Research Journal of Mathematical and Statistical Sciences ISSN 2320-6047
Vol. **5(3),** 1-8, March **(2017)** *Res. J. Mathematical and Statistical Sci.*

An alternative method for choosing a desirable sequential sampling plan

Sandeep Kumar
Department of Statistics, Hindu College/ University of Delhi, India
ch.sandeepkumar@gmail.com

Available online at: www.isca.in, www.isca.me
Received 20th December 2016, revised 5th March 2017, accepted 9th March 2017

Abstract

Several studies deals with Statistical Quality Control of various approaches with respect to Sequential Sampling Plan. The Sequential Sampling Plan is often used to monitor the quality of the product when the sample size needed to terminate the process is a random variable. In this paper, Sequential Sampling Plan analysis have been observed where the quality distribution considered is a Poisson Distribution. The present study deals with a Sequential Sampling Plan have been analysed to make a trade-off between producer's and consumer's risks with respect to OC, ASN, AOQ and ATI.

Keywords: Average Outgoing Quality (AOQ), Average Sample Number (ASN), Average Total Inspection (ATI), Operating Characteristic (OC), Single Sampling Plan (SSP).

Introduction

A good manufacturer for the acceptance purpose always keeps a constant watch on the quality of the products he manufactures. Such alertness enables him to give assurance to the purchaser that the product is according to the given specification of the purchaser. On the other hand, a purchaser is also anxious to satisfy himself about the quality of the products he accepts. In quality control text, the manufacturer and the purchaser are referred to as producer and consumer respectively. In order to assess the quality of the products, the acceptance inspection is based on sampling. All acceptance tests which are destructive in nature must inevitably be done by sampling. In many other instances sampling inspections are used because the cost of 100% inspection is prohibitive. Moreover, when the number of similar items of products to be inspected is very large, sampling inspection is likely to be better than 100% inspection. It has been found that if a scientifically designed sampling plan is used, it provides adequate protection to the producer as well as consumer very economically. In other words, a scientifically designed sampling inspection plan provides protection to consumer against the acceptance of too many defective products, and on the other hand producer is protected against the rejection of too much good product. It is obvious that consumer and producer have completely opposite viewpoints towards the selection of sampling plans i.e., they have conflicting interests. Practically, it has been observed that no sampling plan can give complete protection either to the producer or to the consumer, i.e. a sampling plan is likely to involve some amount of error which will result in producing certain risks on the part of both the producer as well as consumer. These risks will be needful in construction of an acceptance sampling plan. Further variation in the quality of manufactured product in the repetitive process in industries is inevitable. This variation will intern affect the producer's and the consumer's risk.

Sequential sampling is an extension of the double sampling and multiple sampling concepts. In sequential sampling, we take a sequence of samples from the lot and alone the number of samples to be determined entirely by the results of the sampling process. In practice, sequential sampling can theoretically continue indefinitely, until the lot is being inspecting 100%. In practice, sequential sampling plans are usually truncated after the number inspected is equal to three times the number that would have been inspected using a corresponding single sampling plan. If the sample size selected at each stage is greater than one, the process is usually called Group Sequential Sampling. If the sampling size inspected at each stage is one, the procedure is usually called item-by-item sequential sampling. Item-by-item sequential sampling is based on the sequential probability ratio test (SPRT), developed by Wald A.[1].

For organizing a sample acceptance inspection, we need to have a system of rules, called the inspection plans, which enable us to make decisions to accept or reject the lots in the light of sample quality. Consequently, an acceptance sampling plan (ASP) involves the risk of rejecting lots of satisfactory quality and the risk of accepting lots of unsatisfactory quality. While making disposal (accepting or rejecting) of lots with the help of ASP, these faulty decisions go against the interest of the producer and the consumer.

To overcome this difficulty, the sampling inspection is organized so that the risks of erroneous decisions may be minimized. So from practical and economic considerations, different types of ASPs like-single sampling plans, double sampling plans, multiple and sequential sampling plans are used to provide adequate protection to the consumer's and producer's specifications on economic consideration[2]. While all these plans have their own advantages and limitations, still SSP is used in view of its established efficiency over other ASPs in terms of providing pre-assigned protection to producer's and consumer's specifications with reduced sample size. In this regard, the study in Wald A.[1], suggested the ASN function, denoted as $E_\lambda(n)$, as the basis for selecting a desirable SSP out of a class of admissible SSPs.

