6047
Vol. **5(8),** 1-6, August **(2017)** *Res. J. Mathematical and Statistical Sci.*

Merging is possible and continuous for 2-regular graphs

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Available online at: www.isca.in, www.isca.me
Received 23rd April 2017, revised 4th August 2017, accepted 10th August 2017

Abstract

In the previous paper, we show that the merging is possible for smallest 2-regular graphs. Now we work on the merging is possible for all types of undirected similar 2-regular graphs. That is the merging is possible for 2-regular graphs with n vertices. Where n is natural number and representing the vertices of 2-regular graphs for the merging. Always n is greater than or equal to three (n≥3) for 2-regular graphs. That is every regular graph has same number of degree. Here first we prove some results based on the merging of 2-regular graphs with n≥3 vertices. Latter we define continuous merging and after that we will work on the continuous merging for 2-regular graphs with n≥3 vertices using our definitions. Also we prove some results based on continuous merging for 2-regular graphs with n≥3 vertices. We are complete our work using some operators and merging techniques.

Keywords: Regular graph, Merge, Operator, Technique, Continuous, Vertex, Degree.

Introduction

In the previous paper, we prove some results based on the merging of similar smallest 2-regular graphs[1]. Those results are show that the merging is possible for smallest 2-regulaer graphs[2]. Now we think and discuss about the longer merging possibilities conditions for smallest 2-regular graphs[3]. That is the merging is possible for more than two smallest 2-regular graphs. This is called continuous merging for smallest 2-regular graphs[4].

In this paper, first we prove some results based the possibility of merging for more than two smallest 2-regular graphs. If it is possible for three smallest 2-regular graphs, then we prove for finitely many smallest 2-regular graphs.

Later we are providing a condition for infinitely continuous merging for smallest 2-regular graphs.

Also we prove the continuous merging for all types of similar 2-regular graphs[4].

Definitions

Merging technique for regular graphs: Merging technique is a process, which is used for converting of two or more than two regular graphs into a single regular graph[5]. Where given regular graphs are having same number of degree or similar regular graphs (for the both graphs, every vertex has same degree)[5].

We say merging is possible: i. If the resultant graph is regular graph and, ii. Degree of each vertex in resultant graph is equal to degree of each vertex in given regular graphs.

Let G and G' are two regular graphs. If the merging is possible, then merging of G and G' is denoted by G <<>> G' Otherwise G <</>> G' (when merging of G and G' is not possible)[6].

Operator: Operator is the major part of merging. It is used for store/remove edge at the time of merging. Generally, one operator store/remove one edge at the time of merging[4]. When the edge is store by operator then it is denoted by >>OPERATOR_NAME (edge). When edge is leave/remove by operator then it is denoted by OPERATOR_NAME (edge)>>[3].

Continuous merging: If merging is possible for more than two regular graphs, then we say merging is continuous.

Results

Theorem-1: If G_1, G_2, G_3…...... G_{m-1}, G_m are smallest 2-regular graphs, then show that G_1<<>>G_2<<>>G_3<<>>>……………….<<>>G_{m-1}<<>>G_m. where m is any finite nonnegative integer[8].

Or

The continuous merging is possible for finitely many smallest 2-regular graphs.

Proof: Let G_1, G_2, G_3………, $G_{m-1,}$ G_m are smallest 2-regular graphs, then we show that G_1<<>>G_2<<>>G_3<<>>>………………. <<>>G_{m-1}<<>>G_m.

Step-1: Since $G_1=(V_1, E_1)$, where $V_1=\{v_1,v_1',v_1''\}$; $E_1=\{e_1,e_1',e_1''\}$ and $G_2=(V_2, E_2)$, where $V_2=\{v_2,v_2',v_2''\}$; $E_2=\{e_2,e_2',e_2''\}$ are smallest 2-regular graphs.

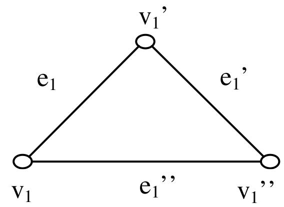

Figure-1: $G_1= (V_1, E_1)$.

