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Solution of an Extended Fractional Programming Problem with Non-Differentiable Function

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Abstract

Method for solving three different problems of extended linear plus linear fractional programming problems in which non-differentiable term occurs in the form of square root of a positive definite quadratic form either in constraints or in objective function or in both is obtained. In the method, model is reduced to fractional programming problem and its solution can be obtained by using programming theorems. Solution of the original problem can then be obtained through the solution of fractional programming problem.

Keywords: Non-differentiable fraction, Optimal solution, Positive definite Quadratic form, Programming problem.

Introduction

Recently much attention has been given[2-4,8,9] to mathematical programming problems with non-differentiable functions/terms either in objective function or in constraints. Many researchers have either derived optimality conditions or established duality relations for linear programming problems and fractional programming problems including a non-differentiable term $(x'Bx)^{\frac{1}{2}}$ either in objective function or in constraints[1,5-7]. Programming problem with non-differentiable term arises in stochastic programming problem in which the coefficients of one of the constraints are random variables.

Here we have considered an extended fractional programming problem with non-differentiable terms. 3 different types of problems are developed here. In type I problem, we consider non-differentiable terms in constraints.

In type II problem, we consider non-differentiable term in objective function. In type III problem, we consider non-differentiable terms in both objective function and constraint. Mathematical formulation of different type problems are as follows:-

I Type Problem:

Minimize $F(x) = (a'x+\gamma)^{K_1} + \dfrac{(d'x+\alpha)}{(c'x+\beta)^{K_2}}$

Subject to $Ax \le b$

$$p'x + (x'Bx)^{\frac{1}{2}} \le 1 \quad (1)$$

and $x \ge 0$

Non-differentiable term occurs only in constraints.

II Type Problem:

$$Minimize\ \ F(x) = \left[a'x+\gamma+(x'Bx)^{\frac{1}{2}}\right]^{K_1} + \frac{\left(d'x+\alpha+(x'Bx)^{\frac{1}{2}}\right)}{(c'x+\beta)^{K_2}}$$

Subject to Ax ≤ b and $x \ge 0$ (2)

Non-differentiable term occurs only in objective function.

III Type Problem:

$$\text{Minimize}\ \ F(x) = \left[a'x+\gamma + (x'Bx)^{\frac{1}{2}}\right]^{K_1} \frac{\left(d'x+\alpha+(x'Bx)^{\frac{1}{2}}\right)}{(c'x+\beta)^{K_2}}$$

Subject to $Ax \le b$

$$p'x + (x'Bx)^{\frac{1}{2}} \le 1 \quad (3)$$

and $x \ge 0$

Non-differentiable term occurs both in objective function and constraints.

Earlier, the available algorithms for quadratic programming can solve such programming problem. Here we have provided an alternative method to solve the extended fractional programming problem with non-differentiable terms by reducing it to a fractional programming problem and its solution can be obtained by using theorems.

Solution of extended fractional programming problem can then be obtained through the solution of fractional programming problem.

Solution

Method of solution is given for all three types of extended fractional programming problems in which a non-linear term occurs in constraints or in objective function or in both in the form of square root of a positive definite quadratic form. The problems under consideration are (1), (2) and (3) in which A is an $(m\ x\ n)$ matrix, B is a symmetric positive definite matrix of order n, decision variable x is an n-vector and *p,d,c,a,b* are vectors and α, β, γ are scalars, Prime (′) denotes the transpose of a vector and k_1,k_2 are positive integers. It is assumed that the general constraint set $S = \left\{x: Ax \le b, p'x + (x'Bx)^{\frac{1}{2}} \le 1, x \ge 0\right\}$ is nonempty and bounded and $\forall\, x \epsilon R, a'x + \gamma \ge 0, (d'x + \alpha) \ge 0\ and\ (c'x + \beta) > 0.$

Solution of I Type of Problem: $Minimize\ F(x) = (a'x + \gamma)^{K_1} + \frac{(d'x+\alpha)}{(c'x+\beta)^{K_2}}$

Subject to $Ax \le b$

$$p'x + (x'Bx)^{\frac{1}{2}} \le 1 \qquad (4)$$

and $x \ge 0$

By using parametric substitution $y = tx$, the above problem reduces to

$$\text{Minimize } G(x,t) = \frac{(a'x + \gamma t)^{K_1}}{t^{K_1}} + \frac{(d'x + \alpha t)t^{K_2}}{t(c'x + \beta t)^{K_2}}$$

$Subject\ to\ \ Ax - bt \le 0$

$$p'x + 1 \le t \qquad (5)$$

$x'Bx \le 1$ and $x \ge 0,\ t > 0$

The following theorems in their generalized form are useful for determining the solution of problem (4).