In view of the above, the present study deals with a sequential sampling plan, when the sample size n needed to terminate the process is a random variable. As such, the ASN function alone cannot be taken as a measure of effectiveness of SSP[3,4]. In this regard, the trend of OC, AOQ and ATI, varying of n for fixed λ, has been suggested as a measure of consistent behavior of n forwards ASN for fixed λ.

In the light of above discussion, OC, AOQ, ATI and ASN of n is proposed as a basis for selecting a desirable sequential sampling plan. Here, in this paper the quality distribution considered is a Poisson distribution. The present study, however, recognizes the fact that in all SSPs. n, the sample size needed to terminate the process is a random variable. As such, the ASN function alone cannot be taken as a measure of effectiveness of SSP[3,4]. In this regard, the volume of $V_\lambda(n)$, variance of n for fixedθ, has been suggested as a measure of the consistent behavior of n towards $E_\lambda(n)$ for fixed λ. Theoretical developments have been highlighted with examples.

Notations

λ = The lot quality in terms of proportion defective. λ_0 = Acceptance quality level (AQL), (Producer's quality specification), λ_1 = Lot tolerance proportion defective (LTPD), (a consumer's quality specification.), L (λ) = Operating characteristics (OC) function. (The probability of accepting a lot of quality *p*.), *n* = The sample size needed to terminate SSP. Thus, *n* is a random sample of size n. $E_p(n)$ = ASN function of fixed λ, α = Producer's risk., β = Consumer's risk. *p.m.f.* = Probability mass function.

Statistical Background

A SSP is characterized by four pre-assigned parameters namely λ_0 (AQL), λ_1 (LTPD), α (Producer's risk) and β (Consumer's risk). SSP can now be developed like a sequential probability ratio test (SPRT) for testing

$H_0 : \lambda = \lambda_0 \quad v/s \quad H_1 : \lambda = \lambda_1$, for pre assigned α and β.

Suppose we have sequentially drawn *n* items from the population with pmf

$$f(x,\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}; \qquad x = 0,1,2,........\infty \tag{1}$$

Thus for pre assigned α and β

The SSP will include the following test procedure: i. If $A_m < m < R_m$ Continue inspection, ii. If $m \le A_m$ Accept H_0 and hence the lot. iii. If $m \ge A_m$ Reject H_0 and hence the lot. Here $A_m = h_0 - ns$ and $R_m = h_1 - ns$

Where, m are the number of observations (m=1, 2, 3,….)

Development of OC function for a desirable SSP

$H_0 : \lambda = \lambda_0 \quad v/s \quad H_1 : \lambda = \lambda_1$

Population recorded from Poisson distribution

$$f(x,\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}; \qquad x = 0,1,2,..........\infty$$

For getting appropriate OC function for the proposed SPRT we consider the expression

$$E\left[\frac{f(x,\lambda_1)}{f(x,\lambda_0)}\right]^{h(\theta)} = 1$$

$$E\left[\frac{e^{-\lambda_1}\lambda_1^{\,x}}{e^{-\lambda_0}\lambda_0^{\,x}}\right]^{h(\theta)} = E\left[e^{h(\lambda_0-\lambda_1)}\left(\frac{\lambda_1}{\lambda_2}\right)^{h(x)}\right] = 1$$

$$\Rightarrow \sum_{x=0}^{\infty} e^{h(\lambda_0-\lambda_1)}\left(\frac{\lambda_1}{\lambda_0}\right)^{hx}\frac{e^{-\lambda}\lambda^x}{x!} = 1$$

$$\Rightarrow e^{h(\lambda_0-\lambda_1)-\lambda}\sum_{x=0}^{\infty}\left(\frac{\lambda_1}{\lambda_0}\right)^{hx}\frac{\lambda^x}{x!} = 1$$

$$\Rightarrow e^{h(\lambda_0-\lambda_1)-\lambda}\sum_{x=0}^{\infty}\frac{\left[\lambda\left(\frac{\lambda_1}{\lambda}\right)^h\right]^x}{x!}e^{-\left[\lambda\left(\frac{\lambda_0}{\lambda_1}\right)^h\right]\left[\lambda\left(\frac{\lambda_1}{\lambda_0}\right)^h\right]} = 1$$

$$\Rightarrow e^{h(\lambda_0-\lambda_1)-\lambda}e^{\lambda\left(\frac{\lambda_0}{\lambda_1}\right)^h}\sum_{x=0}^{\infty}\frac{\left\{\lambda\left(\frac{\lambda_1}{\lambda_0}\right)^h\right\}^x e^{-\left\{\lambda\left(\frac{\lambda_1}{\lambda_0}\right)^h\right\}}}{x!} = 1$$

$$\Rightarrow e^{h(\lambda_0-\lambda_1)-\lambda} e^{\lambda\left(\frac{\lambda_1}{\lambda_0}\right)^h}.1=1$$