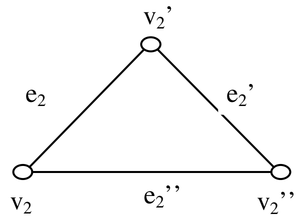

Figure-2: $G_2= (V_2, E_2)$.

Then by Theorem 2: The merging of two smallest 2-regular graphs is possible[6].

Case-1: When the $\alpha_1(e_1')>>$ then this edge joining to $e_1'=(v_1', v_2')$. In this case the $\alpha_2(e_2)>>$ is join to rest special vertices of graphs[7]. i.e. the edge joining to $e_2=(v_1'', v_2)$.

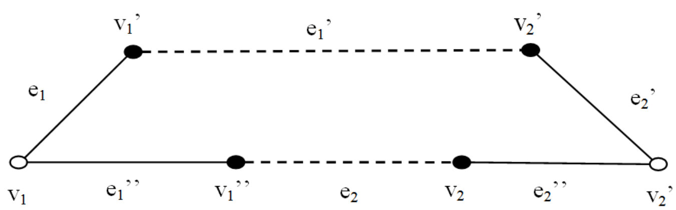

Figure-3: When edge $e_1'= (v_1', v_2')$ and $e_2= (v_1'', v_2)$.

Thus merging of G_1 and G_2 is possible.

Or

Case-2: When the $\alpha_1(e_2)>>$ then this edge joining to $e_2=(v_1', v_2')$. In this case the $\alpha_2(e_1')>>$ is join to rest special vertices of graphs[6]. i.e. the edge joining to $e_1'=(v_1'', v_2)$.

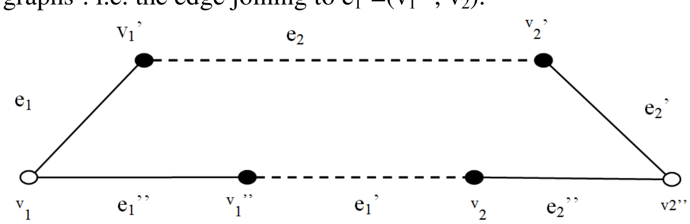

Figure-4: When edge $e_1'= (v_1'', v_2)$ and $e_2= (v_1', v_2')$.

Thus merging of G_1 and G_2 is possible.

Or

Case-3: When the $\alpha_1(e_1')>>$ then this edge joining to $e_1'=(v_1', v_2)$. In this case the $\alpha_2(e_2)>>$ is join to rest special vertices of graphs[6]. i.e. the edge joining to $e_2=(v_1'', v_2')$.

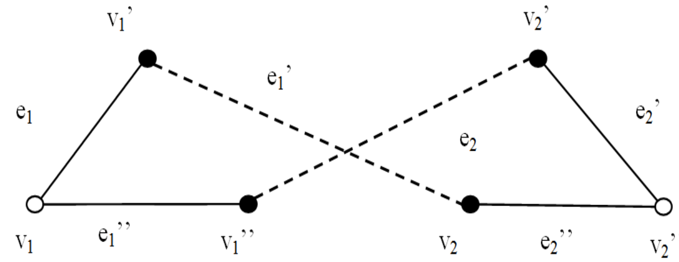

Figure-5: When edge $e_1'= (v_1', v_2)$ and $e_2= (v_1'', v_2')$

Thus merging of G_1 and G_2 is possible.

Or

Case-4: When the $\alpha_1(e_2)>>$ then this edge joining to $e_2=(v_1', v_2)$. In this case the $\alpha_2(e_1')>>$ is join to rest special vertices of graphs[6]. i.e. the edge joining to $e_1'=(v_1'', v_2')$.

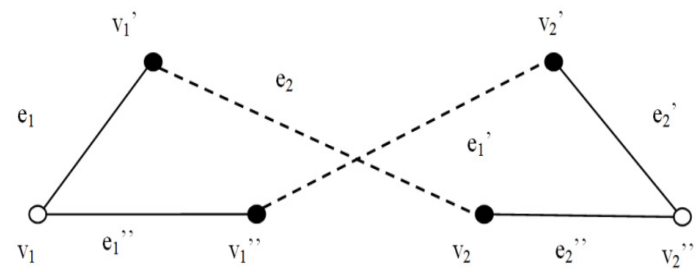

Figure-6: When edge $e_2= (v_1', v_2)$ and $e_1'= (v_1'', v_2')$

Thus merging of G_1 and G_2 is possible.

i.e. $G_1<<>>G_2$ is a 2- regular graph.

