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Estimation of Population mean in Generalized Class of Chain Ratio-Product Type Estimators in Double Sampling Using Two Auxiliary Variables

Ashish Sharma
Department of Statistics, Udai Pratap Autonomous College, Varanasi, U.P. INDIA

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Abstract

The present paper gives the idea of estimating the population mean of the study variable using two auxiliary variables in double sampling. Here we have suggested a class of estimators of population mean in double sampling. The relative bias and mean square error of the proposed class of estimators are also given. An empirical study is carried out for showing the performance of the proposed class of estimators. The comparisons of the proposed class of estimators with relevant estimator are also given on the basis of their mean square error.

Keyword: Two phase sampling, Auxiliary variables, study variable, mean square error, relative bias, chain ratio - product type estimator.

Introduction

We know that the efficiency of Ratio, product and regression type estimators for estimation of population mean of study variable depend on the information on auxiliary variable. When auxiliary variable is positively correlated with study variable, ratio estimator is used. Regression estimator is more useful when there is linear relationship between study and auxiliary variable. The product estimator is also used when there exist high negative correlation. The ratio estimator was developed by Cochran[1] for estimation of population mean of the study variable on the basis of auxiliary information. The product estimator is defined by Robson[2] and it was again defined by Murty[3].

When we do not have proper information on the population mean of the auxiliary variable the method of double sampling scheme is call back automatically. In this method we select a first phase sample whose size is n' from the population whose size is N on which only the auxiliary variable X is observed. Now we select a second phase sample of size n from first phase sample of size n' for estimating the study variable y. This process is called double sampling method which was firstly introduced by Neyman[4]. The chain ratio estimator is given by Chand[5], Kiregyera[6], Singh and Upadhyaya[7], Prasad et al[8], Singh et al[9], Singh and Choudhury[10] and Vishwakarma and Gangele[11].

The estimators

The chain type ratio and product estimator in double sampling using two auxiliary variables is usually given as under

$$\bar{y}_R^{dc} = \bar{y}\frac{\bar{x}'}{\bar{x}}\frac{\bar{Z}}{\bar{z}'}$$
$$\bar{y}_P^{dc} = \bar{y}\frac{\bar{x}}{\bar{x}'}\frac{\bar{z}'}{\bar{Z}} \qquad (1)$$

In the similar way, The exponential chain type ratio and product estimators of population mean given by Singh and Choudhury using two auxiliary variables in double sampling given as

$$\bar{y}_{Re}^{dc} = \bar{y}\exp\left\{\frac{(\bar{x}'/\bar{z}')\bar{Z}-\bar{x}}{(\bar{x}'/\bar{z}')\bar{Z}+\bar{x}}\right\}$$
$$\bar{y}_{Pe}^{dc} = \bar{y}\exp\left\{\frac{\bar{x}-(\bar{x}'/\bar{z}')\bar{Z}}{\bar{x}+(\bar{x}'/\bar{z}')\bar{Z}}\right\} \qquad (2)$$

Singh and Ruiz Espejo[12] suggest the ratio product type estimator in double sampling for population mean using auxiliary variable respectively given by

$$\bar{y}_{RP}^{d} = \bar{y}\left[\alpha\frac{\bar{x}'}{\bar{x}}+(1-\alpha)\frac{\bar{x}}{\bar{x}'}\right] \qquad (3)$$

Where α is real constant.

And Vishwakarma, G.K. and Kumar, M[13] proposed the improved class of chain ratio-product type estimators in double sampling for population mean using two auxiliary variables are given respectively by

$$\bar{y}_{RPe}^{dc}=\bar{y}\left[\alpha\exp\frac{\left(\bar{x}'/\bar{z}'\right)\bar{Z}-\bar{x}}{\left(\bar{x}'/\bar{z}'\right)\bar{Z}+\bar{x}}+(1-\alpha)\exp\frac{\bar{x}-\left(\bar{x}'/\bar{z}'\right)\bar{Z}}{\bar{x}+\left(\bar{x}'/\bar{z}'\right)\bar{Z}}\right] \quad (4)$$

Where α is real constant.

