Research Journal of Mathematical and Statistical Sciences ______________________________ E-ISSN 2320–6047
Vol. **4(1),** 1-11, February **(2016)** *Res. J. Mathematical and Statistical Sci.*

E.P.Q Model for Deteriorating Items with Generalizes Pareto Decay Having Selling Price and Time Dependent Demand

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Available online at: www.isca.in, www.isca.me
Received 27th October 2015, revised 18th December 2015, accepted 30th January 2016

Abstract

In this paper we develop, analyze an E.P.Q model with the assumptions that the life time of commodity is random and follow a Generalized Pareto Distribution. It is assumed that demand is a function of both the time and selling price. Using the differential equations the instantaneous state of inventory is derived .With suitable cost consideration the total cost per unit and profit rate function are obtained. By maximizing the profit rate function, the optimal production quantity and optimal selling price are derived. The sensitivity of model with respect to the parameters and costs is done. This model is much useful for analyzing the situations arising at production processes dealing with perishable commodities.

Keywords: E.P.Q Model, Sensitivity analysis, Generalized Pareto Distribution, Profit Rate Function.

Introduction

Recently much emphasis is given for inventory model with random lifetime. Nahmias, S (1982), Raafat, F (1991) and Goel and Giri (2001) have reviewed the perishable inventory models and their optimal operating strategies. Ghare and Scharader (1963), Shah and Jaiswal (1977), Cohen (1976), Aggarwal (1978), Dave and Shah (1982), Pal (1990), Kalpakam and Sapna (1996), Giri and Chaudhuri (1998) and others developed the inventory models with exponential lifetime. Tadikama1la (1978) developed inventory model with Gamma distribution for deterioration Covert and Philip (1973), Philip (1974), Aggarwal and Goel (1980), Hwang and Hwang (1982), Venkata Subaiah et.al (1999) have developed inventory models with Weibull distribution for lifetime of the commodity. K. Nirupama Devi (2001,2004) has studied the inventory models with the assumptions that the lifetime of the commodity follows a mixture of Weibull distribution. Madhavi, N (2002) has developed inventory models for deteriorating items, with exponential, Weibull and mixture of weibull lifetime distributions having seconds sale. John Mathews (2002) has developed inventory model for deteriorating items with weibull rate of decay and finite replenishment.

Earlier researchers have not made any attempt to develop and analyse inventory models with Generalised Pareto distribution, except the work of Srinivasa Rao, et.al (2005) who developed the inventory models with infinite rate of replenishment. But in many production process the replenishment rate is finite. By considering the above facts an inventory model has been developed and analysed with Generalized Pareto Life time for finite rate of replenishment. The distribution function of the Generalized Pareto distribution is (Pickands, Hoskin et al. (1975))

$$F(t, c, a) = 1 - \left(1 - \frac{ct}{a}\right)^{\frac{1}{c}}$$

For c = 0 and c = 1, and this distribution reduces to the exponential distribution with mean a, and uniform distribution with range [0, a] respectively. This distribution is extensively used in the analysis of extreme events especially in reliability studies when robustness is required against heavier time or lighter time alternatives to an exponential distribution. Darghi-Noubary (1989) recommends Generalised Pareto distribution for use as the distribution of the excess of observed value over an arbitrarily chosen threshold. He pointed out that, "The Generalised Pareto distribution arises as a class of limit distribution for the excess over a threshold, as the threshold increases towards the right hand end distribution (tail). The Generalised Pareto distribution is more suitable for lifetime distribution for some commodities like, food and vegetables, edible oils, natural gas etc. Using the differential equations the instantaneous state of Inventory is derived. The total cost function is obtained by considering the suitable cost and to minimize the optimal order quantity.

Assumptions and Notations: In this section the assumptions and notations used in this paper are given below. i. The demand rate is known, ii. The lead time is zero, iii. Shortages are allowed and fully backlogged, iv. A deteriorated item is lost, v. k, finite production rate, vi. T, the fixed duration of production cycle is known, vii. A, the setup cost for each cycle, viii. Cost of placing an order is zero.

This assumption is made with respective to the producer's perspective and we are minimizing the total production cost. For example in food processing industry like bakery the cost of placing an order with respective to production manger is insignificant for computing the production cost. This assumption simplifies the computational complexity. However this assumption is little deviant from the practical situations. Here it is also considered that the cost of placing and order is mixed with set up cost.

The life time of commodity is random and follows a Gernarlized Pareto distribution having probability density function of the form.

$$f(t) = \begin{cases} \dfrac{1}{a}\left(1-\dfrac{ct}{a}\right)^{\frac{1}{c}-1} & c \neq 0 \\ \dfrac{1}{a}e^{-\frac{t}{a}} & c = 0 \end{cases}$$

a: Scale parameter
c: Shape parameter

The instantaneous rate of deterioration h(t) is

$$h(t) = \frac{1/a\left\{1-\dfrac{ct}{a}\right\}^{\frac{1}{c}-1}}{\left(1-\dfrac{ct}{a}\right)^{1/c}} = \frac{1}{a-ct}$$

Q: The ordering quantity in one cycle.
C: The cost per unit.
h: Inventory holding cost per unit per unit time.
π: The shortage cost per unit per unit time.