Step-2: Now let $X_1 = G_1<<>>G_2$ and let $G_3= (V_3, E_3)$ where $V_3= \{v_3, v_3', v_3''\}$, $E_3= \{e_3, e_3', e_3''\}$. First we draw the $G_3= (V_3, E_3)$.

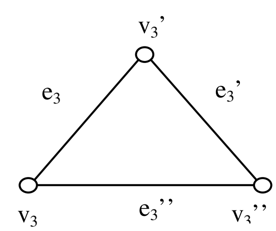

Figure-7: $G_3= (V_3, E_3)$.

Now we show that $X_1<<>>G_3$ is 2-regular graph.

Let α_3 and α_4 be the operators for X_1 and G_3. Now we assume edge e_2' is stored by α_3 and denoted by $>>\alpha_3(e_2')$. Similarly we assume edge e_3 is stored by α_4 and denoted by $>> \alpha_4(e_3)$. After these the vertices v_2', v_2'' in the step1 (all cases) and v_3, v_3' in Figure-7 are special vertices[6]. Then

Case-1: First we are merging G_3 with X_1. Where X_1 is describe in the case1 of step1.

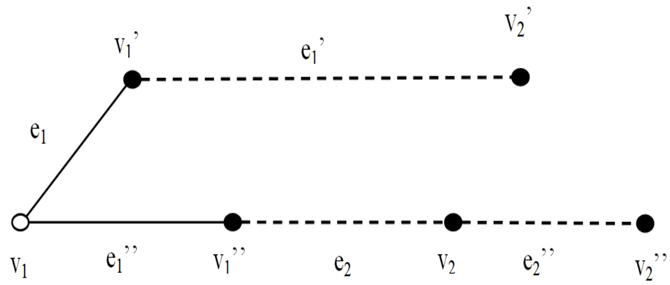

Figure-8a: When edge e_2' of X_1 is stored by α_3

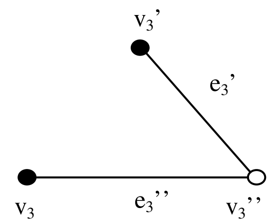

Figure-8b: When edge e_3 of G_3 is stored by α_4

Condition-1: When the $\alpha_3(e_2')$>> then this edge joining to $e_2'=(v_2', v_3')$. In this case the $\alpha_4(e_3)$>> is join to rest special vertices of graphs[6]. i.e. the edge joining to $e_3=(v_2'', v_3)$ (Figure-9).

Condition-2: When the $\alpha_3(e_2')$>> then this edge joining to $e_2'=(v_2'', v_3)$. In this case the $\alpha_4(e_3)$>> is join to rest special vertices of graphs[6]. i.e. the edge joining to $e_3=(v_2', v_3')$ (Figure-10).

Condition-3: When the $\alpha_3(e_2')$>> then this edge joining to $e_2'=(v_2', v_3)$. In this case the $\alpha_4(e_3)$>> is join to rest special vertices of graphs[6]. i.e. the edge joining to $e_3=(v_2'', v_3')$ (Figure-11).

Condition-4: When the $\alpha_3(e_2')$>> then this edge joining to $e_2'=(v_2'', v_3')$. In this case the $\alpha_4(e_3)$>> is join to rest special vertices of graphs[6]. i.e. the edge joining to $e_3=(v_2', v_3)$ (Figure-12).