Taking Log on both the sides

$$h(\lambda_0-\lambda_1)-\lambda+\lambda\left(\frac{\lambda_1}{\lambda_0}\right)^h=0$$

$$\lambda=\frac{h(\lambda_0-\lambda_1)}{1-\left(\frac{\lambda_1}{\lambda_0}\right)^h}$$

$$L(\lambda)=\frac{A^h-1}{A^h-B^h} \quad (2)$$

Development of ASN function for desirable SSP

The SSP is characterized

$H_0:\lambda=\lambda_0 \quad v/s \quad H_1:\lambda=\lambda_1$

Where observations are being sequentially recorded from the Poisson population with parameter λ.

Now for development of SPRT.

$$Z_i=\log\frac{f(x_i,\lambda_1)}{f(x_i,\lambda_0)}$$

$$\Rightarrow Z_i=\log\frac{e^{-\lambda_1}\lambda_1^{x_i}}{e^{-\lambda_0}\lambda_1^{x_i}}$$

$$\Rightarrow Z_i=\log\left[e^{(\lambda_0-\lambda_1)}\left(\frac{\lambda_1}{\lambda_0}\right)^{x_i}\right]$$

$$=(\lambda_0-\lambda_1)+\log\left(\frac{\lambda_1}{\lambda_0}\right)^{x_i}$$

$$Z_i=(\lambda_0-\lambda_1)+x_i\log\left(\frac{\lambda_1}{\lambda_0}\right)$$

$$\sum_{i=1}^{n}Z_i=(\lambda_0-\lambda_1).n+\sum x_i\log\left(\frac{\lambda_1}{\lambda_0}\right)$$

$$\frac{\sum_{i=1}^{n}Z_i}{n}=(\lambda_0-\lambda_1)+\frac{\sum x_i}{n}\log\left(\frac{\lambda_1}{\lambda_0}\right)$$

$$E_\lambda(z)=(\lambda_0-\lambda_1)+\lambda\log\left(\frac{\lambda_1}{\lambda_0}\right)$$

Therefore,

$$E_\lambda(n)=\frac{L(\lambda)\log B+\left[1-L(\lambda)\right]\log A}{(\lambda_0-\lambda_1)+\lambda\log\left(\frac{\lambda_1}{\lambda_0}\right)} \quad (3)$$

The method using average outgoing quality (AOQ) for the selection of desirable SSP

$$X_A=-h_1+S_n \qquad (\text{Acceptance line})$$

$$X_R=h_2+S_n \qquad (\text{Rejection line})$$

$$h_1=\left(\log\frac{1-\alpha}{\beta}\right)K$$

$$h_2=\left(\log\frac{1-\beta}{\alpha}\right)K$$

K= log[$P_2(1-P_1)/P_1(1-P_2)$]

$S\ n$= log [$(1-P_1)/(1-P_2)$]/ K

The probability of acceptance

$$P_a=\frac{h_2}{h_1+h_2}$$

$$=\frac{\left(\log\frac{1-\beta}{\alpha}\right)K}{\left(\log\frac{1-\alpha}{\beta}+\log\frac{1-\beta}{\alpha}\right)K}$$

$$P_a = \frac{\log \frac{1-\beta}{\alpha}}{\log \frac{1-\alpha}{\beta} + \log \frac{1-\beta}{\alpha}}$$

$$P_a = \frac{\log A}{\log A + \log B}$$

Where

$$A = \frac{1-\beta}{\alpha}; \quad B = \frac{\beta}{1-\alpha}$$

So Average outgoing quality

$$AOQ = \lambda P_a \tag{4}$$

The method using average total inspection (ATI) for the selection of desirable SSP.