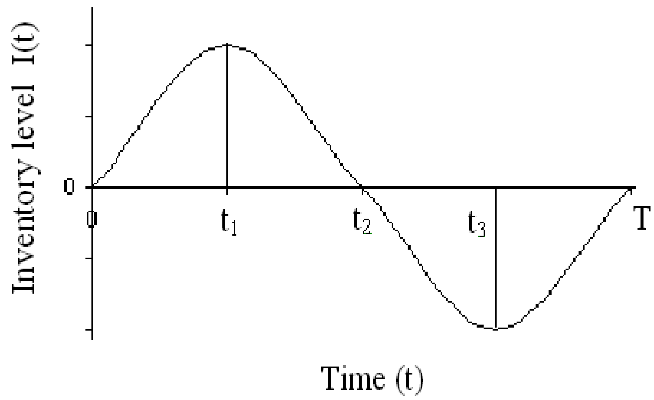
Inventory model with constant demand as power function of time and selling price: In this section demand pattern is considered as uniform throughout the period and demand rate is of form

$$\lambda(t) = \theta_0 + \theta_1 . \frac{r.t^{1_n-1}}{n.T^{1/n}} + \theta_2 s$$

where n is the index parameter and r is demand size, $\theta_0 \theta_1 \theta_2$ are arbitrary consists.

Let the ordering quantity in the cycle length T be Q, the cost price of one unit be C, the inventory holding cost per unit time be h, the shortage cost per unit time be □□

The amount of stock is zero at time t = 0. Production starts at t=0 and stops at $t = t_1$. The stock attains a level S at $t = t_1$. During (t_1, t_2) the inventory level gradually decreases mainly to meet up the demand and partly due to deterioration. By this process the stock reaches zero level at $t = t_2$. Now storages occur and accumulate to the level P at $t = t_3$. Production starts again at $t = t_3$ and backlog is cleared at t = T. The cycle then repeats itself after time T. The schematic diagram showing the inventory level over time is given in figure.



Let I(t) be the inventory level of the system at time $(0 \le t \le T)$. Then the differential equations describing the instantaneous states of I(t) over the cycle of length T are

$$\frac{d}{dt}I(t) + h(t)I(t) = k - \theta_0 - \theta_1 . \frac{r.t^{n_n-1}}{n.T^{1/n}} - \theta_2 s, \quad 0 \le t \le t_1 \quad (3.1)$$

$$\frac{d}{dt}I(t) + h(t)I(t) = -\theta_0 - \theta_1 . \frac{r.t^{1_n-1}}{n.T^{1/n}} - \theta_2 s, \quad t_1 \le t \le t_2 \quad (3.2)$$

$$\frac{d}{dt}I(t) = -\theta_0 - \theta_1 . \frac{r.t^{1_n-1}}{n.T^{1/n}} - \theta_2 s, \quad t_2 \le t \le t_3 \quad (3.3)$$

$$\frac{d}{dt}I(t) + h(t)I(t) = k - \theta_0 - \theta_1 . \frac{r.t^{1_n-1}}{n.T^{1/n}} - \theta_2 s, \quad t_3 \le t \le T \quad (3.4)$$

With the initial conditions I (0) = 0; I (t_2) = 0, I (T) = 0 Substituting h (t) and solving the above differential equations (3.1) (3.2), (3.3) and (3.4), the on hand inventory at time t can be obtained as,

$$I(t) = (a-ct)^{1/c} a^{-1/c}\ k - \theta_0 - \theta_2 .s\ (a-ct)^{1/c} a^{-1/c} \left\{ k\left[t + \frac{t^2}{2a} + \frac{(1+c)t^3}{6a^2} + (1+c)(1+2c)\frac{t^4}{24a^3}\right] - \frac{r}{T^{1/n}}\left[t^{1/n} + \frac{t^{1+1/n}}{a}\frac{1}{n+1} + (1+c)\frac{t^{1/n+2}}{2a^2}\frac{1}{1+2n} + (1+c)(1+2c)\frac{t^{1/n+3}}{6a^3}\frac{1}{1+3n}\right]\right\} \quad 0 \le t \le t_1 \quad (3.5)$$

$$I(t) = (a-ct)^{1/c} a^{-1/c} (\theta_0 + \theta_2 s)\left[(t_2 - t_1) + \left(\frac{t_2^2 - t^2}{2a}\right) + \frac{(1+c)}{6a^2}(t_2^3 - t^3) + \frac{(1+c)(1+2c)}{24a^3}(t_2^4 - t^4)\right]$$

$$+\frac{(1+c)(1+2c)}{6a^3}\frac{(t_2^{1/n+3}-t^{1/n+3})}{(1+3n)}\Bigg] \qquad t_1 \le t \le t_2 \tag{3.6}$$

$$I(t)=-\frac{r}{T^{1/n}}\left[t^{1/n}-t_2^{1/n}\right] \qquad t_2 \le t \le t_3 \tag{3.7}$$

$$I(t)=r\left[1-\left(\frac{t}{T}\right)^{1/n}\right]-k(T-t) \qquad t_3 \le t \le T \tag{3.8}$$

