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An Inventory Model for Log-gamma Distribution Deterioration Rate with Ramp Type Demand and Shortages

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Abstract

In this paper, an inventory model is considered which is reduced not only by demand but also by deterioration. The rate of deterioration is assumed to follow the log-gamma distribution with two parameters. The demand rate is considered as a ramp type function of time. The shortages are allowed and shortages are fully backlogged. The model is derived by minimizing the total inventory cost. The result is demonstrated by considering a numerical example. The model can be used to minimize the total inventory cost for the business enterprises where the demand and deterioration rate are time dependent.

Keywords: Inventory model, Log-gamma distribution, Deterioration, Shortages, Ramp type demand.

Introduction

Inventory refers to idle goods or materials that are held by an organization for use some time in the future. Items carried in inventory include raw materials, purchased parts, work in progress, finished goods and supplies. In general, almost all items deteriorate over time. Deteriorating items are the items that become worst through the passages of time. Keeping this fact in mind, there is a challenging problem for decision makers to maintain the inventory of deteriorating items.

The first inventory model is presented by Harris[1]. Wilson[2], generalized Harris' model and gave a formula for stock control. Whitin[3] has proposed a theory in inventory management in 1957. Ghare and Schrader[4] derived a deterministic inventory model for an exponentially decaying inventory in 1963. Covert and Philip[5] developed an EOQ model for two parameters Weibull distribution deterioration rate of items in 1973.

Some of the work in this field has been established by Chang and Dye[6], Ouyang and Cheng[7], Pal and Mandal[8], Mandal[9], Dash et al.[10], Mishra et al.[11], Sharma and Chaudhary[12] and by many other investigators working in the field. In particular, Chang and Dye[6] established an inventory model with time dependent demand and partial backlogging. Shah and Raykundaliya[13] studied ordering strategy for deteriorating items under trade credit in declining market. Singh and Pattnayak[14] also studied an inventory model for deteriorating items with linear demand, time dependent deterioration and partial backlogging. Tripathy and Mishra[15] also studied an inventory model for deteriorating items with permissible delay in payment. Amutha and Chandrasekaran[16] studied an EOQ model for deteriorating items with quadratic demand and time dependent demand holding cost. Dye et al.[17] developed a deterministic inventory model with a varying rate of deterioration. Roy[18] developed an inventory model for deteriorating items with price dependent demand and variable holding cost. Pareek et al.[19] developed a deterministic inventory model for deteriorating items with time dependent demand and shortages. Skouri et al.[20] given an inventory model for Weibull deterioration rate with ramp type demand and partial backlogging.

In the present paper, an inventory model is developed for deteriorating items by considering deterioration rate as a log-gamma distribution function and demand rate as a ramp type function of time. Shortages are allowed and completely backlogged. In this paper, we made the work of Mandal[9] more realistic by considering deterioration rate as a log-gamma distribution function. Finally, a numerical example is given to illustrate the model.

Assumptions and Notations

Assumptions: The following assumptions are made in developing the proposed model: i. The inventory model is supposed with single item only. ii. The demand rate is variable with respect to time. More precisely, the demand rate is assumed to be ramp-type function of time. iii. The replenishment is instantaneous and its size is finite. iv. Lead time is zero. v. The inventory system is considered over a finite time horizon. vi. The deterioration rate is taken as log-gamma distribution function of time with two parameters (scale and shape).

Notations: The following notations are used for the proposed model: T: The fixed length of each cycle, C_1: The holding cost per unit item per unit time, C_2: The shortage cost per unit item per unit time, C_3: The cost of each deteriorated unit, S: Initial inventory i.e. inventory at t=0, D: Total number of deteriorated items, I (t) : The level of inventory at any time t $(0 \le t \le T)$, $\theta(t)$: The deterioration rate which is taken as log-gamma distribution function of time with two parameters given by

$\theta(t) = \alpha\beta e^{\beta t}$, $\quad 0 < \alpha < 1, \beta > 0, t > 0$

Where: α is a scale parameter and β is shape parameter.

R (t): The demand rate which is taken as ramp-type function of time given by

$R(t) = D_0 \left[t - (t-\mu) H(t-\mu) \right]$

Where: D_0 is initial demand; μ is the time point of ramp type function of time and $H(t-\mu)$ is a Heaviside's function which is defined as follows:

$$H(t-\mu) = \begin{cases} 0 & t \le \mu \\ 1 & t \ge \mu \end{cases}$$

Where: t_1: Time when the inventory level becomes zero. t_1^* : Optimal reorder time. Q: Total production quantity. Q^*: Total optimal production quantity.

