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Estimation of Ratio of two Population means in a Class of Ratio-cum Regression type estimators using Auxiliary character with double Sampling in the presence of Non-response

K.M.Patel and Narendra Kumar
Department of Statistics, Udai Pratap Autonomous College, Varanasi, U.P., INDIA

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Abstract

In this paper, we represent estimation of ratio of two population means in Ratio-cum regression type estimators using auxiliary character with double sampling in the presence of non-response and also study and discuss their properties. After comparing the proposed estimator with relevant estimators, we conclude that the proposed estimators are more efficient on the basis of their mean square error. For numerical support, the empirical study is given.

Keyword: Ratio estimator, regression estimator, study variable, auxiliary variable, double sampling, non-response.

Introduction

The efficiency of Ratio, product and regression type estimators for estimation of ratio of two population means of study characters depend on the information on auxiliary character. When prior information closely related to study variable is available, ratio estimator is used and in case of linear relation ship between study and auxiliary variables exists, regression estimator is more useful. Regression estimator is more suitable than ratio estimator if regression line doesn't pass through origin. Estimation of ratio of two population means is widely used in calculation of growth rate (medical science), crop production rate (agriculture), literacy rate (education) and in the field of socio-economics.

Non-response errors may arise mainly due to faulty sampling frame, biased method of selection of units in the selected sample etc. for the estimation of population mean in the presence of non-response, Hansan and Hurvitz[1] has given a technique of sub sampling from the non-responding units. Further El-Badry[2] suggested a method to sending several waves of questionnaire by mail surveys to reduce the effect of non-response.

Singh[3], Tripathi[4,5], Upadhaya and Singh[6], Srivastava et al[7], Singh et al[8,9] and Singh and Singh[10] have proposed different types of estimators for estimation of ratio of two population means using auxiliary variable. When all the information regarding auxiliary variate are not known then we use double sampling. Srivastava[11], Kiregyera[12,13] have suggested different ratio, regression and product type estimators for estimation of population mean using double sampling.

Using Hansen and Hurwitz technique, in case of known and unknown population means of auxiliary characters have been done by Cochran[14], Rao[15,16], Khare and kumar[17] and Khare and Rehman[18] have proposed conventional and alternative estimators for estimation of population mean in the presence of non-response.

The estimation of ratio of two population means using auxiliary variables in the presence of non-response have been proposed by Khare and pandey[19], Khare and Sinha[20,21,22,23], Khare et al.[24,25], Kumar and Patel[26].

In recent paper, we present a class of ratio-cum-regression type estimators for estimation of raito of two population means using auxiliary character in double sampling in presence of non-response. Also relative bias and mean square error of the suggested estimator are derived for efficiency comparison to the relevant estimators and studied properties. For performance of the suggested estimator, percentage relative efficiencies are also calculated mathematically.

The estimators

Let Y_{ip}(i = 1,2), and X_p(p =1,2,.........,N) be the non- negative value of p-th unit of the population on the study characters y_i(i = 1,2) and the auxiliary character x with their population means $\bar{Y}_i$ (i = 1,2), and $\bar{X}$.

Let us suppose the population of N units can be divided into response class of N_1 units and non-response class of N_2 units such that $N_1 + N_2 = N$ having means $\bar{Y}_{i(1)}$ and $\bar{Y}_{i(2)}$ respectively with study variates y_i (i = 1, 2) and $\bar{X}_1$ and $\bar{X}_2$ respectively with auxiliary variate x.

In Hansen and Hurwitz[1] technique, a sample of n(<N) units is selected out of N population units using simple random

sampling without replacement method and is made a survey schedule each one of the sample. It is observed that in sample of size n(<N), only n_1 units responding and n_2(= n – n_1) units not responding with means $\bar{y}_{i(n_1)}$ and $\bar{y}_{i(n_2)}$ regarding study variates y_i (i = 1, 2) and $\bar{x}_{(n_1)}$ and $\bar{x}_{(n_2)}$ regarding auxiliary variates x.

Thus, a sub sample of r(= n_2/k) units, taken from n_2 non-responding units, is selected randomly with mean $\bar{y}_{i(n_2)r}$ regarding study variates y_i (i = 1, 2) and $\bar{x}_{(n_2)r}$ regarding auxiliary variates x and information from each r units is collected by personal interview.