The stock loss due to deterioration in the interval (0, T) is

$$L(t)=kt_1-\frac{rt^{1/n}}{nT^{1/n}}.t_2 \tag{3.9}$$

The back logged demand at time t is

$$B(t)=\frac{r}{T^{1/n}}\left(t^{1/n}-t_2^{1/n}\right). \qquad t_2 \le t \le t_3 \tag{3.10}$$

The ordering quantity in a cycle of length T is

$$Q = kt_1 + k(T-t_3); \tag{3.11}$$

The total cost per a unit time is sum of the setup cost per a unit time, the unit cost, inventory holding cost, and shortage cost, $K(t_1, t_2, t_3, T, s)$ is obtained as

$$K(t_1, t_2, t_3\, T, s)=\frac{A}{T}+\boxed{\frac{CQ}{T}+\frac{h}{T}\left[\int_0^{t_1}I(t)dt+\int_{t_1}^{t_2}I(t)dt\right]+\frac{\pi}{T}\left[\int_{t_2}^{t_3}I(t)dt+\int_{t_3}^{T}I(t)dt\right]}$$

Substituting equations (3.5), (3.6), (3.7), (3.8) and (3.11) in the above equation integrating and simplifying by neglecting higher power of 1/a

$$K(t_1, t_2, t_3, T,s)=\frac{A}{T}+\frac{Ck\left(t_1+T-t_3\right)}{T}$$

$$+\frac{h}{T}\left\{\begin{array}{l} k\left[\frac{t_1^2}{2}-\frac{t_1^3}{6a}+\frac{t_1^4}{24a^2}(1-2c)-\frac{t_1^5}{120a^3}(2c-1)(3c-1)\right] \\ +\left(\theta_0+\theta_2 s\right)\left[\begin{array}{l}\frac{(2c+1)(3c+1)}{120a^3}t_2^5+\frac{(1+2c)t_2^4}{24a^2} \\ +\frac{t_2^3}{6a}+\frac{t_2^2}{2}\end{array}\right]\end{array}\right\}$$

$$+\frac{h}{T}(\theta_0+\theta_2 s)\left\{\begin{array}{l}\frac{(1+c)(1+2c)}{24a^3}t_2^4.t_1+\frac{(1+c)}{12a^3}t_2^3.t_1^2-\frac{(1+c)t_2^3}{6a^2}-\frac{(1-c)}{12a^3}.t_2^2.t_1^3+\frac{t_2}{4a^2}.t_1^2-\frac{t_2^2.t_1}{2a}+\frac{(1-c)(1-2c)}{24a^3}t_2.t_1^4 \\ -\frac{(1-c)t_2.t_1^3}{6a^2}+\frac{t_2.t_1^2}{2a}-t_2.t_1\end{array}\right\}$$

$$+\frac{h}{T}\cdot\frac{r\,\theta\,1}{T^{1/n}}\left\{\left[\frac{(1+c)(1+2c)t_2^{1/n+3}t_1}{6a^3(1+3n)}-\frac{(1+c)t_2^{1/n+2}t_1^2}{4a^3(1+2n)}\right]+\frac{(1+c)t_2^{1/n+2}t_1}{2a^2(1+2n)}+\frac{(1-c)t_2^{1/n+1}t_1^3}{6a^3(1+n)}-\frac{t_2^{1/n+1}t_1^2}{2a^2(1+n)}+\frac{1}{a(n+1)}.t_2^{1/n+1}.t_1\right\}$$

$$-\frac{(1-c)(1-2c)t_2^{1/n}t_1^4}{24a^3}+\frac{(1-c)t_2^{1/n}t_1^3}{6a^2}-\frac{t_2^{1/n}\,t_1^2}{2a}+t_2^{1/n}\,t_t-\frac{(2c+1)(3c+1)}{24a^3(1+4n)}.t_2^{1/n+4}-\frac{t_2^{1/n+3}(1+2c)}{6a^2(1+3n)}-\frac{t_2^{1/n+2}}{2a(1+4)}-\frac{t_2^{(1/n+1)}}{n+1}$$

$$+\frac{\pi r\,\theta_1}{T^{1/n+1}(n+1)}\left[nt_3^{\,1/n+1}-(n+1)t_2^{\,1/n}\,t_3+t_2^{\,1/n+1}\right]+\frac{\pi}{T}\left[\frac{k}{2}(T-t_3)^2-\frac{r}{n+1}\left(T-(n+1)t_3+n\,\frac{t_3^{1/n+1}}{T^{1/n}}\right)\right] \quad (3.12)$$

Differentiating K (t_1, t_2, t_3, T, s) w.r.t t_1 and equating to zero, we get

$$Ck+h\left\{k\left[t_1-\frac{t_1^2}{2a}+\frac{t_1^3(1-2c)}{6a^2}-\frac{t_1^4(2c-1)(3c-1)}{24a^3}\right]\right.$$
$$-\frac{r}{T^{1/n}}\left[\left(\frac{c[2c(1+n)-3n]}{6a^3(1+n)}\right)t_1^{1/n+3}+\frac{ct_1^{1/n+2}}{2a^2}+\frac{(1+c)(1+2c)t_2^{1/n+3}}{6a^3(1+3n)}\right.$$
$$-\frac{(1+c)t_2^{1/n+2}t_1}{2a^3(1+2n)}+\frac{(1+c)t_2^{1/n+2}}{2a^2(1+2n)}+\frac{(1-c)t_2^{1/n+1}t_1^2}{2a^3(1+n)}$$
$$\left.\left.-\frac{t_2^{1/n+1}t_1}{a^2(1+n)}+\frac{t_2^{1/n+1}}{a(1+n)}-\frac{(1-c)(1-2c)t_2^{1/n}t_1^3}{6a^3}+\frac{(1-c)}{2a^2}t_2^{1/n}t_1^2-\frac{t_2^{1/n}}{a}t_1+t_2^{1/n}\right]\right\}=0 \quad (3.13)$$