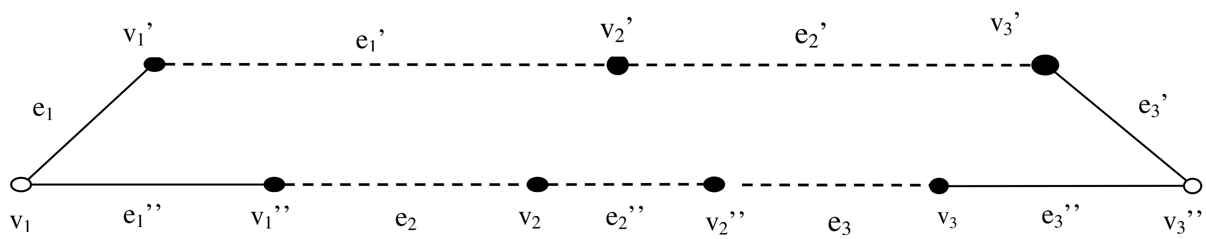

Figure-9: when edge $e_2'= (v_2', v_3')$ and $e_3= (v_2'', v_3)$

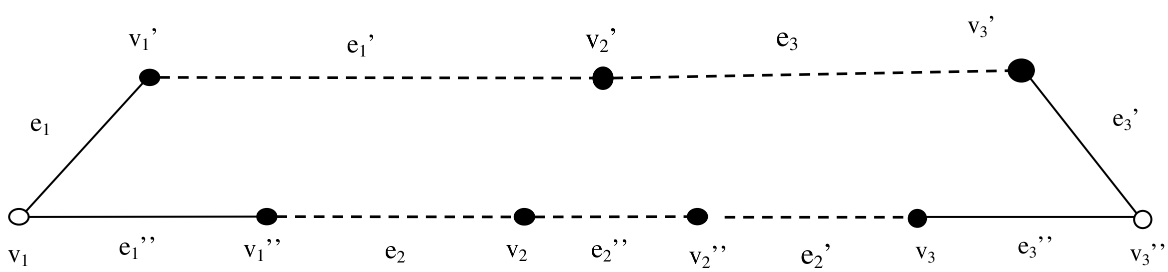

Figure-10: when edge $e_2'= (v_2'', v_3)$ and $e_3= (v_2', v_3')$

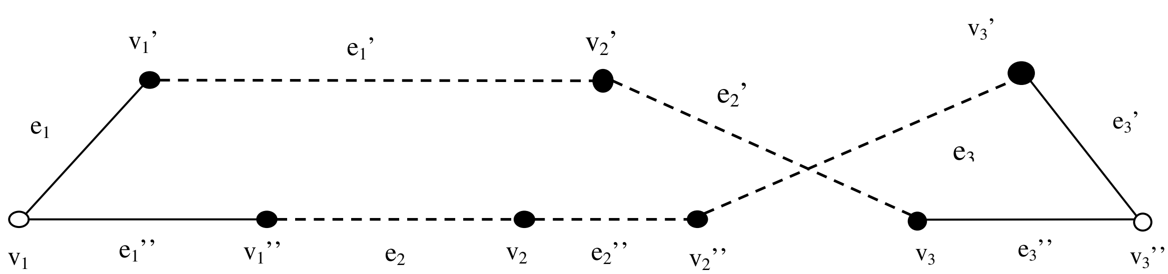

Figure-11: when edge $e_2'= (v_2', v_3)$ and $e_3= (v_2'', v_3')$

Similarly,
Case-2: First we are merging G_3 with X_1. Where X_1 is describe in the case-2 of step-1.

Condition-1: When the $\alpha_3(e_2')$>> then this edge joining to $e_2'=(v_2', v_3')$. In this case the $\alpha_4(e_3)$>> is join to rest special vertices of graphs[6]. i.e. the edge joining to $e_3=(v_2'', v_3)$.

Similarly,
Condition-2: When the $\alpha_3(e_2')$>> then this edge joining to $e_2'=(v_2'', v_3)$. In this case the $\alpha_4(e_3)$>> is join to rest special vertices of graphs[6]. i.e. the edge joining to $e_3=(v_2', v_3')$.

Condition-3: When the $\alpha_3(e_2')$>> then this edge joining to $e_2'=(v_2', v_3)$. In this case the $\alpha_4(e_3)$>> is join to rest special vertices of graphs[6]. i.e. the edge joining to $e_3=(v_2'', v_3')$.