Hence, the Hansen – Hurwitz unbiased estimator for population mean $\bar{Y}_i$ of study characters y_i(i = 1,2) and $\bar{X}$ of auxiliary character x based on (n_1 + r) units are given as

$$\bar{y}_i^* = \frac{n_1}{n}\bar{y}_{i(n_1)} + \frac{n_2}{n}\bar{y}_{i(n_2)r}$$

Where, $\bar{y}_{i(n_1)}$ and $\bar{y}_{i(n_2)r}$ are the unbiased estimator of $\bar{Y}_{i(1)}$ and $\bar{Y}_{i(2)}$ respectively but biased estimator of $\bar{Y}_i$ of y_i(i = 1,2) characters.

$$\bar{x}^* = \frac{n_1}{n}\bar{x}_{(n_1)} + \frac{n_2}{n}\bar{x}_{(n_2)r}$$

Where, $\bar{x}_{(n_1)}$ and $\bar{x}_{(n_2)r}$ are the unbiased estimator of c and $\bar{X}_2$ respectively but biased estimator of $\bar{X}$ of x character.

And also the variance of Hansen- Hurwitz unbiased estimator of study characters y_i(i = 1,2) and auxiliary character x are given by

$$V(\bar{y}_i^*) = fS_{y_i}^2 + f^* S_{y_i}^{*2} \text{ and } V(\bar{x}^*) = fS_x^2 + f^* S_x^{*2}$$

Where, $f = \frac{1}{n} - \frac{1}{N}$ and $f^* = \frac{W_2(k-1)}{n}$

$W_2 = N_2/N$, Stratum weight of non- response class of N_2 units.

$S_{y_i}^2 = \frac{1}{N-1}\sum_{p=1}^{N}(Y_{ip} - \bar{Y}_i)^2$; the mean square of characters y_i(i = 1,2) based on N units of the population.

And, $S_{y_i}^{*2} = \frac{1}{N_2 - 1}\sum_{p=1}^{N_2}(Y_{ip(2)} - \bar{Y}_{i(2)})^2$; the mean square of characters y_i(i = 1,2) based on N_2 non-responding units of the population.

Also, $S_x^2 = \frac{1}{N-1}\sum_{p=1}^{N}(X_p - \bar{X})^2$; the mean square of characters x based on N units of the population.

And, $S_x^{*2} = \frac{1}{N_2 - 1}\sum_{p=1}^{N_2}(X_{p(2)} - \bar{X}_2)^2$; the mean square of characters x based on N_2 non-responding units of the population .

Let $R = \bar{Y}_1 / \bar{Y}_2$; the ratio of two population means of study characters y_i(i = 1,2).

Then, in order to estimate R, Define a conventional estimator $R^* = \bar{y}_1^* / \bar{y}_2^*$; the ratio of two Hansen-Hurvitz unbiased estimators for population means $\bar{Y}_i$ of study characters y_i(i = 1,2).

In this case, when population mean of auxiliary character x is not known. Then in order to estimate $\bar{X}$, a preliminary sample of size n' (> n) is selected from population of N units using SRSWOR technique having mean $\bar{x}'$ regarding auxiliary variate x then a sub sample of preliminary sample of size n is selected using SRSWOR technique having mean $\bar{x}$ regarding auxiliary variate x.

We define a conventional estimator (T_1),when information on both study characters y_i(i = 1,2) and auxiliary character x are incomplete, and a alternative estimator (T_2), when both study characters y_i(i = 1,2) have incomplete but auxiliary character x has complete information, for estimation of ratio of two population means in double sampling in presence of non-response as follow

$$T_1 = R^* \frac{\bar{x}^*}{\bar{x}'} \quad (1)$$

and $$T_2 = R^* \frac{\bar{x}^*}{\bar{x}} \quad (2)$$

Where, $\bar{x} = \frac{1}{n}\sum_{p}^{n} x_p$ and $\bar{x}' = \frac{1}{n'}\sum_{p}^{n'} x_p$