The proposed class of estimator

Now we propose the following generalized form of exponential chain ratio-product type estimator for population mean under double sampling using two auxiliary variables given as under

$$\bar{y}_R=\bar{y}\left[\alpha\exp\frac{\bar{x}'\left(\bar{Z}/\bar{z}'\right)^{\beta}-\bar{x}}{\bar{x}'\left(\bar{Z}/\bar{z}'\right)^{\beta}+\bar{x}}+(1-\alpha)\exp\frac{\bar{x}-\bar{x}'\left(\bar{Z}/\bar{z}'\right)^{\beta}}{\bar{x}+\bar{x}'\left(\bar{Z}/\bar{z}'\right)^{\beta}}\right] \quad (5)$$

Where α and β are constant. $\bar{x}'$ and $\bar{z}'$ are the sample means of X and Z based on the first phase sample of size n' and $\bar{y}$ and $\bar{x}$ are the sample means of Y and X respectively based on second phase sample of size n.

Mean Square Error and Relative Bias

For this purpose let us assume

$\bar{y}=\bar{Y}(1+\xi_1)$, $\bar{x}=\bar{X}(1+\xi_2)$, $\bar{x}'=\bar{X}(1+\xi_3)$, $\bar{z}'=\bar{Z}(1+\xi_4)$

Such that $E(\xi_1)=E(\xi_2)=E(\xi_3)=E(\xi_4)=0$

Putting above values in, the equation (5) express as in terms of $\xi_i's$ and neglecting the power greater than two we have

$$\bar{y}_R=\bar{Y}(1+\xi_1)\left[\alpha\exp\frac{(1+\xi_3)(1+\xi_4)^{-\beta}-(1+\xi_2)}{(1+\xi_3)(1+\xi_4)^{-\beta}+(1+\xi_2)}+(1-\alpha)\exp\frac{(1+\xi_2)-(1+\xi_3)(1+\xi_4)^{-\beta}}{(1+\xi_2)+(1+\xi_3)(1+\xi_4)^{-\beta}}\right]$$

$$R.B.[\bar{y}_R]=E[\bar{y}_R-\bar{Y}]/\bar{Y}$$

$$=\left\{\frac{(2\alpha-1)}{4}(f''C_X^2+\beta f'C_Z^2)+\frac{1}{8}\left(f''C_X^2+\beta^2 f'C_Z^2\right)-\frac{(2\alpha-1)}{2}(f''C_{XY}+\beta f'C_{YZ})\right\}$$

$$MSE[\bar{y}_R]=E[\bar{y}_R-\bar{y}]^2$$

$$=\bar{Y}^2E\left[\xi_1+\frac{(2\alpha-1)}{2}(\xi_3-\xi_2-\beta\xi_4)\right]^2$$

$$=\bar{Y}^2\left[fC_Y^2+f''\frac{(2\alpha-1)}{2}\left\{\frac{(2\alpha-1)}{2}C_X^2-2C_{XY}\right\}+f'\frac{(2\alpha-1)}{2}\beta\left\{\frac{(2\alpha-1)}{2}\beta C_z^2-2C_{YZ}\right\}\right] \quad (6)$$

Where, $f=\frac{1}{n}-\frac{1}{N}$, $f'=\frac{1}{n'}-\frac{1}{N}$, $f''=\frac{1}{n}-\frac{1}{n'}$

And, $E[\xi_1^2]=fC_Y^2$, $E[\xi_2^2]=fC_X^2$, $E[\xi_3^2]=f'C_X^2$, $E[\xi_4^2]=f'C_Z^2$, $E[\xi_1\xi_2]=fC_{XY}$

$E[\xi_1\xi_3]=f'C_{XY}$, $E[\xi_1\xi_4]=f'C_{YZ}$, $E[\xi_2\xi_3]=f'C_X^2$, $E[\xi_2\xi_4]=f'C_{XZ}$, $E[\xi_3\xi_4]=f'C_{XZ}$

And, $C_{XY}=\rho_{XY}C_XC_Y$, $C_{XZ}=\rho_{XZ}C_XC_Z$, $C_{YZ}=\rho_{YZ}C_YC_Z$, $C_Y=S_Y/\bar{Y}$, $C_X=S_X/\bar{X}$, $C_Z=S_Z/\bar{Z}$