According to Wald A.[1], the average total inspection is also easily obtained. Note that the amount of sampling is $\frac{M}{C}$ when a lot is accepted and N when it is rejected. Therefore, the average total inspection is

$$ATI = P_a\left(\frac{M}{C}\right) + (1-P_a)N \tag{5}$$

Where $M = \log \frac{\beta}{1-\alpha}$ and

$$C = E_\lambda(z) = (\lambda_0 - \lambda_1) + \lambda \log\left(\frac{\lambda_1}{\lambda_0}\right)$$

The method using average sample number (ASN) function as a basis for the selection of a desirable SSP.

In the above testing problem, we observed that every SSP is developed with pre-assigned α and β. Let us call such SSP of equal strength (α,β). According to Wald A.[1], we first restrict ourselves to the class of SSP of strength (α,β). Hence, if a SSP exists for which $E_{\lambda_0}(n)$ and $E_{\lambda_1}(n)$ are smaller than the corresponding numbers for any other SSP of the class, then the former SSP may be called as an optimum SSP. To be more specific, if we denote by $n_0(\alpha,\beta)$ the minimum value of $E_{\lambda_0}(n)$ in the class and by $n_1(\alpha,\beta)$ the minimum value of $E_{\lambda_1}(n)$. Then the efficiency of a given SSP in the class under H_0 will be

$$\frac{n_0(\alpha,\beta)}{E_{\lambda_0}(n)} \tag{6}$$

Similarly, its efficiency under H_1 is defined as

$$\frac{n_1(\alpha,\beta)}{E_{\lambda_1}(n)} \tag{7}$$

Clearly, the efficiencies in (6) and (7) lie between 0 and 1.

If $E_\lambda\left(\frac{n}{S_0}\right) = Minimum\ E_\lambda\left(\frac{n}{S}\right)$; for all λ and S. (8)

Then it may be regarded as a uniformly best SSP.

Discussion

As per above discussion, the ASN alone should not be used as the basis for selecting a desirable SSP. Instead, the use of OC, AOQ and ATI are also recommended for comparing SSPs. For pre-assigned α and β, in respect of the consistent behavior of n. The variations of the consistent behavior of the random variable n in respect of variations in the pre-assigned α and β can be analyzed to make a trade-off between α, β, OC, AOQ and ATI for searching a desirable plan.

As per the discussion in above section ASN, AOQ, OC and ATI are helpful for selecting a desirable SSP. With pre-assigned α and β, in respect of the varying the value of λ we observe the behavior of ASN, ATI, AOQ and OC. Now we can compare also the different tables of pre-assigned producer's risk (α) and consumer's risk (β) and analyze for searching a desirable plan. Now the example given below will clarify the concept. Here, R-software and MS-Excel have been used to find the calculations for all the table work and to plot the graphs respectively.

An example: For reaching a trade-off between α, β, OC, AOQ and ATI, we consider four SSPs testing $H_0 : \lambda = 5\ v/s\ H_1 : \lambda = 10$ with four different sets of (α, β) which are $(\alpha = \beta = 0.05)$, $(\alpha = 0.025, \beta = 0.05)$, $(\alpha = 0.05, \beta = 0.025)$ and $(\alpha = \beta = 0.025)$ respectively. Let us call these four SSPs as S_1, S_2, S_3 and S_4 respectively. Now on using the expressions in (2), (3), (4) and (5) for varying λ, we respectively get points on $E_\lambda(n)$, $L(\lambda)$, AOQ and ATI. These points for the four SSPs have been summarized respectively in Table-1, 2 and 3.

For still better analysis of the trends comparative curves of ASN, OC, AOQ and ATI for (S_1, S_2), (S_1, S_3) and (S_1, S_4) are plotted in Figure-1-12.

Table-1: Points on $E_\lambda(n), L(\lambda)$, AOQ and ATI for S_1 and S_2

	$\alpha=\beta=0.05$				$\alpha=0.025,\ \beta=0.05$			
λ	$L(\lambda)$	$E_\lambda(n)$	AOQ	ATI	$L(\lambda)$	$E_\lambda(n)$	AOQ	ATI
3.3333	0.9972	0.1581	1.6666	916.61	0.9993	0.1946	1.8333	917.858
4.2708	0.9840	0.2158	2.1354	961.61	0.9939	0.2565	2.3489	948.27
5.3973	0.9134	0.3181	2.6986	994.23	0.9116	0.3742	2.9685	967.702
6.2596	0.7647	0.3292	3.1298	1012.55	0.8251	0.4146	3.4427	981.309
7.7250	0.3576	0.3332	3.8625	1017.99	0.3885	0.4300	4.2487	989.729
8.8170	0.1465	0.3342	4.0885	1018.81	0.1516	0.4324	4.3494	994.474
9.3973	0.0871	0.3132	3.6986	1018.98	0.0878	0.4244	3.7685	997.204
10.624	0.0286	0.2787	3.3123	1019.99	0.0277	0.3957	3.4435	998.988
11.271	0.0161	0.2173	2.6354	1020.96	0.0154	0.3315	2.7989	999.055
12.626	0.0050	0.1233	1.9129	1021.56	0.0047	0.2177	2.0442	1000.99
13.333	0.0028	0.6259	1.5666	1022.19	0.0025	0.9413	1.7333	1001.73