Using above situation, Khare and Sinha[21] have suggested a improved classes of conventional (T_3) and alternative estimators (T_4) for estimation of ratio of two population means in double sampling in presence of non-response as follow

$$T_3 = f_1(R^* \frac{\bar{x}^*}{\bar{x}'}) \quad (3)$$

and $$T_4 = f_2(R^* \frac{\bar{x}^*}{\bar{x}}) \quad (4)$$

The proposed class of estimators: On the basis of above information and condition, we propose a class of ratio-cum regression type estimators for estimation of ratio of two population means(R) using auxiliary character with double sampling in presence of non-response is given as

$$T_R = [R^* + k_1(\bar{x} - \bar{x}^*) + k_2(\bar{x}' - \bar{x})][2 - \left(\frac{a\bar{x}' + b}{a\bar{x}^* + b}\right)] \quad (5)$$

Where, $R^* = \dfrac{\bar{y}_1^*}{\bar{y}_2^*}$ and k_1, k_2, a and b are either real number or functions of known parameters.

To obtain Relative Bias and Mean Square Error: For this purpose ,let us assume

$\bar{y}_1^* = \bar{Y}_1(1+\psi_1)$, $\bar{y}_2^* = \bar{Y}_2(1+\psi_2)$, $\bar{x} = \bar{X}(1+\psi_3)$,

$\bar{x}' = \bar{X}(1+\psi_4)$ and $\bar{x}^* = \bar{X}(1+\psi_5)$

Such that- E[ψ_1] =E[ψ_2] =E[ψ_3] =E[ψ_4] =E[ψ_5]= 0

Using above substitution, the equation (5) express as in terms of ψ_i's and neglecting 3rd and higher terms, we have

$$T_R = [R(1+\psi_1)(1+\psi_2)^{-1} + k_1\bar{X}(\psi_3 - \psi_5) + k_2\bar{X}(\psi_4 - \psi_3)][2 - \{(1+\delta\psi_4)(1+\delta\psi_5)^{-1}\}]$$
$$= [R(1+\psi_1 - \psi_2 + \psi_1\psi_2 + \psi_2^2) + k_1\bar{X}(\psi_3 - \psi_5) + k_2\bar{X}(\psi_4 - \psi_3) - R\delta(\psi_4 - \psi_5)$$
$$- R\delta\psi_1(\psi_4 - \psi_5) + R\delta\psi_2(\psi_4 - \psi_5) - k_1\bar{X}\delta(\psi_3 - \psi_5)(\psi_4 - \psi_5) - k_2\bar{X}\delta(\psi_4 - \psi_5)(\psi_4 - \psi_3)$$
$$+ R\delta(\delta - 1)(\psi_4 - \psi_5)^2/2]$$

$$R.B.[T_R] = E[T_R - R]/R$$
$$= R.B.[R^*] - \delta[f''A + f^*B] - (k_1\bar{X}\delta/R)f^*C_x^{*2} - (k_2\bar{X}\delta/R)f''C_x^2 \quad (6)$$
$$+ \frac{\delta(\delta-1)}{2}[f''C_x^2 + f^*C_x^{*2}]$$

$$M.S.E.[T_R] = E[T_R - R]^2$$
$$= E[R(\psi_1 - \psi_2) + k_1\bar{X}(\psi_3 - \psi_5) + k_2\bar{X}(\psi_4 - \psi_3) - R\delta(\psi_4 - \psi_5)]^2$$
$$= M.S.E.[R^*] + (k_2\bar{X} - \delta R)^2 f''C_x^2 + 2R(k_2\bar{X} - \delta R)f''A + (k_1\bar{X} - \delta R)^2 f^*C_x^{*2}$$
$$+ 2R(k_1\bar{X} - \delta R)f^*B$$

$$= M.S.E.[R^*] + f''[\theta^2 C_x^2 + 2R\theta A] + f^*[\phi^2 C_x^{*2} + 2R\phi B] \quad (7)$$

Where, $\theta = (k_2\bar{X} - \delta R)$ $\quad \phi = (k_1\bar{X} - \delta R)$ and $\delta = \dfrac{a\bar{X}}{a\bar{X} + b}$

The optimum value of δ is given as

$$\delta_{opt} = \frac{f''(k_2\bar{X}C_x^2 + RA) + f^*(k_1\bar{X}C_x^{*2} + RB)}{R(f''C_x^2 + f^*C_x^{*2})}$$