Differentiating K (t_1, t_2, t_3, T, s) w.r.t t_2 and equating to zero, we get

$$h\left\{\frac{(1+2c)t_2^{1/n+2}}{6a^2}\left(1+\frac{(3c+1)t_2}{4a}\right)+t_2^{1/n}\left(1+\frac{t_2}{2a}\right)-\frac{(1+c)(1+2c)t_2^{1/n+2}t_1}{6a^3}\right.$$
$$-\frac{(1+c)t_2^{1/n+1}t_1}{2a^2}\left(1-\frac{t_1}{2a}\right)+\frac{t_2^{1/n}t_1^2}{2a^2}\left(1-\frac{(1-c)t_1}{3a}\right)$$
$$\left.-\frac{(1-c)t_2^{1/n-1}t_1^3}{6a^2}\left(1-\frac{(1-2c)t_1}{4a}\right)-t_2^{1/n-1}t_1\left(1-\frac{t_1}{2a}\right)-\frac{t_2^{1/n}.t_1}{a}\right\}+\pi\,t_2^{1/n-1}(t_2-t_3)=0$$

$$h\left[(\theta_0+\theta_2 s)\frac{(1+2c)}{6a^2}.t_2^3\left(1+\frac{(3c+1)}{4a}.t_2\right)+t_2\left(1+\frac{t_2}{2a}\right)-\frac{(1+c)(1+2c)}{6a^3}t_2^3.t_1-\frac{(1+c)t_2^2t_1}{2a^2}\left(1-\frac{t_1}{2a}\right)+\frac{t_2t_1^2}{2a^2}\right.$$
$$\left.1-\left(\frac{(1-c).t_1}{3a}\right)-\frac{(1-c)}{6a^2}.t_1^3\left[1-\frac{(1-2c).t_1}{4a}\right]-t_1\left(1-\frac{t_1}{2a}\right)-\frac{t_2t_1}{a}\right]$$
$$+\frac{h.\theta_1 r}{T^{1/n}n}\left\{\frac{(1+2_c)t_2^{1/n+2}}{6a^2}\left(1+\frac{(3c+1)t_2}{4a}\right)+t_2^{1/n}\left(1+\frac{t_2}{2a}\right)-\frac{(1+c)(1+2c)t_2^{1/n+2}t_1}{6a^3}\right\}$$
$$+\frac{h\theta_1 r}{T^{1/n}n}\left\{-\frac{(1+c)t_2^{1/n+1}t_1}{2a^2}\left(1-\frac{t_1}{2a}\right)+\frac{t_2^{1/n}t_1^2}{2a^2}\left(1-\frac{(1-c)t_1}{3a}\right)\right.$$
$$\left.-\frac{(1-c)t_2^{1/n-1}t_1^3}{6a^2}\left(1-\frac{(1-2c)t_1}{4a}\right)-t_2^{1/n-1}t_1\left(1-\frac{t_1}{2a}\right)-\frac{t_2^{1/n}.t_1}{a}\right\} \quad (3.14)$$
$$+\frac{\pi r\theta_1}{r.T^{1/n}}t_2^{1/n-1}(t_2-t_3)+\pi(\theta_0+\theta_2 s)(t_2-t_3)=0$$

By differentiating K (t_1, t_2, t_3, T, s) w.r.t t_3 and equating to zero, we get

$$Ck+\pi\left[(k-\theta_0-\theta_2 s)(T-t_3)-r\theta_1\frac{(T^{1/n}-t_2^{1/n})}{T^{1/n}}\right]+\pi(\theta_0+\theta_2 s)(t_2-t_3)=0 \quad (3.15)$$

By differentiating K (t_1, t_2, t_3, T, s) w.r.t s and equating to zero, we get

$$h\left\{\begin{matrix}\frac{(2c+1)(3c+1)}{120.a^3}t_2^5+\frac{(1+2c)}{24a^2}t_2^4+\frac{t_2^3}{6a}+\frac{t_2^2}{2}-\frac{(1+c)(1+2c)}{24a^3}t_2^4\,t_1+\frac{(1+c)t_2^{\,3}}{12a^3}t_1^2-\frac{(1+c)t_2^3}{6a^2}t_1\\ -\frac{(1-c)t_2^2}{12a^3}t_1^3+\frac{1}{4a^2}t_2^2t_1^2-\frac{t_2^2\,t_1}{2a}+\frac{(1-c)(1-2c)}{24a^3}t_2.t_1^4-\frac{(1-c)t_2.t_1^3}{6a^2}+\frac{t_2.t_1^2}{2a}-t_2t_1\end{matrix}\right\}+\frac{T}{2}\left[(t_3-t_2)^2-(T-t_3)^2\right]=0$$

Solving equations (3.13), (3.14), (3.15) for various values of a, c, r, n, C, h, k, π, the optimal values of t_1^*, t_2^*, t^*_3 and Q*, K are computed.