Mathematical Formulation and Solution

Let us take Q be the total inventory produced at the starting of each production cycle and after fulfilling backorders, we have an amount S as the initial inventory. Due to market demand and deterioration of items, the stock level reduces from time t=0 to t_1. At the time t_1, the stock level becomes zero and shortages occurring during the period (t_1, T) are completely backlogged. The total number of backlogged items is replaced by the next replenishment.

The rate of change of inventory I (t) at any time t during the stock period (0, t_1) and shortage period (t_1, T) is described by the following differential equations:

$$\frac{dI(t)}{dt} + \theta(t) I(t) = -R(t), \quad 0 \le t \le t_1 \qquad (1)$$

and

$$\frac{dI(t)}{dt} = -R(t), \quad t_1 \le t \le T \qquad (2)$$

Where: the deterioration rate $\theta(t)$ follows the log-gamma distribution function given by

$$\theta(t) = \alpha\beta e^{\beta t}, \quad 0 < \alpha < 1, \beta > 0 \qquad (3)$$

The boundary conditions for the above system can be taken as

$$I(t_1) = 0 \quad and \; I(0) = S \qquad (4)$$

Here, we consider μ < t_1 and therefore, Equations (1) and (2) become

$$\frac{dI(t)}{dt} + \alpha\beta e^{\beta t} I(t) = -D_0 t, \quad 0 \le t \le \mu \qquad (5)$$

$$\frac{dI(t)}{dt} + \alpha\beta e^{\beta t} I(t) = -D_0 \mu, \quad \mu \le t \le t_1 \qquad (6)$$

and

$$\frac{dI(t)}{dt} = -D_0 \mu, \quad t_1 \le t \le T \qquad (7)$$

The above Equations (5) - (7) are linear differential equations. While solving equations (5) and (6), we find integrating factor (I.F.) as follows:

$$I.F. = e^{\int \alpha\beta e^{\beta t} dt} = \exp\left(\alpha e^{\beta t}\right) = 1 + \alpha e^{\beta t} + \frac{\alpha^2}{!2} e^{2\beta t} + ...$$

Since, $0 < \alpha < 1$. Therefore, neglecting the second and higher order terms of α, the solutions of (5) and (6) using condition (4) are given by

$$I(t) = -D_0 \left(\frac{t^2}{2} + \frac{\alpha}{\beta} t e^{\beta t} - \frac{\alpha}{\beta^2} e^{\beta t} - \frac{\alpha t^2}{2} e^{\beta t} \right) + \left\{ S(1+\alpha) - \frac{D_0 \alpha}{\beta^2} \right\} \left(1 - \alpha e^{\beta t}\right), 0 \le t \le \mu \qquad (8)$$

$$I(t) = \left\{ S(1+\alpha) - \frac{D_0 \alpha}{\beta^2} \right\} \left(1 - \alpha e^{\beta t}\right) + D_0 \left(\frac{\mu^2}{2} + \frac{\alpha}{\beta^2} e^{\beta\mu} - \frac{\alpha\mu^2}{2} e^{\beta t} \right) - D_0 \mu \left(t + \frac{\alpha}{\beta} e^{\beta t} - \alpha t e^{\beta t} \right), \mu \le t \le t_1 \qquad (9)$$

The solution of equation (7) is given by

$$I(t) = -D_0 \mu (t - t_1), \; t_1 \le t \le T \qquad (10)$$

Using condition I (t_1) =0 in equation (9) and neglecting the second and higher order terms of α , we get

$$S = D_0 \mu \left\{ t_1 (1-\alpha) + \frac{\alpha}{\beta} e^{\beta t_1} \right\} - D_0 \left\{ \frac{\mu^2}{2} (1-\alpha) + \frac{\alpha}{\beta^2} e^{\beta\mu} - \frac{\alpha}{\beta^2} \right\} \qquad (11)$$

The total number of deteriorated items in the stock period $[0, t_1]$ is given by

$$D = S - \int_0^{t_1} R(t) dt = S - \left[\int_0^{\mu} D_0 t dt + \int_{\mu}^{t_1} D_0 \mu dt \right] \qquad (12)$$

Integrating the integrals appearing in (12) and using equation (11), we get

$$D = D_0\mu\left(\frac{\alpha}{\beta}e^{\beta t_1} - \alpha t_1 + \frac{\alpha\mu}{2} - \frac{\alpha}{\beta^2\mu}e^{\beta\mu} + \frac{\alpha}{\beta^2\mu}\right) \quad (13)$$

