Research Journal of Mathematical and Statistical Sciences ______________________________ISSN 2320–6047
Vol. **2(7),** 4-6, July **(2014)** *Res. J. Mathematical and Statistical Sci.*

ON Q-Fuzzy Ideal and Q-Fuzzy Quotient Near-Rings

Bhimraj Basumatary[1] and Gopi Kanta Barthakur[2]
[1]Research Scholar, Department of Mathematical Sciences, Bodoland University, Kokrajhar, Assam, INDIA
[2]Department of Mathematics, G. K. B. College, Morigaon, Assam, INDIA

Available online at: www.isca.in, www.isca.me
Received 20th May 2014, revised 15th June 2014, accepted 10th July 2014

Abstract

In this paper, we shall study Q-fuzzy ideal and Q-fuzzy quotient near-ring and investigate some of there properties and we prove some characterizations of a near-ring in terms of Q-fuzzy quotient near-ring and Q-fuzzy ideal.

Keywords: Q-fuzzy ideal, Q-fuzzy quotient near-ring.

Introduction

Zadeh[1] introduce fuzzy set in 1965. The idea of the fuzzy ideal[2] in near-ring[3] is discussed by Zaid[2]. Solarairaju *et al.*[4] introduce the new structures of Q-fuzzy groups. On the other hand Muhammad Akram[5] introduces the T-fuzzy Ideals and quotient near-ring. In this paper, we shall study quotient near-rings via Q-fuzzy[3] ideals and study some of their properties. Generally in this work we follow a paper published by Muhammad Akram[5] to prove theorems.

Preliminaries

Definition: A near-ring[3] is a set R which is non empty with two binary operation "+" and "." Which holds the condition, (R, +) is group, (R, .) is semi[1] group and multiplicative is distributive with respect to addition.

Definition: Let us consider a non empty set A. Then a function μ : R → [0, 1] is a fuzzy[1] subset of A.

Definition: A function μ : G×Q →[0, 1] is called Q-fuzzy[3] set in G, where Q be a set and G be group respectively.

Definition: Consider a function f from a set A to B and a Q-fuzzy[3] set μ in A. Then μ is a Q- fuzzy[3] set in B defind by

$$f(\mu)(y, q) = \begin{cases} \sup_{x\in f^{-1}(y)} \mu(x,q) : f^{-1}(y) \neq \phi \\ 0 : otherwise \end{cases}$$

Definition: Let Im(λ) denote the image set of λ. Let λ be a Q-fuzzy[3] set in a set R. For t in [0, 1] the set λ_t={ x∈R, q∈Q; λ(X, Q)≥t} is called Q-level[3] subset of λ .

Definition: Consider μ a Q- fuzzy[3] subset in a near-ring[5] R, then μ is Q- fuzzy[3] subnear-ring[5] of R if it holds the conditions
1. μ(x-y, q) ≥ min{ μ(x, q) , μ(y, q) }
2. μ(xy, q) ≥ min{ μ(x, q) , μ(y, q) }

Definition: A Q-fuzzy[3] subnear-ring μ in R is called Q-fuzzy[3] ideal[6] if
1. μ(y +x-y, q) ≥ μ(x, q)
2. μ(xy, q) ≥ μ(y, q)
3. μ((x+ z)y-xy, q) ≥ μ(z, q)

Theorem: If we consider a onto homomorphism[5] function f : A → B of near-rings. Consider μ be a Q- fuzzy[3] ideal[6] in A, we get, then a Q- fuzzy[3] ideal[6] f(u) in B.
Proof: Consider v, w be two elements in the set B.

Since f is onto homomorphism, then as Muhammad Akram[5] we are clear to show
{b – c |b∈f^{-1}(v), c∈f^{-1}(w)} is subset of {x| x∈ f^{-1}(v-w)}.