Illustration: Consider the case of deriving the economic production quantity, production down time and production up time for a food processing industry which manufactures bred and bun. In this industry the product is of deteriorating nature and the life time is random. After discussion with production managers and workers we collected the data on the life time of commodity and found that it follows a Generalised Pareto Distribution through a frequency curve. Assuming the life time of commodity follows a Generalised Pareto Distribution and using the data collected over a cycle period of 400 hrs, the deteriorating distribution parameters are estimated through the method of maximum likely hood estimation and the estimates are c = 0.2, a = 105. A chi-square test for goodness of fit also carried and found that the Generalised Pareto Distribution gives a good fit to the data. The discussion with production manger and manufactures revealed that the estimates for production cost of a unit bread , setup cost , holding cost per a unit time and penalty cost per a unit time are C = Rs.3, h = Rs.0.5, π = Rs.0.6, A = Rs. 1200, and T = 400 hrs. The demand parameters are also estimated as r = 550, n = 5, the rate of production per a hour is k = 5 units, with these parameters, using the above model the optimal production schedule is computed and found that the optimal down time is t_1^* = 94.4 hrs, the optimal start-up time t_3^* = 391 hrs, with these two values the economic quantity for cycle is 517 units, and the minimum production cost per unit time is Rs. 14. From these optimal values, the production manger of the unit has continuing production and manufacturing the production scheduling. A sensitivity analysis is also performed for checking the effectiveness of the model with respect to the deteriorating parameters a , c ; demand parameters r , n ; cost parameters C , h , π ; and production rate k on optimum polices for different values of the parameters. The following values for the parameters are considered.

A = 100, 105 ,110 ; c = -1 , 0.2 , 0.25 , 0.3 , 0.35 ; r = 550 ,560 ,570 ; n = 5 ,6 ,8 ,9 ; C = Rs. 3 ,3.5 ,4 ,4.5 ; h = Rs.0.5 ,0.52 ,0.53 ,0.55 ; π = Rs.0.6 , 0.62 , 0.63 , 0.65 ; k = 5 , 6 , 7 ,8 ; T = 400 hrs and A = Rs. 1200.

Using the equations (3.13), (3.14), and (3.15) we obtain the optimal values of t_1, t_2, and t_3. By substituting these values in (3.12), (3.11) we computed the expected minimum cost per unit time and economic production quantity per cycle and presented in table 1.

A careful perusal of table 1 reveals that the deteriorating parameters has a significant influence on optimal production scheduling as ‘a’ increases the optimal value of production down time and economic order quantity and minimum cost per unit time are decreasing and production up time is increasing, when other cost and parameters are fixed. With respect to the other deteriorating parameter ‘c’ the production up time and economic order quantity are increasing for an increase in c. However the production down time is decreasing insignificantly. With respect to cost parameters there is significant impact on the optimal values of the production schedule. An increase in holding cost per unit will increase the optimal values of startup time, shutdown time and economic production quantity.

As the shortage cost π increases, the optimal values of the production down time, the production uptime, economic order quantity are increasing, when other parameters remain fixed. As the production cost of the unit increasing the optimal values of the production schedule behave similarly. If the demand parameter r increases, the optimal values of t_1 is increasing, where as the optimal values of t_3 is decreasing. With respective the parameter n the optimal values of t_1 and Q are decreasing and optimal value t_3 is increasing for an increase in n. This decrease is highly significant. So we conclude that the production manger effectively the estimate the parameters of the demand rate for optimal production scheduling.

Inventory model with demand as power function of time and selling price and without shortages: The differential equations governing the instantaneous state of inventory level of the system at time t of the model are

$$\frac{d}{dt}I(t) + h(t)I(t) = k - \theta_0 - \frac{\theta_1 r.t^{1/n-1}}{n.T^{1/n}} - \theta_2 s \quad 0 \le t \le t_1 \qquad (4.1)$$

$$\frac{d}{dt}I(t) + h(t)I(t) = -\frac{r\,t^{\frac{1}{n}-1}}{n\,T^{1/n}} = \theta_0 - \frac{\theta_1 r.t^{1/n-1}}{n.T^{1/n}} - \theta_2 s \quad t_1 \le t \le T \ (4.2)$$

with the initial conditions I(0)=0, I(T)=0.

The on hand inventory at time t is obtained as,

$$I(t) = (a-ct)^{1/c}\, a^{-1/c} \left\{ \begin{array}{l} k\left[t + \frac{t^2}{2a} + \frac{1+c}{6a^2}t^3 + (1+c)(1+2c)\frac{t^4}{24a^3} \right] \\ -\frac{r\theta}{T^{1/n}}\left[t^{1/n} + \frac{t^{1+1/n}}{a}\frac{1}{(n+1)} + (1+c)\frac{t^{1/n+2}}{2a^2}\frac{1}{1+2n} + (1+c)(1+2c)\frac{t^{1/n+3}}{6a^3}\frac{1}{1+3n} \right] \end{array} \right\}$$