Now, the average total inventory cost per unit time is given by

$$C(S,t_1) = \frac{C_1}{T}\int_0^{t_1} I(t)dt - \frac{C_2}{T}\int_{t_1}^{T} I(t)dt + \frac{C_3 D}{T}$$
$$or \quad C(S,t_1) = \frac{C_1}{T}\left[\int_0^{\mu} I(t)\,dt + \int_{\mu}^{t_1} I(t)\,dt\right] - \frac{C_2}{T}\int_{t_1}^{T} I(t) + \frac{C_3 D}{T} \quad (14)$$

Substituting the value of I (t) from (8) - (10) and the value of D from (13) in (14) and then integrating, we get

$$C(t_1) = \frac{D_0\mu C_1}{T}\left(\frac{t_1^2}{2} - \frac{\mu^2}{6} + \frac{3\alpha}{\beta^3\mu}e^{\beta\mu} - \frac{3\alpha}{\beta^3\mu} + \frac{\alpha t_1}{\beta} - \frac{\alpha\mu}{2\beta} - \frac{2\alpha}{\beta^2}e^{\beta t_1} + \frac{\alpha t_1}{\beta}e^{\beta t_1} - \frac{\alpha}{\beta^2}e^{\beta\mu}\right)$$
$$+ \frac{D_0\mu C_2}{2T}(T-t_1)^2 + \frac{D_0\mu C_3}{T}\left(\frac{\alpha}{\beta}e^{\beta t_1} - \alpha t_1 + \frac{\alpha\mu}{2} - \frac{\alpha}{\beta^2\mu}e^{\beta\mu} + \frac{\alpha}{\beta^2\mu}\right) \ldots\ldots(15)$$

The condition for minimization of C (t_1) is $\frac{dC(t_1)}{dt_1} = 0,$ which gives

$$C_1\left(t_1 + \frac{\alpha}{\beta} - \frac{\alpha}{\beta}e^{\beta t_1} + \alpha t_1 e^{\beta t_1}\right) + C_2(t_1 - T) + C_3\left(\alpha e^{\beta t_1} - \alpha\right) = 0 \quad (16)$$

Now, we solve (16) for t_1 as follows:
Let $g(t_1) =$

$$C_1\left(t_1 + \frac{\alpha}{\beta} - \frac{\alpha}{\beta}e^{\beta t_1} + \alpha t_1 e^{\beta t_1}\right) + C_2(t_1 - T) + C_3\left(\alpha e^{\beta t_1} - \alpha\right) \quad (17)$$

Therefore, we have
$g(0) = -C_2 T < 0$ and $g(T) > 0$ so that $g(0)\,g(T) < 0$.

Also, we find

$$g'(t_1) = C_1\left(1 + \alpha\beta t_1 e^{\beta t_1}\right) + C_2 + C_3\alpha\beta e^{\beta t_1} > 0 \quad (18)$$

It implies that $g(t_1)$ is a monotonic increasing function and equation (16) has unique solution at $t_1 = t_1^*$ $for\ t_1^* \in (0,T)$.
Again, we have

$$\frac{d^2C(t_1)}{dt_1^2}\Big|_{t_1 = t_1^*} = \frac{D_0\mu}{T}\left\{C_3\alpha\beta e^{\beta t_1^*} + C_1\left(1 + \alpha\beta e^{\beta t_1^*}\right)\right\} > 0 \quad (19)$$

Which is the sufficient condition for minimization of the average inventory cost $C(t_1)$.

Now, putting $t_1 = t_1^*$ in the equation (11), the optimum value of S is given by

$$S^* = D_0\mu\left\{t_1^*(1-\alpha) + \frac{\alpha}{\beta}e^{\beta t_1^*}\right\} - D_0\left\{\frac{\mu^2}{2}(1-\alpha) + \frac{\alpha}{\beta^2}e^{\beta\mu} - \frac{\alpha}{\beta^2}\right\} \quad (20)$$

At the end of the cycle, the total backlogged item is

$$\int_{t_1}^{T} D_0\mu dt = D_0\mu(T - t_1)$$

Therefore, the optimum value of total production quantity Q is given by

$Q^* = S^* + D_0\mu(T - t_1^*)$ or

$$Q^* = D_0\mu\left\{\frac{\alpha}{\beta}e^{\beta t_1^*} - \alpha t_1^* - \frac{\mu}{2}(1-\alpha) - \frac{\alpha}{\beta^2\mu}e^{\beta\mu} + \frac{\alpha}{\beta^2\mu} + T\right\} \quad (21)$$