The mean square error of the above existing estimators

$$MSE(\bar{y}_R^{dc})=\bar{Y}^2\left[fC_Y^2+f''C_X^2+f'C_Z^2-2f''C_{YX}-2f'C_{YZ}\right]$$

$$MSE(\bar{y}_P^{dc})=\bar{Y}^2\left[fC_Y^2+f''C_X^2+f'C_Z^2+2f''C_{YX}+2f'C_{YZ}\right]$$

$$MSE(\bar{y}_{Re}^{dc})=\bar{Y}^2\left[fC_Y^2+1/4\left\{f''C_X^2+f'C_Z^2\right\}-\left\{f''C_{YX}+f'C_{YZ}\right\}\right]$$

$$MSE(\bar{y}_{Pe}^{dc})=\bar{Y}^2\left[fC_Y^2+1/4\left\{f''C_X^2+f'C_Z^2\right\}+\left\{f''C_{YX}+f'C_{YZ}\right\}\right]$$

$$MSE(\bar{y}_{RP}^{d})=\bar{Y}^2\left[fC_Y^2+4\alpha^2f''C_X^2-4\alpha f''\left\{C_X^2+C_{YX}\right\}+f''\left\{C_X^2+2C_{YX}\right\}\right] \quad (7)$$

$$MSE(\bar{y}_{RPe}^{dc})=\bar{Y}^2\left[fC_Y^2+(2\alpha-1)^2/4\left\{f''C_X^2+f'C_Z^2\right\}-(2\alpha-1)\left\{f''C_{YX}+f'C_{YZ}\right\}\right] \quad (8)$$

Optimum value of α and β

$$\frac{\partial}{\partial\alpha}MSE(\bar{y}_{RP}^{d})=0$$

$$\frac{\partial}{\partial\alpha}MSE(\bar{y}_{RPe}^{dc})=0$$

$$\frac{\partial}{\partial\alpha}MSE(\bar{y}_{R})=0 \quad (9)$$

$$\frac{\partial}{\partial\beta}MSE(\bar{y}_{R})=0$$

The optimum value of α which minimizes the MSE of $\bar{y}_{RP}^{d}$ is given as under

$$\alpha_{opt}=1/2\left[1+\rho_{YX}\frac{C_Y}{C_X}\right] \quad (10)$$

Similarly, The optimum value of α which minimizes the MSE of $\bar{y}_{RPe}^{dc}$ is given as under

$$\alpha_{opt}=\frac{f''(2C_{YX}+C_X^2)+f'(2C_{YZ}+C_Z^2)}{2(f''C_X^2+f'C_Z^2)} \quad (11)$$

In the same way

The optimum value of α and β which minimizes the MSE of $\bar{y}_R$ is given as under

$$\alpha_{opt} = 1/2\left[1 + \frac{2C_{XY}}{C_X^2}\right]$$

And (12)

$$\beta_{opt} = \left[\frac{C_{YZ}C_X^2}{C_{XY}C_Z^2}\right]$$

Putting the above optimum values of α and β in their respective MSE we get the min (MSE) of these estimators

$$MSE(\bar{y}_{RP}^{d})_{\min} = \bar{Y}^2\left[fC_Y^2 - f''\rho_{YX}^2 C_Y^2\right] \quad (13)$$

$$MSE(\bar{y}_{RPe}^{dc})_{\min} = \bar{Y}^2\left[fC_Y^2 - \frac{\left(f''C_{YX} + f'C_{YZ}\right)^2}{(f''C_X^2 + f'C_Z^2)}\right] \quad (14)$$

$$MSE(\bar{y}_R)_{\min} = \bar{Y}^2\left[fC_Y^2 - f''\frac{C_{XY}^2}{C_X^2} - f'\frac{C_{YZ}^2}{C_Z^2}\right] \quad (15)$$

Efficiency comparison

Theoretical comparison of proposed class of estimator ($\bar{y}_R$) over other estimators

$$where, V(\bar{y}) = f\bar{Y}^2C_Y^2$$

$$MSE(\bar{y}_R) < V(\bar{y}) if$$

$$f''(2\alpha-1)/2[\{(2\alpha-1)/2\}C_x^2 - 2C_{xy}] + f'(2\alpha-1)\beta/2[\{(2\alpha-1)\beta/2\}C_z^2 - 2C_{yz}] < 0$$

Or, $f''(2\alpha-1)/2[\{(2\alpha-1)/2\}C_x^2 - 2C_{xy}] < 0$

either $2C_{xy}/C_x^2 < \alpha-1/2 < 0$ Or $2C_{xy}/C_x^2 > \alpha-1/2 > 0$

and, $f'(2\alpha-1)\beta/2[\{(2\alpha-1)\beta/2\}C_z^2 - 2C_{yz}] < 0$

either $2C_{yz}/C_z^2 < \alpha-1/2 < 0$; $\beta > 0$ Or $2C_{yz}/C_z^2 > \alpha-1/2 > 0$; $\beta > 0$

$$MSE(\bar{y}_R) < MSE(\bar{y}_R^{dc}) if$$

$$f''\{\frac{(2\alpha-1)}{2} - 1\}[\{\frac{(2\alpha-1)}{2} + 1\}C_x^2 - 2C_{xy}] + f'\{\frac{(2\alpha-1)\beta}{2} - 1\}[\{\frac{(2\alpha-1)\beta}{2} + 1\}C_z^2 - 2C_{yz}] < 0$$