$0 \le t \le t_1$ (4.3)

$$I(t) = (a-ct)^{1/c}\frac{r\theta_1 a^{-1/c}}{T^{1/n}}\left[\left(T^{1/n}-t^{1/n}\right)+\frac{\left(T^{1/n+1}-t^{1/n+1}\right)}{(1+n)a}+\frac{(1+c)}{2a^2}\frac{\left(T^{1/n+2}-t^{1/n+2}\right)}{(1+2n)}+\frac{(1+c)(1+2c)}{6a^3}\frac{\left(T^{1/n+3}-t^{1/n+3}\right)}{(1+3n)}\right]+$$

$$(a-ct)^{1/c}a^{-1/c}(\theta_0+\theta_2 s)\left[(T-t_1)+\left(\frac{T^2-t^2}{2a}+\right)+\frac{1+c}{6a^2}\left(T^3-T^3\right)+\frac{(1+c)(1+2c)}{24a^3}\left(T^4-t^4\right)\right] \quad t_1 \le t \le T \qquad (4.4)$$

The stock loss due to deterioration in the interval (0, T) is $L(t) = kt_1 - \left(\theta_0 - \frac{\theta_1 r.t^{1/n-1}}{n.T^{1/n}} - \theta_2 s\right)T$ (4.5)

The ordering quantity in a cycle of length T is obtained as $Q = kt_1$ (4.6)

For obtaining the optimal policies of the perishable inventory model having deterministic demand as power function of time and without shortages, the total cost per unit time $k(t_1, T)$ is obtained as

$$K(t_1, T, s) = \frac{A}{T}+\frac{CQ}{T}+\frac{h}{T}\int_0^{t_1} I(t)dt+\frac{h}{T}\int_{t_1}^{T} I(t)dt \qquad (4.7)$$

Differentiating $K(t_1,T)$ with respect to t_1 and equating to zero, one can get
Differentiating $K(t_1, t_2, t_3, T)$ w.r.t t_1 and equating to zero, we get

$$Ck + h\left\{k\left[t_1-\frac{t_1^2}{2a}+\frac{t_1^3(1-2c)}{6a^2}-\frac{t_1^4(2c-1)(3c-1)}{24a^3}\right]\right.$$
$$-\frac{r}{T^{1/n}}\left[\left(\frac{c[2c(1+n)-3n]}{6a^3(1+n)}\right)t_1^{1/n+3}+\frac{ct_1^{1/n+2}}{2a^2}+\frac{(1+c)(1+2c)t_2^{1/n+3}}{6a^3(1+3n)}\right.$$
$$-\frac{(1+c)t_2^{1/n+2}t_1}{2a^3(1+2n)}+\frac{(1+c)t_2^{1/n+2}}{2a^2(1+2n)}+\frac{(1-c)t_2^{1/n+1}t_1^2}{2a^3(1+n)}$$
$$\left.\left.-\frac{t_2^{1/n+1}t_1}{a^2(1+n)}+\frac{t_2^{1/n+1}}{a(1+n)}-\frac{(1-c)(1-2c)t_2^{1/n}t_1^3}{6a^3}+\frac{(1-c)}{2a^2}t_2^{1/n}t_1^2-\frac{t_2^{1/n}}{a}t_1+t_2^{1/n}\right]\right\} = 0 \qquad (4.8)$$

Differentiating $K(t_1, T)$ with respect to s and equating to zero, one can get

$$\frac{h}{T}.\theta_2\left\{\begin{array}{l}\frac{(2c+1)(3C+1)}{120.a^3}T^5+\frac{(1+2c)}{24a^2}t_2^4+\frac{t_2^3}{6a}+\frac{t_2^2}{2}-\frac{(1+C)(1+2C)}{24a^3}t_2^4\,ot+\frac{(1+C)}{12a^3}t_1^2-\frac{(1+C)t_2^3}{6a^2}t_1\\ -\frac{(1-C)t_2^2}{12a^3}t_1^3+\frac{1}{4a^2}t_2^2t_1^2-\frac{t_2^2t_1}{2a}+\frac{(1-c)(1-2c)}{24a^3}t_2.t_1^4-\frac{(1-c)t_2.t_1^3}{6a^2}+\frac{t_2.t_1^2}{2a}-t_2t_1\end{array}\right\} = 0$$

For various values of a, c, r and n the optimal values of t^*_1 and Q^* are computed by solving the equations (4.8) and (4.6). ($\theta_0 = 0, \theta_1 = 1, \theta_2 = 0$)

Numerical Illustration of the Model without Shortages: In this section we illustrate the optimal operating policies of the model without shortages by applying it to a food processing industry with the same values for the deteriorating parameters, demand parameters, replenishment parameters, cost value and cycle time give in the example of section 4. Using the equations (4.8) and (4.6) we computed the optimal shutdown time, optimal production stopping time 181.5 hrs and the economic production quantity is 908 units.

The comparison of with shortages and without shortage models for the same parameters values reveals that the economic production quantity of without shortage model is more than that of with shortages model. And production cost per unit time for the without shortage model is Rs.52, where as for with shortage model it is Rs.14. this shows that without shortage model is more economical than that of with shortage model.

For the sensitivity of the parameters costs and cycle time on optimal operating policies are completed. The optimal value of production down time. Economic order quantity and cost per unit time for different values of the parameters and costs are shown in table 2. From table 2 it is observed that the same phenomenon of the earlier model is exhibited for all parameters.