Theorem 1: If (x_s, t_s) is an optimal solution of reduced fractional programming problem (5), then $t_s > 0$.

In this theorem we are given that (x_s, t_s) is an optimal solution of reduced fractional programming problem, then we have to prove that $t_s > 0$. Here we suppose that $x_s \ne 0\ \ and\ \ t_s = 0$ be an optimal solution of reduced fractional programming problem and take an element $\bar{x}$ from a set of S, which is bounded, but if some positive number is added in it, then addition will be also a real number and contained is S, i.e., $x_\mu = \bar{x} + \mu x_s$ is in S of all $\mu \ge 0$ and in that case S is unbounded, which contradicts our assumption that S is bounded. Hence $x_s \ne 0\ and\ t_s = 0$ cannot be an optimal solution of reduced fractional programming problem. Now, we take both variables $x_s = 0\ and\ \ t_s = 0$ be an optimal solution of reduced fractional programming problem. Then by using the definition of an optimal solution, i.e., it must satisfy objective function, so

$$\text{Minimize } G(x_s,t_s) = \frac{(a'x_s + \gamma t_s)^{K_1}}{{t_s}^{K_1}} + \frac{(d'x_s + \alpha t_s){t_s}^{K_2}}{t_s(c'x_s + \beta t_s)^{K_2}}$$

or

$$G(x_s,t_s) = \frac{(a'x_s + \gamma t_s)^{K_1}(c'x_s + \beta t_s)^{K_2} t_s + {t_s}^{K_1+K_2}(d'x_s + \alpha t_s)}{{t_s}^{K_1+1}(c'x_s + \beta t_s)^{K_2}} = \infty$$

$\Rightarrow x_s = 0, t_s = 0$ is not feasible for reduced fractional programming problem. Hence $t_s > 0$.

Theorem 2: If x_1 is an optimal solution of extended fractional programming problem (4) then $\exists\ \ t_1 = \frac{1}{(x_1'Bx_1)^{\frac{1}{2}}} > 0$ such that (t_1x_1, t_1) is an optimal solution of reduced fractional programming problem (5).

Let x_1 be an optimal solution of extended fractional programming problem (4) and take $t = t_1 = \frac{1}{(x_1'Bx_1)^{\frac{1}{2}}} > 0$.

Then (t_1x_1, t_1) is feasible solution of reduced fractional programming problem (5).

If (x_s, t_s) is an optimal solution of reduced fractional programming problem (5), then

$$G(x_s,t_s) = \frac{(a'x_s + \gamma t_s)^{K_1}(c'x_s + \beta t_s)^{K_2} t_s + (d'x_s + \alpha t_s){t_s}^{K_1+K_2}}{{t_s}^{K_1+1}(c'x_s + \beta t_s)^{K_2}}$$

$$\le \frac{(a'x_1t_1 + \gamma t_1)^{K_1}(c'x_1t_1 + \beta t_1)^{K_2} t_1 + (d'x_1t_1 + \alpha t_1){t_1}^{K_1+K_2}}{{t_1}^{K_1+1}(c'x_1t_1 + \beta t_1)^{K_2}}$$

$$\le \frac{{t_1}^{K_1+K_2+1}(a'x_1 + \gamma)^{K_1}(c'x_1 + \beta)^{K_2} + (d'x_1 + \alpha)t^{K_1+K_2+1}}{(c'x_1 + \beta)^{K_2}\quad {t_1}^{K_1+K_2+1}}$$

$$\le (a'x_1 + \gamma)^{K_1} + \frac{(d'x_1 + \alpha)}{(c'x_1 + \beta)^{K_2}}$$

$$G(x_s,t_s) \le F(x_1) \qquad \text{(i)}$$

Since $t_s > 0$ from theorem 1, and $x_s \ge 0$ by non-negative constraints, so it is obvious that $\left(\frac{x_s}{t_s}\right)$ is feasible for (4) and hence