Where, $R = \dfrac{\bar{Y}_1}{\bar{Y}_2}$, $f' = \dfrac{1}{n'} - \dfrac{1}{N}$, $f'' = \dfrac{1}{n} - \dfrac{1}{n'}$,

$A = (C_{xy_2} - C_{xy_1})$ and $B = (C_{xy_2}^* - C_{xy_1}^*)$

Also, $E[\psi_1^2] = fC_{y_1}^2 + f^*C_{y_1}^{*2}$, $E[\psi_2^2] = fC_{y_2}^2 + f^*C_{y_2}^{*2}$,
$E[\psi_3^2] = fC_x^2$, $E[\psi_4^2] = f'C_x^2$
$E[\psi_5^2] = fC_x^2 + f^*C_x^{*2}$, $E[\psi_1\psi_2] = fC_{y_1y_2} + f^*C_{y_1y_2}^*$,
$E[\psi_1\psi_3] = fC_{xy_1}$,
$E[\psi_1\psi_4] = f'C_{xy_1}$, $E[\psi_1\psi_5] = fC_{xy_1} + f^*C_{xy_1}^*$,
$E[\psi_2\psi_3] = fC_{xy_2}$, $E[\psi_2\psi_4] = f'C_{xy_2}$,
$E[\psi_2\psi_5] = fC_{xy_2} + f^*C_{xy_2}^*$, $E[\psi_3\psi_4] = f'C_x^2$,
$E[\psi_3\psi_5] = fC_x^2$, $E[\psi_4\psi_5] = f'C_x^2$
And, $C_{xy_i} = \rho_{xy_i}C_xC_{y_i}$, $C_{xy_i}^* = \rho_{xy_i}^*C_x^*C_{y_i}^*$, $C_{y_i} = S_{y_i}/\bar{Y}_i$,
$C_{y_i}^* = S_{y_i}^*/\bar{Y}_i$,(i = 1,2), $C_x = S_x/\bar{X}$ and $C_x^* = S_x^*/\bar{X}$

For optimum value of k_1 and k_2, we have to minimize expression (3) w.r.t. φ and θ respectively, we have

$$\theta_{\min} = -\frac{RA}{C_x^2} \text{ and } \phi_{\min} = -\frac{RB}{C_x^{*2}}$$

Putting the optimum value of θ and φ , we get

$$M.S.E.[T_R]_{\min} = M.S.E.[R^*] - R^2\{f''[A^2/C_x^2] + f^*[B^2/C_x^{*2}]\}$$

Table-1 presents some members of the proposed class of estimators TR for different values of k1, k2, a and b in equation (5).

Table-1

Member of Estimators	k_1	k_2	**a**	**b**
$T_R^1 = R^*$	0	0	0	1
$T_R^2 = [R^* + k_1(\bar{x} - \bar{x}^*) + k_2(\bar{x}' - \bar{x})][2 - \left(\frac{\bar{x}'}{\bar{x}^*}\right)]$	k_1	k_2	1	0
$T_R^3 = [R^* + k_1(\bar{x} - \bar{x}^*) + k_2(\bar{x}' - \bar{x})]$	k_1	k_2	0	1
$T_R^4 = [R^* + k_1(\bar{x} - \bar{x}^*) + k_2(\bar{x}' - \bar{x})][2 - \left(\frac{\bar{x}' + C_x}{\bar{x}^* + C_x}\right)]$	k_1	k_2	1	C_x
$T_R^5 = [R^* + k_1(\bar{x} - \bar{x}^*) + k_2(\bar{x}' - \bar{x})][2 - \left(\frac{\bar{x}' + C_x^*}{\bar{x}^* + C_x^*}\right)]$	k_1	k_2	1	C_x^*
$T_R^6 = [R^* + \beta_1^{**}(\bar{x} - \bar{x}^*) + \beta_2^{**}(\bar{x}' - \bar{x})][2 - \left(\frac{a\bar{x}' + b}{a\bar{x}^* + b}\right)]$	β_1^{**}	β_2^{**}	a	b
$T_R^7 = [R^* + C_x^{**}(\bar{x} - \bar{x}^*) + C_x^{***}(\bar{x}' - \bar{x})][2 - \left(\frac{a\bar{x}' + b}{a\bar{x}^* + b}\right)]$	C_x^{**}	C_x^{***}	a	b
$T_R^8 = [R^* + \rho^{**}(\bar{x} - \bar{x}^*) + \rho^{***}(\bar{x}' - \bar{x})][2 - \left(\frac{a\bar{x}' + b}{a\bar{x}^* + b}\right)]$	ρ^{**}	ρ^{***}	a	b