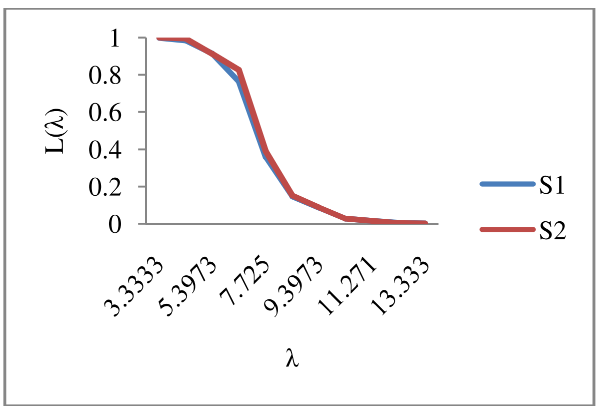

Figure-1: OC Curve for S_1 and S_2.

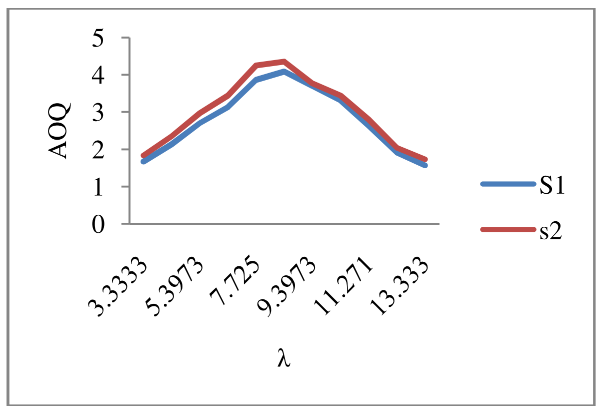

Figure-3: AOQ Curve for S_1 and S_2.

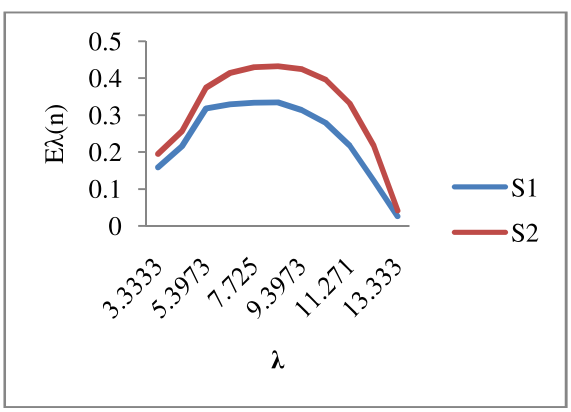

Figure-2: ASN Curve for S_1 and S_2.

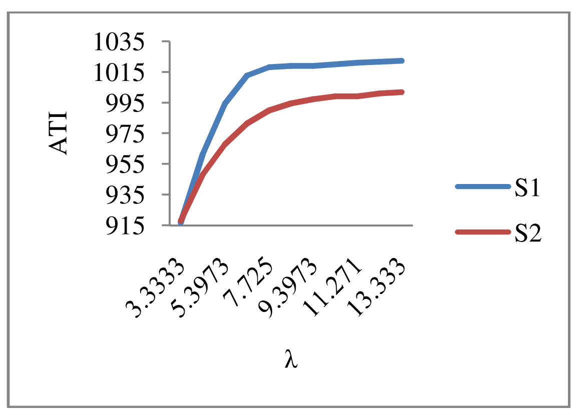

Figure-4: ATI Curve for S_1 and S_2.

Table-2: Points on $E_\lambda(n), L(\lambda)$, AOQ and ATI for S_1 and S_3.