Condition-4: When the $\alpha_3(e_2')$>> then this edge joining to $e_2'=(v_2'', v_3')$. In this case the $\alpha_4(e_3)$>> is join to rest special vertices of graphs[6]. i.e. the edge joining to $e_3=(v_2', v_3)$.

Case-3: First we are merging G_3 with X_1. Where X_1 is describe in the case-3 of step-1.

Condition-1: When the $\alpha_3(e_2')$>> then this edge joining to $e_2'=(v_2', v_3')$. In this case the $\alpha_4(e_3)$>> is join to rest special vertices of graphs[6]. i.e. the edge joining to $e_3=(v_2'', v_3)$.

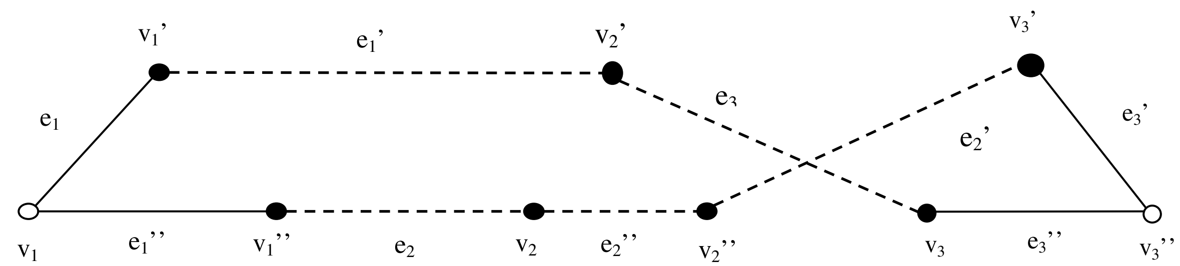

Figure-12: when edge $e_2'= (v_2'', v_3')$ and $e_3= (v_2', v_3)$

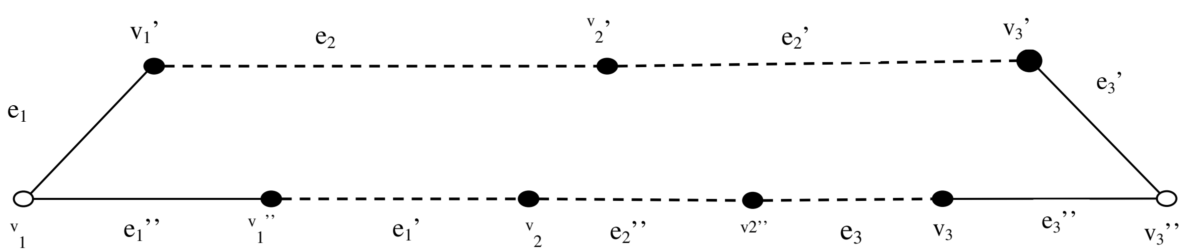

Figure-13: when edge $e_2'= (v_2', v_3')$ and $e_3= (v_2'', v_3)$

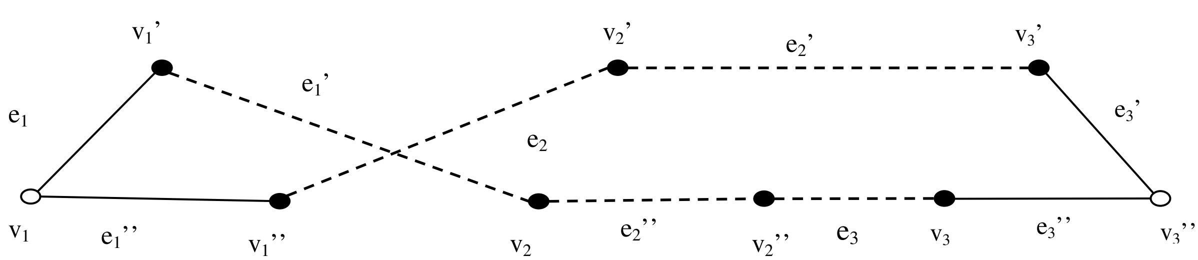

Figure-14: when edge $e_2'= (v_2', v_3')$ and $e_3= (v_2'', v_3)$

Similarly,
Condition-2: When the $\alpha_3(e_2')$>> then this edge joining to $e_2'=(v_2'', v_3)$. In this case the $\alpha_4(e_3)$>> is join to rest special vertices of graphs[6]. i.e. the edge joining to $e_3=(v_2', v_3')$.