$\beta_1^{**} = (\beta_1^* + \beta_1)/\bar{Y}_1$, $\beta_2^{**} = (\beta_2^* - \beta_2)/\bar{Y}_2$, $C_x^{**} = (C_x^* + C_x)/2$,
$C_x^{***} = (C_x^* - C_x)/2$,
$\rho^{**} = (\rho_{xy_1} - \rho_{xy_2})/(\rho_{xy_1} + \rho_{xy_2} + 2\rho_{y_1y_2})$ and $\rho^{***} = (\rho_{xy_1}^* - \rho_{xy_2}^*)/(\rho_{xy_1}^* + \rho_{xy_2}^* + 2\rho_{y_1y_2}^*)$
Where, $\beta_i^* = \rho_{xy_i}^*C_{y_i}^*/C_x^*$ and $\beta_i = \rho_{xy_i}C_{y_i}/C_x$ (i = 1 , 2)

The relative bias and mean square error of members of the proposed estimator TR, mention in table 1 , can be expressed by suitably putting the values of k1, k2, a and b, in equation (6) & (7) respectively, which are given as

$$R.B.[T_R^1] = R.B.[R^*] = f[C_{y_2}^2 - C_{y_1y_2}] + f^*[C_{y_2}^{*2} - C_{y_1y_2}^*]$$

$$M.S.E.[T_R^1] = M.S.E.[R^*] = f[C_{y_1}^2 + C_{y_2}^2 - 2C_{y_1y_2}] + f^*[C_{y_1}^{*2} + C_{y_2}^{*2} - 2C_{y_1y_2}^*]$$

$$R.B.[T_R^2] = R.B.[R^*] - f''[(k_2\overline{X}/R)C_x^2 + A] - f^*[(k_1\overline{X}/R)C_x^{*2} + B]$$

$$M.S.E.[T_R^2] = M.S.E.[R^*] + (k_2\overline{X} - R)f''[(k_2\overline{X} - R)C_x^2 + 2RA] + (k_1\overline{X} - R)f^*[(k_1\overline{X} - R)C_x^{*2} + 2RB]$$

$$R.B.[T_R^3] = R.B.[R^*]$$

$$M.S.E.[T_R^3] = M.S.E.[R^*] + (k_2\overline{X})f''[(k_2\overline{X})C_x^2 + 2RA] + (k_1\overline{X})f^*[(k_1\overline{X})C_x^{*2} + 2RB]$$

$$R.B.[T_R^4] = R.B.[R^*] - \eta[f''A + f^*B] - (k_1\overline{X}\eta/R)f^*C_x^{*2} - (k_2\overline{X}\eta/R)f''C_x^2 + \frac{\eta(\eta-1)}{2}[f''C_x^2 + f^*C_x^{*2}]$$

$$M.S.E.[T_R^4] = M.S.E.[R^*] + (k_2\overline{X} - \eta R)f''[(k_2\overline{X} - \eta R)C_x^2 + 2RA] + (k_1\overline{X} - \eta R)f^*[(k_1\overline{X} - \eta R)C_x^{*2} + 2RB]$$

$$R.B.[T_R^5] = R.B.[R^*] - \eta^*[f''A + f^*B] - (k_1\overline{X}\eta^*/R)f^*C_x^{*2} - (k_2\overline{X}\eta^*/R)f''C_x^2 + \frac{\eta^*(\eta^*-1)}{2}[f''C_x^2 + f^*C_x^{*2}]$$

$$M.S.E.[T_R^5] = M.S.E.[R^*] + (k_2\overline{X} - \eta^* R)f''[(k_2\overline{X} - \eta^* R)C_x^2 + 2RA] + (k_1\overline{X} - \eta^* R)f^*[(k_1\overline{X} - \eta^* R)C_x^{*2} + 2RB]$$