The optimum value of the average total inventory cost $C(t_1)$ *is* given by

$$C(t_1^*) = \frac{D_0\mu C_1}{T}\left(\frac{t_1^{*2}}{2} - \frac{\mu^2}{6} + \frac{3\alpha}{\beta^3\mu}e^{\beta\mu} - \frac{3\alpha}{\beta^3\mu} + \frac{\alpha t_1^*}{\beta} - \frac{\alpha\mu}{2\beta} - \frac{2\alpha}{\beta^2}e^{\beta t_1^*} + \frac{\alpha t_1^*}{\beta}e^{\beta t_1^*} - \frac{\alpha}{\beta^2}e^{\beta\mu}\right)$$
$$+ \frac{D_0\mu C_2}{2T}(T-t_1^*)^2 + \frac{D_0\mu C_3}{T}\left(\frac{\alpha}{\beta}e^{\beta t_1^*} - \alpha t_1^* + \frac{\alpha\mu}{2} - \frac{\alpha}{\beta^2\mu}e^{\beta\mu} + \frac{\alpha}{\beta^2\mu}\right) \quad (22)$$

Case of Absence of Deterioration

In the absence of deterioration (i.e. α =0), the equation (16) takes the form

$$(C_1 + C_2)t_1 - C_2 T = 0 \quad (23)$$

Which is linear equation in t_1 for which the optimum value of t_1 is given by

$$t_1^* = \frac{C_2 T}{C_1 + C_2} \quad (24)$$

Again, putting α =0 in the Equation (20) and (21) we get

$$S^* = D_0\mu\left(\frac{C_2 T}{C_1 + C_2} - \frac{\mu}{2}\right) \quad (25)$$

and

$$Q^* = D_0\mu\left(T - \frac{\mu}{2}\right) \quad (26)$$

and

Similarly, putting α =0 in (22) the average total inventory cost is given by

$$C(t_1^*) = \frac{D_0\mu C_1}{2T}\left(t_1^{*2} - \frac{\mu^2}{3}\right) + \frac{D_0\mu C_2}{2T}(T - t_1^*)^2 \quad (27)$$

Numerical Example

Let us consider an inventory model with the following values of parameters: C_1=\$3.0 per unit per year, C_2=\$16.0 per unit per year, C_3=\$6.0 per year, β=2, α = 0.002, D_0=100units, μ=0.14year, T=1year.

The solution of the non-linear equation (16) by using Newton-Raphson method, starting with the value of t_1=0.8 and above values of parameters, the optimum value of t_1 is given by

t_1^* =0.838 year.

Taking t_1^* =0.838 year, we get the following optimal values:
Q^*=13.057 units, S^*=10.789 units, C (t_1^*) =\$17.971 per year.

Conclusion

In the present paper, we developed an inventory model by considering deterioration rate as a log-gamma distribution function and demand rate as a ramp type function of time. Shortages are allowed and completely backlogged. In the present paper, deterioration cost, inventory holding cost and shortage cost are considered. The model is solved analytically by minimizing the total inventory cost. Finally, the proposed model has been verified graphically as shown below:

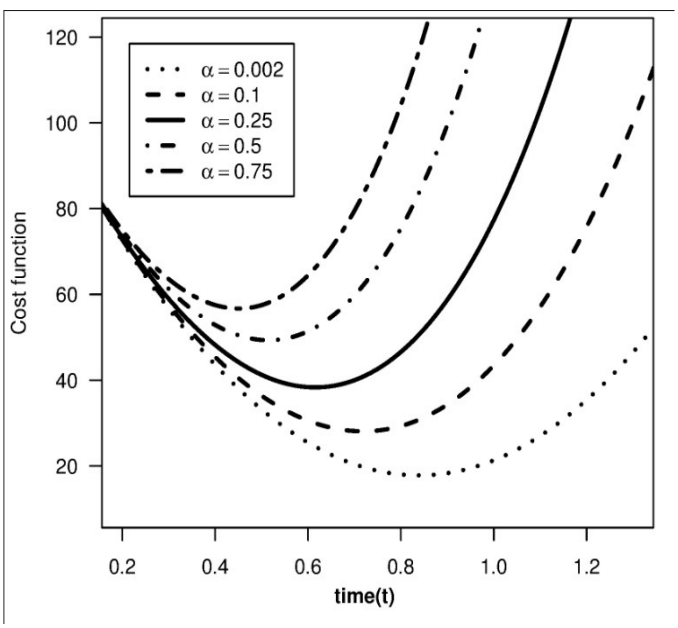

Figure-1
Showing variation of cost function with time

The present inventory model can be extended in several ways. For example, shortages can be considered at beginning. Moreover, the system may be generalized for partial backlogging shortages.