Or, $f''\{\frac{(2\alpha-1)}{2} - 1\}[\{\frac{(2\alpha-1)}{2} + 1\}C_x^2 - 2C_{xy}] < 0$

either $2C_{xy}/C_x^2 < \alpha+1/2 < 2$
Or $2C_{xy}/C_x^2 > \alpha+1/2 > 2$ And,

$$f'\{\frac{(2\alpha-1)\beta}{2} - 1\}[\{\frac{(2\alpha-1)\beta}{2} + 1\}C_z^2 - 2C_{yz}] < 0$$

either $(2C_{yz}/C_z^2)-1 < (\alpha-1/2)\beta < 1$ Or $(2C_{yz}/C_z^2)-1 > (\alpha-1/2)\beta > 1$

$$MSE(\bar{y}_R) < MSE(\bar{y}_{Re}^{dc}) if$$

$$f''\{\frac{(2\alpha-1)}{2} - 1/2\}[\{\frac{(2\alpha-1)}{2} + 1/2\}C_x^2 - 2C_{xy}] + f'\{\frac{(2\alpha-1)\beta}{2} - 1/2\}[\{\frac{(2\alpha-1)\beta}{2} + 1/2\}C_z^2 - 2C_{yz}] < 0$$

Or, $f''\{\frac{(2\alpha-1)}{2} - 1/2\}[\{\frac{(2\alpha-1)}{2} + 1/2\}C_x^2 - 2C_{xy}] < 0$

either $2C_{xy}/C_x^2 < \alpha < 1$ Or $2C_{xy}/C_x^2 > \alpha > 1$

And, $f'\{\frac{(2\alpha-1)\beta}{2} - 1/2\}[\{\frac{(2\alpha-1)\beta}{2} + 1/2\}C_z^2 - 2C_{yz}] < 0$

either $(2C_{yz}/C_z^2)-1/2 < (\alpha-1/2)\beta < ½$ Or $(2C_{yz}/C_z^2)-1/2 > (\alpha-1/2)\beta > 1/2$

$$MSE(\bar{y}_R) < MSE(\bar{y}_{RPe}^{dc}) if$$

$$f'\{\frac{(2\alpha-1)}{2}(\beta-1)\}[\{\frac{(2\alpha-1)}{2}(\beta+1)\}C_z^2 - 2C_{yz}] < 0$$

either $(2C_{yz}/C_z^2) < (\alpha-1/2) < 0$; $\beta > 1$ Or $(2C_{yz}/C_z^2) > (\alpha-1/2) > 0$; $\beta > 1$

A Numerical comparison: Here we take an example of following population data set for checking the merit of proposed class of estimator of population mean.
Population 1(Source: Cochran (1977)
The variables are given as under
y – Number of placebo children.
x – Number of paralytic polio cases in the 'placebo' group.
z – Number of paralytic polio cases in the 'not inoculated' group.

N=34, n' = 15, n=10 $\bar{Y}$ = 4.92 $\bar{X}$ = 2.59
$\bar{Z}$ =2.91 C_Y =1.0123 C_X =1.2318 C_Z = 1.0720 ρ_{YX}=0.7326
ρ_{YZ}=0.6430 ρ_{XZ}=0.6837.

The formula for percent relative efficiencies (PREs) is give as under $PRE(.,\bar{y}) = \frac{MSE(\bar{y})}{MSE(.)} \times 100$

Table-1
Mean square error and percent relative efficiency of the estimators with respect to $\bar{y}$

Estimators	Population (I)	
	MSE	PRE
$\bar{y}$	1.75106	100
$\bar{y}_R^{dc}$	1.27900	136.9087
$\bar{y}_P^{dc}$	6.74470	25.9621
$\bar{y}_{Re}^{dc}$	0.94980	184.3612
$\bar{y}_{Pe}^{dc}$	3.68270	47.5483
$\bar{y}_{RP}^{d}$	1.30730	133.9450
$\bar{y}_{RPe}^{dc}$	0.92519	189.2652
$\bar{y}_R$	0.92517	189.2693

Conclusion

From table1, we conclude that the proposed class of estimators ($\overline{y}_R$) is more efficient in comparison to other existing estimators but it is equally precise to estimator ($\overline{y}^{dc}_{RPe}$) .

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