	$\alpha=\beta=0.05$				$\alpha=0.05,\ \beta=0.025$			
λ	$L(\lambda)$	$E_\lambda(n)$	AOQ	ATI	$L(\lambda)$	$E_\lambda(n)$	AOQ	ATI
3.3333	0.9972	0.1581	1.6666	916.61	0.9973	0.1903	1.5000	937.80
4.2708	0.9840	0.2158	2.1354	961.61	0.9844	0.2515	1.9218	1003.21
5.3973	0.9134	0.3181	2.6986	994.23	0.9116	0.2927	2.4288	1058.64
6.2596	0.7647	0.3292	3.1298	1012.55	0.7481	0.4090	2.8168	1092.25
7.7250	0.3576	0.3332	3.8625	1017.99	0.2941	0.4170	3.4762	1110.73
8.8170	0.1465	0.3342	4.0885	1018.81	0.0948	0.4124	3.7677	1117.41
9.3973	0.0871	0.3132	3.6986	1018.98	0.0491	0.3943	3.2288	1120.14
10.624	0.0286	0.2787	3.3123	1019.99	0.0121	0.3531	2.4811	1121.92
11.271	0.0161	0.2173	2.6354	1020.96	0.0059	0.2763	2.0718	1122.98
12.626	0.0050	0.1233	1.9129	1021.56	0.0014	0.1561	1.6816	1123.91
13.333	0.0028	0.6259	1.5666	1022.19	0.0006	0.5973	1.3820	1124.65

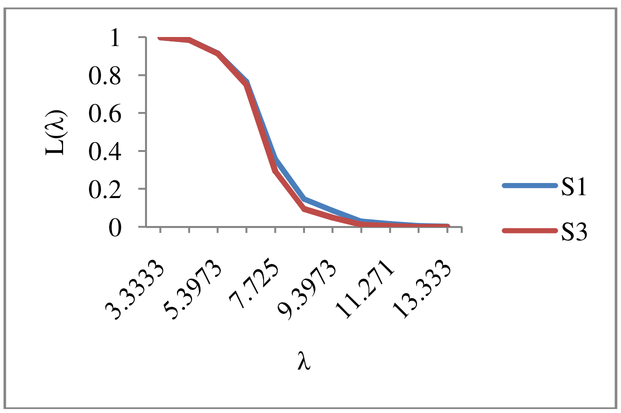

Figure-5: OC Curve for S_1 and S_3.

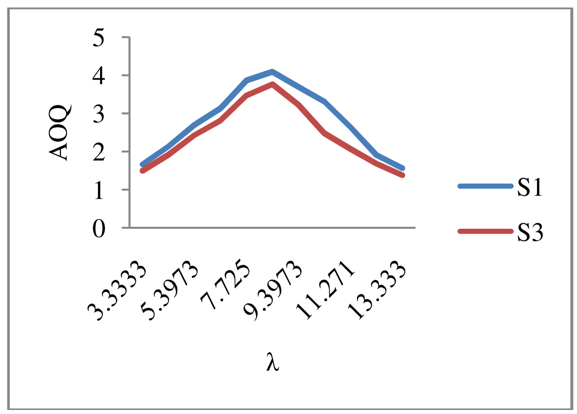

Figure-7: AOQ Curve for S_1 and S_3.

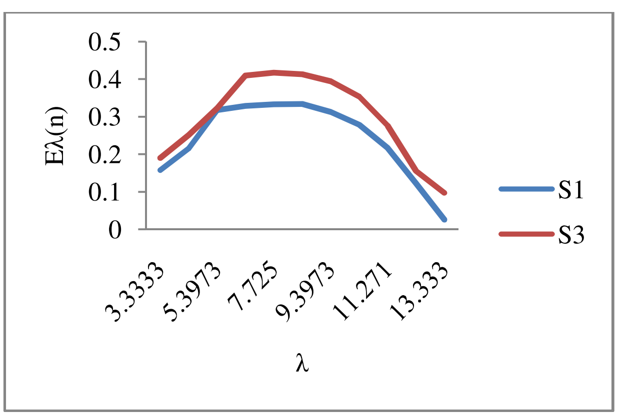

Figure-6: ASN Curve for S_1 and S_3.