$$F(x_1) = (a'x_1 + \gamma)^{K_1} + \frac{(d'x_1 + \alpha)}{(c'x_1 + \beta)^{K_2}} \le F\left(\frac{x_s}{t_s}\right)$$

$$\le \left(\frac{a'x_s}{t_s} + \gamma\right)^{K_1} + \frac{\left(\frac{d'x_s}{t_s} + \alpha\right)}{\left(\frac{c'x_st_1}{t_s} + \beta\right)^{K_2}}$$

$$\le \frac{(a'x_s + \gamma t_s)^{K_1}}{{t_s}^{K_1}} + \frac{(d'x_s + \alpha t_s){t_s}^{K_2}}{t_s(c'x_s + \beta t_s)^{K_2}}$$

$$F(x_1) \le G(x_s,t_s) \qquad \text{(ii)}$$

(i) and (ii) $\Longrightarrow$ $F(x_1) = G(x_s,t_s)$

or

$$F(x_1) = (a'x_1 + \gamma)^{K_1} + \frac{(d'x_1 + \alpha)}{(c'x_1 + \beta)^{K_2}}$$

$$= \frac{(a'x_1t_1 + \gamma t_1)^{K_1}}{{t_1}^{K_1}} + \frac{(d'x_1t_1 + \alpha t_1){t_1}^{K_2}}{t_1(c'x_1t_1 + \beta t_1)^{K_2}} = G(x_1t_1, t_1)$$

Hence (t_1x_1, t_1) is an optimal solution of reduced fractional programming problem (5).

Theorem 3: If (x_s, t_s) is an optimal solution of reduced fractional programming problem (5) then $\frac{x_s}{t_s}$ is an optimal solution of extended fractional problem (4).

Suppose that x_1 is an optimal solution of extended fractional programming problem (4); then there exists a $t = t_1 = \frac{1}{(x_1'Bx_1)^{\frac{1}{2}}} > 0$ such that (t_1x_1, t_1) is feasible solution of reduced fractional programming problem (5).

Since (x_s, t_s) is an optimal solution of reduced fractional programming problem (5), so

$$G(x_s, t_s) = \frac{(a'x_s + \gamma t_s)^{K_1}}{{t_s}^{K_1}} + \frac{(d'x_s + \alpha t_s){t_s}^{K_2}}{t_s(c'x_s + \beta t_s)^{K_2}}$$

$$\leq \frac{(a'x_1t_1 + \gamma t_1)^{K_1}}{{t_1}^{K_1}} + \frac{(d'x_1t_1 + \alpha t_1){t_1}^{K_2}}{t_1(c'x_1t_1 + \beta t_1)^{K_2}}$$

$$\leq (a^1x_1 + \gamma)^{K_1} + \frac{(d^1x_1 + \alpha)}{(c^1x_1 + \beta)^{K_2}}$$

$$G(x_s, t_s) \leq F(x_1) \qquad \text{(iii)}$$

Since $t_s > 0$ from theorem 1, and $x_s \geq 0$ by non-negative constraints, so it is obvious that $\frac{x_s}{t_s}$ is feasible for extended fractional programming problem (4) and we have

$$F\left(\frac{x_s}{t_s}\right) = \left(a'\frac{x_s}{t_s} + \gamma\right)^{K_1} + \frac{\left(d'\frac{x_s}{t_s} + \alpha\right)}{\left(c'\frac{x_s}{t_s} + \beta\right)^{K_2}}$$

$$= \frac{(a'x_s + \gamma t_s)^{K_1}}{{t_s}^{K_1}} + \frac{(d'x_s + \alpha t_s){t_s}^{K_2}}{t_s(c'x_s + \beta t_s)^{K_2}}$$

$$= G(x_s, t_s) \leq F(x_1)$$

Hence $\left(\frac{x_s}{t_s}\right)$ is an optimal solution of extended fractional programming problem (4). Now, it is clear from the above theorems that for solving the extended fractional programming problem (4), it is sufficient to solve the reduced fractional programming problem (5). The reduced fractional programming problem (5) can be solved by either existing methods of convex programming or the method given by earlier researchers.