Table-1
Optimal values of t_1*, t_2*, t_3*, K and Q*

a	c	k	h	r	C	n	π	T	t_1*	t_2*	t_3*	K	Q*
100 105 110	0.2	5	0.5	550	3	5	0.6	400	94.676 94.4526 94.238	201.335 203.228 205.005	390.888 391.0677 391.235	17.313 16.932 16.5796	518.9395 516.92 515.015
100	0.25 0.3 0.35	5	0.5	550	3	5	0.6	400	96.517 98.763 101.590	200.4367 199.883 199.803	390.8023 390.749 390.741	17.324 17.3033 17.237	528.5767 540.068 554.245
100	0.2	6 7 8	0.5	550	3	5	0.6	400	80.414 69.5369 61.0537	191.567 184.104 178.281	392.44 393.705 394.740	21.624 24.988 27.675	527.793 530.820 531.509
100	0.2	5	0.52 0.53 0.55	550	3	5	0.6	400	94.175 93.9325 93.459	199.170 198.1205 196.083	390.68 390.5799 390.3825	17.291 17.227 17.2403	517.47 516.763 515.38
100	0.2	5	0.5	550 560 570	3	5	0.6	400	94.676 96.154 97.615	201.335 202.345 203.344	390.888 390. 729 390.5725	17.313 17.12 16.919	518.9395 527.12 535.216
100	0.2	5	0.5	550	3.5 4 4.5	5	0.6	400	93.462 92.261 91.07	200.65 199.98 199.32	391.656 392.425 393.195	17.955 18.585 19.263	509.028 499.176 489.38
100	0.2	5	0.5	550	3	6 8 9	0.6	400	91.122 86.306 84.603	199.297 196.504 195.509	392.94 395.648 396.589	12.763 6.526 4.29	490.904 453.289 440.07
100	0.2	5	0.5	550	3	5	0.62 0.63 0.65	400	95.3377 95.6634 96.3047	203.285 204.2401 206.109	390.911 390.925 390.95	17.669 17.843 18.1874	522.12 523.6915 526.153
105 110 115	-1	5	0.5	550	3	5	0.6	400	87.44762 85.94079 85.177	209.7924 211.6364 213.60208	391.6804 391.84981 392.029	16.831 16.47413 16.13127	478.8108 470.4548 465.7476
100	-0.9 -0.8 -0.6	5	0.5	550	3	5	0.6	400	86.59594 84.87407 84.140	211.142 214.5348 219.9034	391.8045 392.1136 392.594	16.944 16.69185 16.3384	473.9571 463.802 457.726
100	-1	6 8 9	0.5	550	3	5	0.6	400	117.5535 125.058 126.944	222.3467 226.798 227.97	394.842 397.624 398.500	23.34683 36.44 43.1317	736.2674 1019.4 1156
100	-1	5	0.55 0.56 0.58	550	3	5	0.6	400	105.366 105.579 105.9857	211.125 210.375 208.919	391.803 391.734 391.599	17.294 17.3017 17.311	567.817 569.228 571.929
100	-1	6	0.5	560 580 600	3	5	0.6	400	116.823 115.2302 113.4024	221.9305 221.0313 220.0156	394.6265 394.18 393.731	23.0236 12.388 21.7714	733.183 726.263 718.024
100	-1	5	0.5	550	4 5 6	5	0.6	400	109.193 112.465 115.0538	218.0613 220.1411 221.88	394.097 395.94 397.769	18.6506 20.099 21.56	575.480 582.578 586.419
100	-1	5	0.5	550	3	6 8 9	0.6	400	110.1317 115.587 117.2278	218.61 221.99 223.031	394.463 397.19 398.08	12.959 7.349 5.405	578.34 591.966 595.703
100	-1	5	0.5	550	3	5	0.61 0.62 0.63	400	103.56 102.58 102.088	215.602 216.0429 216.4238	392.128 392.088 392.04	17.37 17.529 17.679	559.177 553.96 550.213

Table-2
Optimal values of t_1*, K, Q*

a	c	h	r	k	C	n	T	t_1^*	K	Q^*
100 105 110	0.2	0.5	550	5	3	5	400	185.028 181.559 177.774	57.296 54.932 52.891	925.141 907.793 888.868
100	0.25 0.3 0.35	0.5	550	5	3	5	400	195.046 207.335 223.426	58.098 57.729 55.453	975.231 1037.0 1117.0
100	0.2	0.55 0.6 0.65	550	5	3	5	400	185.466 185.831 186.14	62.031 63.765 71.497	927.331 929.156 930.702
100	0.2	0.5	560 580 600	5	3	5	400	186.052 187.991 189.796	57.124 56.756 56.359	930.261 939.956 948.982
100	0.2	0.5	550	6 7 8	3	5	400	173.566 162.346 151.652	69.49 80.659 90.836	1041 1136 1213
100	0.2	0.5	550	5	4 5 6	5	400	183.425 181.826 180.231	59.599 61.882 64.145	917.125 909.13 901.157
100	0.2	0.5	550	5	3	6 8 9	400	173.202 155.761 149.085	50.47 40.004 35.91	866.011 778.806 745.427
100 105 110	-0.2	0.5	550	5	3	5	400	137.084 134.377 132.119	40.069 39.21 38.375	685.422 671.886 660.597
100	-.19 -.18 -.15	0.5	550	5	3	5	400	137.885 138.702 140.389	40.444 40.83 41.633	689.423 693.511 701.944
100	-0.2	0.55 0.6 0.65	550	5	3	5	400	137.675 138.171 138.594	43.26 46.45 49.638	688.373 690.853 692.968
100	-0.2	0.5	560 580 600	5	3	5	400	138.568 140.031 144.3	39.931 39.782 39.261	692.839 700.155 721.502
100	-0.2	0.5	550	6 8 9	3	5	400	122.488 101.299 93.317	48.198 61.039 66.209	734.931 810.393 839.854
100	-0.2	0.5	550	5	4 5 6	5	400	134.965 132.911 130.916	41.769 43.443 45.092	674.824 664.553 654.578
100	-0.2	0.5	550	5	3	6 7 8	400	127.445 120.367 114.875	33.373 28.088 23.817	637.226 601.835 574.374