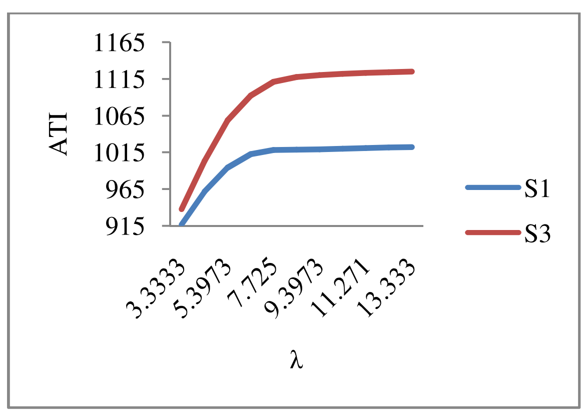

Figure-8: ATI Curve for S_1 and S_3.

Table-3: Points on $E_\lambda(n), L(\lambda)$, AOQ and ATI for S_1 and S_4

	$\alpha=\beta=0.05$				$\alpha=\beta=0.025$			
λ	$L(\lambda)$	$E_\lambda(n)$	AOQ	ATI	$L(\lambda)$	$E_\lambda(n)$	AOQ	ATI
3.3333	0.9972	0.1581	1.6666	916.61	0.9993	0.1949	1.6666	919.779
4.2708	0.9840	0.2158	2.3154	961.61	0.9941	0.2572	2.3154	970.235
5.3973	0.9134	0.3181	2.6986	994.23	0.9492	0.3758	2.6986	1004.71
6.2596	0.7647	0.3292	3.1298	1012.55	0.8119	0.4147	3.1298	1017.39
7.7250	0.3576	0.3332	3.8625	1017.99	0.3224	0.4302	3.8625	1021.92
8.8170	0.1465	0.3342	4.0885	1018.81	0.0984	0.4324	4.2085	1022.68
9.3973	0.0871	0.3132	3.6986	1018.98	0.0496	0.4235	3.8986	1023.48
10.625	0.0286	0.2787	3.3123	1019.99	0.0118	0.3916	3.6123	1024.35
11.271	0.0161	0.2173	2.6354	1020.96	0.0056	0.3185	2.8354	1025.53
12.626	0.0050	0.1233	1.9129	1021.56	0.0013	0.1864	2.3129	1026.57
13.333	0.0028	0.6259	1.5666	1022.19	0.0006	0.7154	1.9666	1027.39

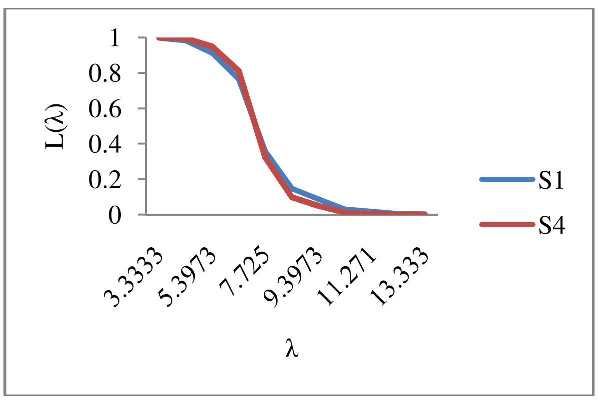

Figure-9: OC Curve for S_1 and S_4.

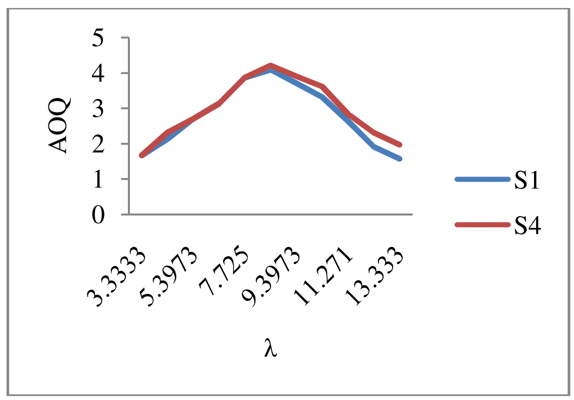

Figure-11: AOQ Curve for S_1 and S_4.

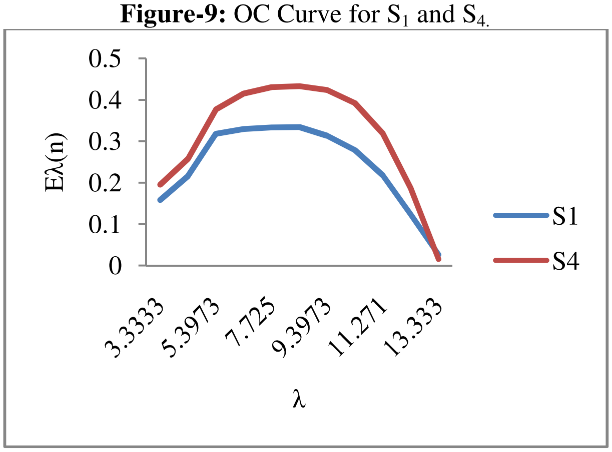

Figure-10: ASN Curve for S_1 and S_4.