Now as definition[1] 2.9 of f(μ)(x, q) we have

$$f(\mu)(v-w, q) = \sup_{x\in f^{-1}(v-w)} \mu(x,q)$$

$$\geq \sup_{\substack{b\in f^{-1}(v)\\ c\in f^{-1}(w)}} \mu(b-c,q)$$

$$\geq \min\left\{\sup_{b\in f^{-1}(v)} \mu(b,q), \sup_{c\in f^{-1}(w)} \mu(c,q)\right\}$$

= min {f(μ)(v, q), f(μ)(w, q)}

Now following defination[1] 2.9

$$f(\mu)(vw, q) = \sup_{x\in f^{-1}(vw)} \mu(x,q)$$

$$\geq \sup_{\substack{b\in f^{-1}(v)\\ c\in f^{-1}(w)}} \mu(bc,q)$$

$$\geq \min\left\{\sup_{b\in f^{-1}(v)} \mu(b,q), \sup_{c\in f^{-1}(w)} \mu(c,q)\right\}$$

f(μ) is Q-fuzzy[3] sub near-ring[5].

Now also we have

$$f(\mu)(v+w-v, q) = \sup_{x\in f^{-1}(v+w-v)} \mu(x,q)$$

$\geq \sup_{\substack{b\in f^{-1}(v)\\ c\in f^{-1}(w)}} \mu(b+c-b,q)$

$\geq \sup_{c\in f^{-1}(w)} \mu(c,q)$

= f(μ)(w, q)

Again as just we did, it can show easily

f(μ)(vw, q) = $\sup_{x\in f^{-1}(vw)} \mu(x,q)$

$\geq \sup_{\substack{b\in f^{-1}(v)\\ c\in f^{-1}(w)}} \mu(bc,q)$

$\geq \sup_{c\in f^{-1}(w)} \mu(c,q)$

= f(μ)(w, q)

Now from the result above it is clear that
f (μ)((v+z)w-vw, q) ≥ f(μ)(z, q).
Hence f(μ) is Q-fuzzy[3] ideal[6].

Theorem 3.2 Consider an ideal[6] A of a near ring[5] R. Consider a Q- fuzzy[3] ideal[6] μ of R, let us construct Q- fuzzy[3] set Ψ of R/A such that
Ψ (a+A, q) = $\sup_{x\epsilon A}$ μ(a+x, q)
then Ψ is Q-fuzzy[3] ideal[6] of the quotient[5] near- ring[5] R/A with respect to A.
Proof: Consider two elements a, b of R so that (a+A) is equal to (b+A). Then we have
b =a+y for some y in A. Now as Muhammad Akram[5] we try to show Ψ is well define
Ψ (b+A, q)= $\sup_{x\epsilon A}$ μ(b+x, q)
= $\sup_{x\epsilon A}$ μ(a+y+x, q)
= $\sup_{x+y=z\epsilon A}$ μ(a+z, q)
= Ψ(a+A, q).

Consider (x+A), (y+A) be two elements of R/A, now following definition[6] 2.6, 2.9 and 2.12 we do the following steps
Ψ ((x+A)-(y+A), q)=Ψ((x-y)+A, q)
= $\sup_{z\epsilon A}$ μ ((x-y)+z, q)
= $\sup_{u-v=z\epsilon A}$ μ ((x-y) + (u-v), q)
= $\sup_{x\epsilon A}$ μ ((x+u)-(y+v), q)
≥ min{$\sup_{x\epsilon A}$ μ(x+u, q), $\sup_{x\epsilon A}$ μ(y+v, q)}

Also similarly following the definition[6] 2.9
Ψ ((x+A)(y+A), q)=Ψ(xy+A, q)
= $\sup_{Z\epsilon A}$ μ (xy+z, q)
= $\sup_{z=uv\epsilon A}$ μ (xy+uv, q)
≥ min {$\sup_{u,v\epsilon A}$ μ(x+u, q), $\sup_{x\epsilon A}$ μ(y+v, q)}

This shows that Ψ is Q- fuzzy[3] sub near-ring in R/A.
Thus we can show Ψ is an Q- fuzzy[3] ideal[6].

Theorem 3.3 Consider an ideal[6] A of a near- ring[5] R. We can have then one to one mapping between then set of Q-fuzzy[3] ideals[6] μ of R so that μ(0, q) is equal to μ(s, q) for all "s" in A and Ψ set of all Q-fuzzy[3] ideals[6] of R/A.