Condition-3: When the $\alpha_3(e_2')$>> then this edge joining to $e_2'=(v_2', v_3)$. In this case the $\alpha_4(e_3)$>> is join to rest special vertices of graphs[6]. i.e. the edge joining to $e_3=(v_2'', v_3')$.

Condition-4: When the $\alpha_3(e_2')$>> then this edge joining to $e_2'=(v_2'', v_3')$. In this case the $\alpha_4(e_3)$>> is join to rest special vertices of graphs[6]. i.e. the edge joining to $e_3=(v_2', v_3)$.

Case-4: First we are merging G_3 with X_1. Where X_1 is describe in the case-4 of Step-1.

Condition-1: When the $\alpha_3(e_2')$>> then this edge joining to $e_2'=(v_2', v_3')$. In this case the $\alpha_4(e_3)$>> is join to rest special vertices of graphs[6]. i.e. the edge joining to $e_3=(v_2'', v_3)$.

Similarly,
Condition-2: When the $\alpha_3(e_2')$>> then this edge joining to $e_2'=(v_2'', v_3)$. In this case the $\alpha_4(e_3)$>> is join to rest special vertices of graphs[6]. i.e. the edge joining to $e_3=(v_2', v_3')$.

Condition-3: When the $\alpha_3(e_2')$>> then this edge joining to $e_2'=(v_2', v_3)$. In this case the $\alpha_4(e_3)$>> is join to rest special vertices of graphs[6]. i.e. the edge joining to $e_3=(v_2'', v_3')$.

Condition-4: When the $\alpha_3(e_2')$>> then this edge joining to $e_2'=(v_2'', v_3')$. In this case the $\alpha_4(e_3)$>> is join to rest special vertices of graphs[6]. i.e. the edge joining to $e_3=(v_2', v_3)$.

Now by apply the definition of merging technique [1] on the resultant graphs. Then, in each condition of all cases we get the resultant graph is a regular graph and degree of each vertex is equal to $\mathbf{X_1}$ or $\mathbf{G_3}$**.**

Thus the resultant graphs are **2-**regular graphs.

Thus the merging is possible for $\mathbf{X_1}$ and $\mathbf{G_3}$.

i.e. X_1<<>> G_3 or G_1<<>>G_2<<>> G_3. Since $X_1 = G_1$<<>>G_2.

Now we assume in t-1[th] step the merging of $G_t = (V_t, E_t)$ where $t<m$ and $V_t = \{v_t, v_t', v_t''\}$; $E_t= \{e_t, e_t', e_t''\}$ is possible with X_{t-2}, where $X_{t-2}= G_1$<<>>G_2<<>>G_3<<>>>……………….<<>>G_{t-1}.

Thus G_1<<>>G_2<<>>G_3<<>>>…………….<<>>G_{t-1}<<>>G_t.

Step t: Now let $X_{t-1}=G_1$<<>>G_2<<>>G_3 <<>>>………………. <<>>G_{t-1}<<>>G_t and $G_{t+1} = (V_{t+1}, E_{t+1})$ where $V_{t+1} = \{v_{t+1}, v_{t+1}', v_{t+1}''\}$; $E_{t+1}= \{e_{t+1}, e_{t+1}', e_{t+1}''\}$ we show are the merging is possible for X_{t-1} and G_{t+1}(By strong induction principle)[8] &(By strong mathematical induction method)[9].

Case-1: We are merging G_{t+1} with X_{t-1}. Where X_{t-1} is describe in the case1 of t-1[th] step.

Here there are $4^{t-1}/4$ cases. We are not discussing all the cases. Since the cases are depending on the value of t.

In this case we are getting a 2-regular graph (in all cases we getting 2-regular graphs).