Solution of II Type Problem: Consider the problem, in which non-differentiable term occurs only in objective function:

$$Minimize \;\; F(x) = \left[a'x + \gamma + (x'Bx)^{\frac{1}{2}}\right]^{K_1} + \frac{\left(d'x + \alpha + (x'Bx)^{\frac{1}{2}}\right)}{(c'x + \beta)^{K_2}}$$

$$Subject\ to \;\; Ax \leq b \qquad (6)$$

and $\quad x \geq 0$

By using parametric substitution $y = tx$ and $t = \frac{1}{(x'Bx)^{\frac{1}{2}}} > 0$, the above problem reduces to

$$Minimize\ G(x,t) = \left(a'\frac{x}{t} + \gamma + \frac{1}{t}\right)^{K_1} + \frac{\left(d'\frac{x}{t} + \alpha + \frac{1}{t}\right)}{\left(c'\frac{x}{t} + \beta\right)^{K_2}}$$

or

$$G(x,t) = \frac{(a'x + \gamma t + 1)^{K_1}}{t^{K_1}} + \frac{(d'x + \alpha t + 1)t^{K_2}}{t(c'x + \beta t)^{K_2}}$$

$$Subject\ to \;\; Ax - bt \leq 0$$

$$x'Bx \leq 1$$

and $\quad x \geq 0, t > 0 \qquad (7)$

Theorem 4: If (x_s, t_s) is an optimal solution of reduced fractional programming problem (7) then $t_s > 0$.
It follows from Theorem 1.

Theorem 5: If x_1 is an optimal solution of extended fractional programming problem (6) then $\exists \;\; t_1 = \frac{1}{(x_1'Bx_1)^{\frac{1}{2}}} > 0$ is an optimal solution of reduced fractional programming problem (7).

Let x_1 be an optimal solution of extended fractional programming problem (6) and take $t = t_1 = \frac{1}{(x_1'Bx_1)^{\frac{1}{2}}} > 0$. Then (t_1x_1, t_1) is feasible solution of reduced fractional programming problem. if (x_s, t_s) is an optimal solution of reduced fractional programming problem (7), then

$$G(x_s, t_s) = \frac{(a'x_s + \gamma t_s + 1)^{K_1}}{{t_s}^{K_1}} + \frac{(d'x_s + \alpha t_s + 1){t_s}^{K_2}}{t_s(c'x_s + \beta t_s)^{K_2}}$$

$$\leq \frac{(a'x_1t_1 + \gamma t_1 + 1)^{K_1}}{{t_1}^{K_1}} + \frac{(d'x_1t_1 + \alpha t_1 + 1){t_1}^{K_2}}{t_1(c'x_1t_1 + \beta t_1)^{K_2}}$$

$$\leq \left(a'x_1 + \gamma + \frac{1}{t_1}\right)^{K_1} + \frac{\left(d'x_1 + \alpha + \frac{1}{t_1}\right)}{(c'x_1 + \beta)^{K_2}}$$

$$\leq \left[a'x_1 + \gamma + (x_1'Bx_1)^{\frac{1}{2}}\right]^{K_1} + \frac{\left[d'x_1 + \alpha + (x_1'Bx_1)^{\frac{1}{2}}\right]}{(c'x_1 + \beta)^{K_2}}$$

$$G(x_s, t_s) \leq F(x_1) \qquad \text{(iv)}$$

Since $t_s > 0$ from Theorem 1, and $x_s \geq 0$ by non-negative constraints, so it is obvious that $\left(\frac{x_s}{t_s}\right)$ is feasible for (6) and hence $F(x_1) = \left[a'x_1 + \gamma + (x_1'Bx_1)^{\frac{1}{2}}\right]^{K_1} + \frac{\left[d'x_1+\alpha+(x_1'Bx_1)^{\frac{1}{2}}\right]}{(c'x_1+\beta)^{K_2}}$

$$\leq F\left(\frac{x_s}{t_s}\right)$$

$$\leq \left[a'\frac{x_s}{t_s} + \gamma + \frac{(x_s'Bx_s)^{\frac{1}{2}}}{t_s}\right]^{K_1} + \frac{\left[d'\frac{x_s}{t_s} + \alpha + \frac{(x_s'Bx_s)^{\frac{1}{2}}}{t_s}\right]}{\left(c'\frac{x_s}{t_s} + \beta\right)^{K_2}}$$