Where, $\eta = \dfrac{\overline{X}}{\overline{X} + C_x}$ and $\eta^* = \dfrac{\overline{X}}{\overline{X} + C_x^*}$

$$R.B.[T_R^6] = R.B.[R^*] - \delta[f''A + f^*B] - (\beta_1^{**}\overline{X}\delta/R)f^*C_x^{*2} - (\beta_2^{**}\overline{X}\delta/R)f''C_x^2 + \frac{\delta(\delta-1)}{2}[f''C_x^2 + f^*C_x^{*2}]$$

$$M.S.E.[T_R^6] = M.S.E.[R^*] + (\beta_2^{**}\overline{X} - \delta R)f''[(\beta_2^{**}\overline{X} - \delta R)C_x^2 + 2RA] + (\beta_1^{**}\overline{X} - \delta R)f^*[(\beta_1^{**}\overline{X} - \delta R)C_x^{*2} + 2RB]$$

$$R.B.[T_R^7] = R.B.[R^*] - \delta[f''A + f^*B] - (C_x^{**}\overline{X}\delta/R)f^*C_x^{*2} - (C_x^{***}\overline{X}\delta/R)f''C_x^2 + \frac{\delta(\delta-1)}{2}[f''C_x^2 + f^*C_x^{*2}]$$

$$M.S.E.[T_R^7] = M.S.E.[R^*] + (C_x^{***}\overline{X} - \delta R)f''[(C_x^{***}\overline{X} - \delta R)C_x^2 + 2RA] + (C_x^{**}\overline{X} - \delta R)f^*[(C_x^{**}\overline{X} - \delta R)C_x^{*2} + 2RB]$$

$$R.B.[T_R^8] = R.B.[R^*] - \delta[f''A + f^*B] - (\rho^{**}\overline{X}\delta/R)f^*C_x^{*2} - (\rho^{***}\overline{X}\delta/R)f''C_x^2 + \frac{\delta(\delta-1)}{2}[f''C_x^2 + f^*C_x^{*2}]$$

$$M.S.E.[T_R^8] = M.S.E.[R^*] + (\rho^{***}\overline{X} - \delta R)f''[(\rho^{***}\overline{X} - \delta R)C_x^2 + 2RA] + (\rho^{**}\overline{X} - \delta R)f^*[(\rho^{**}\overline{X} - \delta R)C_x^{*2} + 2RB]$$

The optimum value of δ is given as

$$\delta_{opt} = \frac{f''(k_2\overline{X}C_x^2 + RA) + f^*(k_1\overline{X}C_x^{*2} + RB)}{R(f''C_x^2 + f^*C_x^{*2})}$$

Where, (k_1,k_2) stand for $(\beta_1^{**}/C_x^{**}/\rho^{**}, \beta_2^{**}/C_x^{***}/\rho^{***})$.

Also the expression of relative bias and mean square error of estimators, mention in (1), (2), (3) and (4), have been derived and given as

$$R.B.[T_1] = R.B.[R^*] - [f''A + f^*B]$$

$$M.S.E.[T_1] = M.S.E.[R^*] + R^2[f''(C_x^2 - 2A) + f^*(C_x^{*2} - 2B)]$$

$$R.B.[T_2] = R.B.[R^*] - f''A$$

$$M.S.E.[T_2] = M.S.E.[R^*] + R^2 f''(C_x^2 - 2A)$$

$$M.S.E.[T_3]_{\min} = M.S.E.[R^*] - R^2\{f''[A^2/C_x^2] + f^*[B^2/C_x^{*2}]\}$$

$$M.S.E.[T_4]_{\min} = M.S.E.[R^*] - R^2 f''[A^2/C_x^2]$$

Efficiency comparison

Theoretical comparison of proposed estimator (T_R) over other estimators

Comparison with R^*

$\theta[\theta + 2R\lambda] < 0$ when θ > 0 then θ < -2Rλ; when θ < 0 then θ > -2Rλ

and, $\phi[\phi + 2R\lambda^*] < 0$ when φ > 0 then φ < $-2R\lambda^*$; when φ < 0 then φ > $-2R\lambda^*$