Proof: Let μ be Q-fuzzy[3] ideal[6] of R so following theorem 3.1 and 3.2 and from definition 2.4 and 2.6 we are clear to show Ψ(a+A, q)=$\sup_{x\epsilon A}$ μ(a, q) is a Q-fuzzy[3] ideal[6] of R/A.
Since, we have μ(0, q)=μ(s, q)

Also from definition[2] 2.12
μ(a+s, q)≥μ(a, q).
Also, μ(a, q) = μ(a+s-s, q) ≥ μ(a+s, q)
Thus we have μ(a+s, q)=μ(a, q), for all s ϵ A.

Thus, Ψ(a+A, q) is equal to μ(a, q).
Hence the corresponding μ ⟼ Ψ is one to one.
Let Ψ be Q-fuzzy[3] ideal[6] of R/A. Consider μ as a Q- fuzzy[3] set in R so that for all "a" in A μ(a, q) is equal to Ψ(a+A, q).

Now, for x, y ϵR, we have from definition 2.6 and from theorems 3.1 and 3.2 it follows
μ(x-y, q)= Ψ((x-y)+A, q)
= Ψ((x+A)-(y+A), q)
≥ min { Ψ((x+A), q), Ψ((y+A), q)}
= min{ Ψ((x+A)-(y+A), q)}

μ(xy, q)= Ψ((xy)+A, q)
= Ψ((x+A)(y+A), q)
≥ Ψ(x+A, q)
=μ(x, q).

Thus μ is Q-fuzzy[3] ideal[6] of R. clearly μ(a, q) is equal to Ψ(a+A, q) which equal to Ψ(A, q), for all a in A. This indicates that μ(0, q) is equal to μ(s, q) for all s ϵA.

Theorem 3.4 Let us consider A be an ideal[6] of a near-ring[5] R. We can have then a Q-fuzzy[3] ideal[6] μ of R so that μ(0, a) is t and λ_t is A, for tϵ[0, 1] where λ_t is called Q-level[3] subset of λ.
Proof: Following definition 2.6 and theorems 3.1, 3.2, 3.3 the proof is straight forward[2].

Theorem 3.5 Consider a Q-fuzzy[3] ideal[6] μ of a near-ring[5] R also μ(0, a) is t. Then Ψ is a Q-fuzzy[3] ideal[6] of R/λ_t, where Ψ is constructed as Ψ(x+λ_t, q) =μ(x, q) for all xϵR and λ_t is called Q-level[3] subset of λ.
Proof: Similarly following definition 2.6 and theorems 3.1, 3.2, 3.3 and 3.4 proof is straight forward[2].

Conclusion

In this paper we have defined Q-fuzzy subnear-ring, Q-fuzzy ideal. With the help of Q-fuzzy subnear-ring and Q-fuzzy ideal, we have discussed on Q-fuzzy quotient near-ring and proved some theorems on Q-fuzzy quotient near-ring. We hope that this work will help for further work of fuzzy set.

References

1. Zadeh L.A., Fuzzy sets, *Information and Control*, **8,** 338-353 **(1965)**

2. Zaid A., On fuzzy sub near-rings, *Fuzzy Sets and System*, **44**, 139-146 **(1991)**

3. Mourad O.M., On Q-Fuzzy R-Subgroups of Near-Rings, *International Mathematical Forum*, **8**, 387-393 **(2013)**

4. Solairaju A., Nagarajan R. A new structure and construction of Q-fuzzy group, *Advance in Fuzzy Mathematics*, **4**, 512-517 **(2009)**

5. Akram M., On T-Fuzzy Ideals in Nearings, *International Journal of Mathematics and Mathematical Sciences*, doi:10.1155/2007/73514 **(2007)**

6. Bhakad S.K., Das P., Fuzzy subrings and ideals redefined, *Fuzzy Sets and System*, **81**, 383-393 **(1996)**

7. Bhakad S.K., Das P., On the definition of a fuzzy subgroup, *Fuzzy Sets and System*, **51**, 235-241 **(1992)**