References

1. Harris F.W. (1915). #Operations and Cost.# A.W. Shaw Company Chicago, 48-54.
2. Wilson R.H. (1934). #A Scientific Routine for Stock Control.# *Harvard Business Review,* 13(1), 116-128.
3. Whitin T.M. (1957). #Theory of Inventory Management.# Princeton University Press, Princeton, 62-72.
4. Ghare P.M. and Schrader G.P. (1963). #A Model for Exponentially Decaying Inventory.# *Journal of Industrial Engineering,* 14, 238-243.
5. Covert R.P. and Philip G.C. (1973). #An EOQ Model for items with Weibull Distribution Deterioration.# *AIIF Trans,* 5, 323-326.
6. Chang H.J. and Dye C.Y. (1999) #An EOQ Model for Deteriorating Items with Time Varying Demand and Partial Backlogging.# *Journal of Operational Research Society,* 50, 1176-1182.
7. Ouyang L.Y., Ks Wu and Cheng M.C. (2005). #An Inventory Model for Deteriorating Items with Exponential Declining Demand and Partial Backlogging.# *Yugoslav Journal of Operations Research,* 15(2), 277-288.
8. Pal A.K. and Mandal B. (1997). #An EOQ Model for Deteriorating Inventory with Alternating Demand Rates.# *Korean Journal of Computer and Applied Mathematics,* 4(2), 397-407.
9. Mandal B. (2010). #An EOQ Inventory Model for Weibull Distributed Deteriorating under Ramp Type Demand and Shortages.# *Opsearch,* 42(2), 158-165.
10. Dash B.P., Singh T. and Pattnayak H. (2014) #An Inventory Model for Deteriorating Items with Exponential Declining Demand and Time-Varying Holding Cost.# *American Journal of Operational Research,* 4, 1-7.
11. Mishra V. K., Singh L.S. and Kumar R. (2013). #An Inventory Model for Deteriorating Items with Time Dependent Demand and Time Varying Holding Cost under Partial Backlogging.# *Journal of Industrial Engineering International,* 9(4).
12. Sharma V. and Chaudhary R.R. (2013). #An Inventory Model for Deteriorating Items with Weibull Deterioration with Time Dependent Demand and Shortages.# *Research Journal of Management Sciences,* 2(3), 28-30.
13. Shah N.H. and Raykundaliya N. (2010). #Retailer's Pricing and Ordering Strategy for Weibull Distribution Deterioration under Trade Credit in Declining Market.# *Applied Mathematical Sciences,* 4(21), 1011-1020.
14. Singh T. and Pattnayak H. (2013). #An EOQ Model for Deteriorating Items with Linear Demand, Variable Deterioration and Partial Backlogging.# *Journal of Service Science and Management,* 6(2), 186-190.

15. Tripathy C.K. and Mishra U. (2010). #Ordering Policy for Weibull Deteriorating Items for Quadratic Demand with Permissible Delay in Payment.# *Applied Mathematical Science,* 4(44), 2181-2191.

16. Amutha R. and Chandrasekaran E. (2013). #An EOQ Model for Deteriorating Items with Quadratic Demand and Time Dependent Holding Cost.# *International Journal of Emerging Science and Engineering,* 1(5), 5-6.

17. Dye C.Y., Ouyang L.Y. and Hsieh T.P. (2007). #Deterministic Inventory Model for Deteriorating Items with Capacity Constraints and Time-proportional Backlogging Rate.# *European Journal of Operational Research,* 178(3), 789-807.

18. Roy A. (2008). #An Inventory Model for Deteriorating Items with Price Dependent Demand and Time Varying Holding Cost.# *Adv. Modeling Opt,* 10, 25-37.

19. Pareek S., Mishra V.K. and Rani S. (2009). #An Inventory Model for Time Dependent Deteriorating Item with Salvage Value and Shortages.# *Math Today,* 25, 31-39.

20. Skouri K., Konstantaras I., Papachristos S. and Ganas I. (2009). #Inventory Models with Ramp Type Demand Rate, Partial Backlogging and Weibull Deterioration rate.# *European Journal of Operational Research,* 192, 79-92.