Comparison with T_1

$(\theta - R)[(\theta - R) + 2R\lambda] < 0$ when θ > R then (θ-R) < -2Rλ; when θ < R then (θ-R) > -2Rλ

and, $(\phi - R)[(\phi - R) + 2R\lambda^*] < 0$ when φ > R then (φ-R) < $-2R\lambda^*$; when φ < R then (φ-R) > $-2R\lambda^*$

Comparison with T_2

$(\theta - R)[(\theta - R) + 2R\lambda] < 0$ when θ > R then (θ-R) < -2Rλ; when θ < R then (θ-R) > -2Rλ

and, $\phi[\phi + 2R\lambda^*] < 0$ when φ > 0 then φ < $-2R\lambda^*$; when φ < 0 then φ > $-2R\lambda^*$

Comparison with T_3

We observe that , these two estimators have almost same significance for optimum values of constants .

Comparison with T_4

$B\lambda^* > 0$

Where, $\lambda = A/C_x^2$ and $\lambda^* = B/C_x^{*2}$

A mathematical comparison

For performance of our proposed classes of estimator, we have to calculate mean square error mathematically and compare with relevant estimators using the data which has been used by Khare and Sinha and belong to the data on physical growth of upper socio –economic group of 95 schools going children of Varanasi under an ICMR study, Department of pediatrics, BHU, during 1983-84. In this data

Let x and y_i (i = 1,2) ,the auxiliary and study characters, are defined as
Y_1: The height of children in c.m.
Y_2 : The weight of children in k.g.
x : The chest circumference of children in c.m.
the values of parameters related to the study characters Y_2 (i =1,2) and auxiliary character, when first 25% (i.e. 24 children) units has been considered as non-response units, are given as :

N = 95	$n' = 75$	n = 55
$Y_1 = 115.9526$	$\overline{Y}_2 = 19.4968$	$\overline{X} = 55.8611$
$C_{y_1} = 0.05146$	$C_{y_2} = 0.15613$	$C_x = 0.05860$
$C^*_{y_1} = 0.04402$	$C^*_{y_2} = 0.12075$	$C^*_x = 0.05402$
$\rho_{y_1x} = 0.620$	$\rho_{y_2x} = 0.846$	$\rho_{y_1y_2} = 0.713$
$\rho^*_{y_1x} = 0.401$	$\rho^*_{y_2x} = 0.729$	$\rho^*_{y_1y_2} = 0.678$
$N_2 = 24$	f = 0.007656	f’ = 0.00280701
$f^*/(k-1) = 0.004593$	f” = 0.004848	R = 5.947263

Table 2: the mean square error(MSE) and relative efficiency (RE) of the estimators for different values of k

Conclusion

After analyzing of above table-2, we conclude that the proposed class of estimators $[T_R]$ is more efficient than relevant estimators T_1, T_2 and T_4 as well as its members T_R^1, T_R^6, T_R^7 and T_R^8 but it is equally precise to estimator T_3 for different values of k and fixed value of n and n’. Also it is observed that the MSE and percent RE of the proposed class of estimators $[T_R]$ increase with increase of the value of k.

Table-2

			k			
estimators		**2**		**3**		**4**
	MSE	**RE(%)**	**MSE**	**RE(%)**	**MSE**	**RE(%)**
T_R^1	0.005728	100	0.007241	100	0.008753	100
T_1	0.003542	161.7073	0.004294	168.6349291	0.005045	173.4988
T_2	0.004303	133.1048	0.005816	124.4947327	0.007329	119.439
$[T_3]_{min}$	**0.003202**	**178.8745**	**0.00391**	**185.170123**	**0.004618**	**189.5354**
$[T_4]_{min}$	0.004007	142.9543	0.005519	131.1824451	0.007032	124.4749
T_R^6	0.003203	178.8511	0.003911	185.147846	0.004619	189.514
T_R^7	0.003209	178.4893	0.003918	184.8038744	0.004627	189.1836
T_R^8	0.003205	178.6933	0.003914	184.9978256	0.004622	189.3699
$[T_R]_{min}$	**0.003202**	**178.8745**	**0.00391**	**185.170123**	**0.004618**	**189.5354**

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