$$\leq \frac{\left(a'x_s + \gamma t_s + (x_s'Bx_s)^{\frac{1}{2}}\right)^{K_1}}{{t_s}^{K_1}} + \frac{{t_s}^{K_2}\left(d'x_s + \alpha t_s + (x_s'Bx_s)^{\frac{1}{2}}\right)}{t_s(c'x_s + \beta t_s)^{K_2}}$$

$$\leq \frac{(a'x_s + \gamma t_s + 1)^{K_1}}{{t_s}^{K_1}} + \frac{(d'x_s + \alpha t_s + 1){t_s}^{K_2}}{t_s(c'x_s + \beta t_s)^{K_2}}$$

$$F(x_1) \leq G(x_s, t_s) \qquad \text{(v)}$$

(iv) and (v) $\Rightarrow F(x_1) \leq G(x_s, t_s)$

$$F(x_1) = \left[a'x_1 + \gamma + (x_1'Bx_1)^{\frac{1}{2}}\right]^{K_1} + \frac{\left[d'x_1 + \alpha + (x_1'Bx_1)^{\frac{1}{2}}\right]}{(c'x_1 + \beta)^{K_2}}$$

$$F(x_1) = \frac{(a'x_1t_1 + \gamma t_1 + 1)^{K_1}}{{t_1}^{K_1}} + \frac{(d'x_1t_1 + \alpha t_1 + 1)}{t_1(c'x_1 + \beta)^{K_2}}$$

$$= \frac{(a'x_1t_1+\gamma t_1+1)^{K_1}}{{t_1}^{K_1}} + \frac{{t_1}^{K_2}(d'x_1t_1+\alpha t_1+1)}{t_1(c'x_1t_1+\beta t_1)^{K_2}}$$

$$= G(x_1t_1, t_1)$$

Hence (t_1x_1, t_1) is an optimal solution of reduced fractional programming problem (7).

Theorem 6: If (x_s, t_s) is an optimal solution of reduced fractional programming problem (7) then $\left(\frac{x_s}{t_s}\right)$ is an optimal solution of extended fractional programming problem (6).

Suppose that x_1 is an optimal solution of extended fractional programming problem (6), then $\exists\ t = t_1 = \frac{1}{(x_1'Bx_1)^{\frac{1}{2}}} > 0$ such that (t_1x_1, t_1) is feasible solution of reduced fractional programming problem (7). Since (x_s, t_s) is an optimal solution of reduced fractional programming problem (7), so

$$G(x_s, t_s) = \frac{(a'x_s + \gamma t_s + 1)^{K_1}}{{t_s}^{K_1}} + \frac{(d'x_s + \alpha t_s + 1){t_s}^{K_2}}{t_s(c'x_s + \beta t_s)^{K_2}}$$

$$\leq \frac{(a'x_1t_1 + \gamma t_1 + 1)^{K_1}}{{t_1}^{K_1}} + \frac{(d'x_1t_1 + \alpha t_1 + 1){t_1}^{K_2}}{t_1(c'x_1t_1 + \beta t_1)^{K_2}}$$

$$\leq \left[a'x_1 + \gamma + \frac{1}{t_1}\right]^{K_1} + \frac{\left[d'x_1 + \alpha + \frac{1}{t_1}\right]}{(c'x_1 + \beta)^{K_2}}$$

$$\leq \left[a'x_1 + \gamma + (x_1'Bx_1)^{\frac{1}{2}}\right]^{K_1} + \frac{\left[d'x_1 + \alpha + (x_1'Bx_1)^{\frac{1}{2}}\right]}{(c'x_1 + \beta)^{K_2}}$$

$$G(x_s, t_s) \leq F(x_1) \qquad \text{(vi)}$$

Since $t_s > 0$ from Theorem 1, and $x_s \geq 0$ by non-negative constraints, so it is obvious that $\left(\frac{x_s}{t_s}\right)$ is feasible for extended fractional programming problem (6) and we have

$$F\left(\frac{x_s}{t_s}\right) = \left[a'\left(\frac{x_s}{t_s}\right) + \gamma + \frac{(x_s'Bx_s)^{\frac{1}{2}}}{t_s}\right]^{K_1} + \frac{\left[d'\left(\frac{x_s}{t_s}\right) + \alpha + \frac{(x_s'Bx_s)^{\frac{1}{2}}}{t_s}\right]}{\left(c'\left(\frac{x_s}{t_s}\right) + \beta\right)^{K_2}}$$