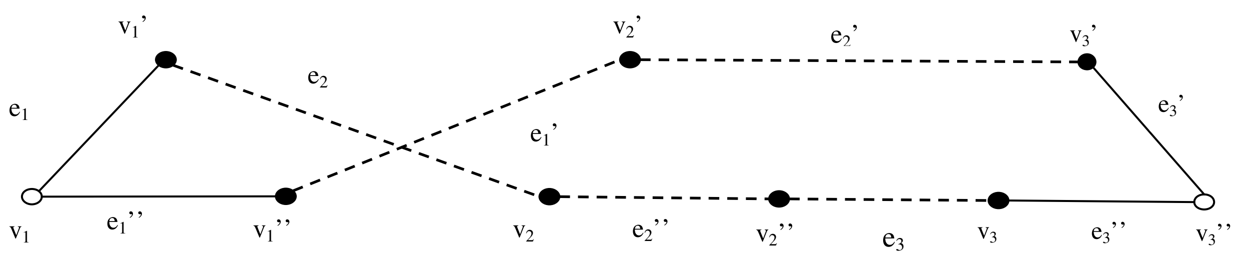

Figure-15: when edge $e_2'= (v_2', v_3')$ and $e_3= (v_2'', v_3)$

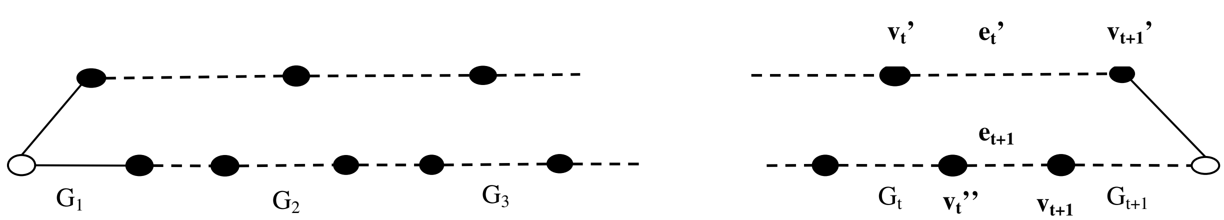

Figure-16: The merging of X_{t-1} and G_{t+1}

Thus the merging of $\mathbf{X_{t-1}}$ and $\mathbf{G_{t+1}}$ is possible**.**

i.e $G_1<<>>G_2<<>>G_3<<>>>\ldots\ldots\ldots\ldots<<>>G_{t-1}<<>>G_t <<>> G_{t+1}$ is possible. Since

$X_{t-1}= G_1<<>>G_2<<>>G_3<<>>>\ldots\ldots\ldots\ldots<<>>G_{t-1}<<>>G_t$

Thus by the strong induction method the merging is possible for all smallest 2-regular graphs.

That is if G_1, G_2, G_3……... G_{m-1}, G_m are smallest 2-regular graphs, then $G_1<<>>G_2<<>>G_3<<>>>\ldots\ldots\ldots\ldots<<>>G_{m-1} <<>>G_m$ is possible. Where, $m\geq 2$ is finite nonnegative integer.

Hence the continuous merging is possible for finitely many smallest 2-regular graphs.

Theorem-2: If G_1, G_2, G_3……... G_{m-1}, G_m are similar 2-regular graphs with finitely many vertices, then show that $G_1<<>>G_2<<>>G_3<<>>>\ldots\ldots\ldots\ldots<<>>G_{m-1}<<>>G_m$. where m is any finite nonnegative integer.

Or

If $n\geq 3$ is represent the number of vertices of similar 2-regular graphs. Then the continuous merging is possible for finitely many similar 2-regular graphs.

Proof: We know that the smallest 2-regular graph contains three vertices and the continuous merging is possible for smallest 2-regular graph. Thus the continuous merging is possible for n=3. (By Theorem 1: The continuous merging is possible for finitely many smallest 2-regular graphs.)

Now, if the number of vertices n>3. Then by the definition of merging technique, the continuous merging is possible for those types of 2-regular graphs.

Hence the continuous merging is possible for all types of similar 2-regular graphs.

Conclusion

The above results are show that the continuous merging is possible for finitely many smallest 2-regular graphs and also the continuous merging is possible for all types of similar 2-regular graphs.

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