Table-3
Sensitivity analysis with shortages

Variation in parameters		Percentage change in parameters						
		-15	**-10**	**-5**	**0**	**5**	**10**	**15**
a	K Q	18.67057 525.02811	18.17617 523.09095	17.7259 521.01953	17.313 518.93954	16.93229 516.92472	16.57967 515.01509	16.25185 513.22959
c	K Q	17.29271 513.83527	17.30045 515.4873	17.30723 517.18784	17.313 518.93954	17.31772 520.74531	17.32133 522.60829	17.32378 524.53194
k	K Q	13.22189 505.2078	14.68229 510.74121	16.04326 515.25861	17.313 518.93954	18.49921 521.92879	19.609 524.34223	20.64889 526.27244
h	K Q	17.28656 525.01028	17.31543 522.88095	17.3237 520.85911	17.313 518.93954	17.28477 517.11671	17.2403 515.38498	17.18071 513.73883
r	K Q	18.44719 448.05589	18.16132 472.32905	17.78149 495.9658	17.313 518.93954	16.76101 541.22417	16.13057 562.79559	15.42659 583.6319
C	K Q	16.72415 527.91148	16.92156 524.91538	17.11784 521.92474	17.313 518.93954	17.50704 515.95978	17.69997 512.98543	17.89178 510.01647
n	K Q	21.75355 546.7895	20.15136 536.68634	18.67597 527.43712	17.313 518.93954	16.05016 511.10666	14.87687 503.86417	13.78397 497.1483
∏	K Q	15.59351 503.32594	16.18941 508.784	16.762 513.97859	17.313 518.93954	17.84396 523.69154	18.35623 528.2552	18.85104 532.64797

Table-4
Sensitivity analysis without shortages

Variation in parameters		Percentage change in parameters						
		-15	**-10**	**-5**	**0**	**5**	**10**	**15**
a	K Q	67.87676 960.76341	63.5503 952.17929	60.10843 940.15286	57.296 925.141	54.93247 907.79306	52.89064 888.86752	51.083 869.1296
c	K Q	56.43737 898.85012	56.74887 907.35197	57.03587 916.10948	57.296 925.141	57.52687 934.46789	57.72561 944.1126	57.88925 954.10127
k	K Q	47.47832 822.72876	50.8142 858.35673	54.08695 892.4896	57.296 925.141	60.44108 956.33019	63.52182 986.08167	66.5313 1014.43
h	K Q	50.19003 920.89511	52.55929 922.46712	54.92793 923.87428	57.296 925.141	59.6637 926.2879	62.03097 927.33066	64.3979 928.28301
r	K Q	58.34259 874.54306	58.07791 893.25077	57.72459 910.13507	57.296 925.141	56.80327 938.782	56.25536 951.14099	55.65982 962.37608
C	K Q	56.25323 928.75451	56.60128 927.54967	56.94889 926.34524	57.296 925.141	57.64275 923.93761	57.989 922.73443	58.3348 921.53168
n	K Q	63.37632 980.25505	61.24601 960.6141	59.22127 942.29236	57.296 925.141	55.46407 909.03594	53.71936 893.87076	51.05627 879.55524

Conclusions

This paper proposes a continues production Inventory model for deteriorating items with and without shortages. Here we constrained deterioration of economic production quantity, production down time, production up time, the case of Generalized Pareto rate of decay and time dependent demand. The Generalized Pareto Distribution used for life time of commodity also includes uniform and exponential as particular cases. We have utilized the differential equation and unconstrained optimal techniques under stochastic environment to obtain the solution of the model. Two examples are included for illustrating the utility of the models in food processing industry. It is observed that the deteriorating parameters has significant effect on the optimal operating policy models. The model with shortages is much economical than without shortages.

Acknowledgements: The authors are very much thankful to the referees and the Editor for their constructive comments which have helped to the quality of paper to the present level.

Changes incorporated to the revised version of the paper: The paper is thoroughly revised by incorporating the suggestions given by the referee. The item wise modifications made in the text as follows:

A conclusion session is included, the validation of results has demonstrated through the data from food processing industry. Instead of considering the cost of placing the order which was assumed to be zero, a set up cost is included to consider all the layout costs. The English language style is improved.

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