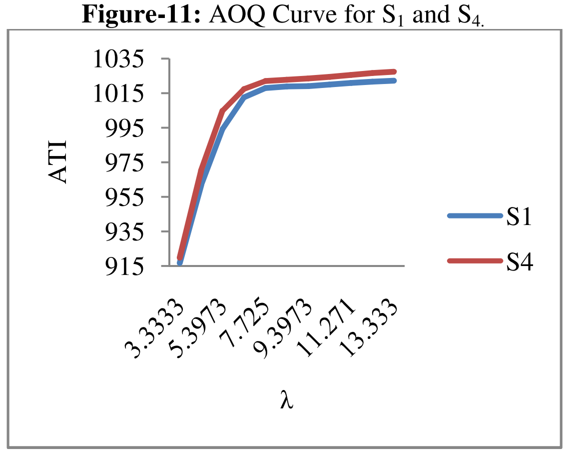

Figure-12: ATI Curve for S_1 and S_4.

Conclusion

Analysis of the results in Table-1, 2 and 3 reveal the following: i. For varying λ, the ASN of a SSP tends to be uniformly higher as any (or both) of the risk's (α and β) is (are) lowered. ii. For varying λ, the OC of a SSP tends to be uniformly higher when both are lowered or only Producer's risk (α) is lowered. iii. For varying λ, the ATI of a SSP tends to be uniformly higher when both (α, β) or β are lowered and uniformly smaller when α is lowered. iv. For varying λ, the AOQ of a SSP tends to be: (a) Uniformly higher as α is lowered. (b) Uniformly smaller asβ is lowered. (c) Constant when both are same.

However, according to the above results the change in consistent behavior of n is substantial when the value of λ is not consistent. We now consider under H_0 or H_1. The following trends in Table-1, 2 and 3, a trade-off between α, β, ASN, ATI, AOQ and OC under consistent behavior of n can be reached to be in search of a desirable SSP. Here one can easily restrict to the class of admissible SSPs of strength. Then, among SSPs and SSP providing maximum consistency of n is considered as most desirable can be obtained. Further such parameters (OC, ATI, AOQ) can also be obtained to assure pre-assigned ASN and consistency of n for a SSP by making suitable variations in the producer's and consumer's risk. However future research, for varying λ, this consistent behavior may be analyzed only by using a relative measure of dispersion, called the co-efficient of variation (C.V.).

References

1. Wald A. (1973). Sequential Analysis. Courier Corporation.
2. Dodge H.F. and Romig H.G. (1959). Sampling inspection tables, Single and Double sampling. 2nd edition, John Wiley, New York.
3. Hall J.E. (1979). Minimum Variance and VOQL Sampling Plans. *Tachometric*, 21(4), 555-565.
4. Vellaisamy P. and Sankar S. (1995). Minimum variance double sampling rectifying inspection plans. *Jour. Indian Statist. Assoc.,* 31, 35-46.
5. Duncan A.J. (1974). Quality Control and Industrial Statistics. Irwin: Homewood, IL
6. Ekambaram S.K. (1960). The statistical basis of Quality control charts. Asia publishing house.
7. Brush G.G., Cautin H. and Lewin B.R. (1981). Outgoing quality distributions for MIL-STD-105 D sampling plans. *Journal of Quality Technology*, 13, 254-263.
8. Govil A.K. (1983). Reliability Engineering. Tata Mc-Graw Hill publishing company limited, New Delhi.
9. Grant E.L. and Leavenworth R.S. (1946). Statistical Quality Control. Mc-Graw Hill, New York.
10. Montgomery D.C. (2007). Introduction to Statistical Quality Control. 6th ed. John Wiley & Sons.
11. Sharma K.K. and Rana R.S. (1990). Roubstness of sequential gamma life testing procedures. *Microelectron Reliab*., 30(6), 1145-1153.
12. Shewhart W.A. (1931). Economic control of quality of manufactured products. ASQ Quality Press.