$$= \frac{\left[a'x_s + \gamma t_s + (x_s'Bx_s)^{\frac{1}{2}}\right]^{K_1}}{{t_s}^{K_1}} + \frac{{t_s}^{K_2}\left[d'x_s + \alpha t_s + (x_s'Bx_s)^{\frac{1}{2}}\right]}{t_s(c'x_s + \beta t_s)^{K_2}}$$

$$= \frac{(a'x_s + \gamma t_s + 1)^{K_1}}{{t_s}^{K_1}} + \frac{{t_s}^{K_2}[d'x_s + \alpha t_s + 1]}{t_s(c'x_s + \beta t_s)^{K_2}}$$

$$= G(x_s, t_s) \leq F(x_1)$$

Hence $\left(\frac{x_s}{t_s}\right)$ is an optimal solution of extended fractional programming problem (6). It is then clear from theorems 4, 5 and 6 that from the solution of reduced fractional programming problems (7), we get a solution of extended fractional programming problem (6).

Solution of III Type Problem: Consider the problem in which non-differentiable term occurs both in objective function and constraints.

$$Minimize\quad F(x) = \left[a'x + \gamma + (x'Bx)^{\frac{1}{2}}\right]^{K_1} + \frac{\left(d'x + \alpha + (x'Bx)^{\frac{1}{2}}\right)}{(c'x + \beta)^{K_2}}$$

Subject to $Ax \leq b$

$$p'x + (x'Bx)^{\frac{1}{2}} \leq 1 \text{ and } \quad x \geq 0 \qquad (8)$$

It is assumed that the general constraint set $S = \left\{x: Ax \leq b, p'x + (x'Bx)^{\frac{1}{2}} \leq 1, x \geq 0\right\}$ is nonempty and bounded and that $\forall\ x \epsilon R, a'x + \gamma \geq 0, (d'x + \alpha) \geq 0\ and\ (c'x + \beta) > 0.$

By using parametric substitutions $y = tx\ and\ t = \frac{1}{(x_1'Bx_1)^{\frac{1}{2}}}$ the above problem reduces to

$$Minimize\ \ G(x,t) = \frac{\left[a'\frac{x}{t} + \gamma + \frac{1}{t}\right]^{K_1}}{1} + \frac{\left[d'\frac{x}{t} + \alpha + \frac{1}{t}\right]}{\left(c'\frac{x}{t} + \beta\right)^{K_2}}$$

$$= \frac{(a'x + \gamma t + 1)^{K_1}}{t^{K_1}} + \frac{(d'x + \alpha t + 1)}{t(c'x + \beta t)^{K_2}}$$

$Subject\ to\ \ Ax - bt \leq 0$

$p'x + 1 \leq t$
$x'Bx \leq 1$
and $x \geq 0,\ t > 0$ (9)

Theorem 7: If (x_s, t_s) is an optimal solution of reduced fractional programming problem (9), then $t_s > 0$.
It follows from Theorem 1.

Theorem 8: If x_1 is an optimal solution of extended fractional programming problem (8), then $\exists\ t = \ t_1 = \frac{1}{(x_1'Bx_1)^{\frac{1}{2}}} > 0$ such that (t_1x_1, t_1) is an optimal solution of reduced fractional programming problem (9). One constraint $p'x + 1 \leq t$ is extra in problem (9), then problem (7) and (t_1x_1, t_1) is satisfied, so (t_1x_1, t_1) is feasible solution of reduced fractional programming problem (9). Rest of the pat follows from theorem 5.

Theorem 9: If (x_s, t_s) is an optimal solution of reduced fractional programming problem (9), then $\left(\frac{x_s}{t_s}\right)$ is an optimal solution of extended fractional programming problem (8).

It follows from theorem 6.

It is then clear from theorems 7, 8 and 9 that from the solution of reduced fractional programming (9), we get a solution of extended fractional programming problem (8).

Conclusion

The models discussed here are reduced to fractional programming problem and its solution can be obtained by using programming theorems. Solution of the original problem can then be obtained through the solution of fractional programming problem.

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