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Construction of New Series of Super-Saturated Designs

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Abstract

A factorial design is said to be a 'super-saturated design' if the number of factors are more than the number of design points. For efficient utilization of resources and to study the effects active factors which are fewer, these designs reduces the experimental cost and time significantly. In this paper, an attempt is made to construct a new series of super-saturated designs. The designs are illustrated with suitable examples.

Keywords: Row-column designs, cyclic designs, super-saturated designs, $E(s^2)$-optimality.

Introduction

In a factorial design, if the number of factors is more than the number of design points then the design is said to be a super-saturated design. When the experimentation is very expensive and the number of factors is more but there are few factors, which are active, then we seek for the designs with less number of design points. These designs are more economical and flexible because of their run size.

Satterthwaite initially made an attempt to construct saturated designs randomly and suggested the random balance designs[1]. Booth and Cox proposed a systematic method for the construction of super-saturated designs[2]. After Booth and Cox,, no attempts were made till Lin [3]. Later several researchers made attempts on the construction of super-saturated designs.

For selecting the best among the available super-saturated designs the criteria of E(s2)-optimality is used. It can be evaluated from the cross product terms of the super saturated design. Let s_{ij} be the sum of cross products between two different factors i and j of a design. If the expected value of s^2 i.e. the mean of s_{ij}^2 of all pairs (i, j) for (i≠j) is minimum, then the design is said to be $E(s^2)$-optimal design.

Construction of Super-Saturated Design Using Row-Column Design

Row-Column design is one of the most popular incomplete block designs. The arrangement of 'v' treatments in 'b' blocks such that each treatment is repeated 'r' times and each block contains k treatment (k<v) and the treatment in each block are arranged in 'p' rows and 'q' columns. In the row column design, experimental units are grouped in two directions, one in row wise and the other one in column wise.

Consider a row-column design with parameters v, b, r, k, p and q. let the total number of columns be qb. Assume that v > qb. Construct a design of order v x qb such that corresponding to each treatment, put +1 if the treatment appears in the column of a block and put -1 if the treatment not appears in that column. The resulting design is a super-saturated design in qb factors with v design points. The method is illustrated through the following example

Example-2.1 Consider a row-column design with parameters v=7, b=7, k=4, r= 4, p=q=2 as:

Blocks	**B_1**		**B_2**		**B_3**		**B_4**		**B_5**		**B_6**		**B_7**	
Columns	C_1	C_2	C_1	C_2	C_1	C_2	C_1	C_2	C_1	C_2	C_1	C_2	C_1	C_2
R_1	A	C	B	D	C	E	D	F	E	G	F	A	G	B
R_2	E	B	F	C	G	D	A	E	B	F	C	G	D	A

The resulting super-saturated design is

$$X=\begin{bmatrix} +1 & -1 & -1 & -1 & -1 & -1 & +1 & -1 & -1 & -1 & -1 & +1 & -1 & +1 \\ -1 & +1 & +1 & -1 & -1 & -1 & -1 & -1 & +1 & -1 & -1 & -1 & -1 & +1 \\ -1 & +1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 & -1 & +1 & -1 & -1 & -1 \\ -1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 & -1 & +1 & -1 \\ +1 & -1 & -1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & +1 & -1 & -1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & +1 & -1 & -1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 \end{bmatrix}$$

The $E(s^2)$ optimal value for the supersaturated design is 4.80

Method of Construction of Super-Saturated Design Using Cyclic Design

The 'v' treatments are arranged in 'b' blocks with each of size 'k' (k>2) such that each treatment replicated 'r' times. The number of blocks can be generated are ${}^{v}C_{k}$. These blocks can be divided into number of cyclic sets. A cyclic set will be such that the treatments in the i^{th} block of the set can be obtained by adding unity with reduced modulo 'v' when necessary to the treatments in the $(i-1)^{th}$ block. These designs may be balanced or partially balanced incomplete block designs.

The cyclic designs can be obtained by the cyclic development of a set of initial blocks (one or more blocks). It is said to be resolvable if the blocks can be grouped in such a way that each group contains every treatments or object exactly once, thus forming a complete replicate[4]. Let 'v' be the number of treatments and 'b' be the number of blocks of a cyclic resolvable incomplete block design developed through the 'q' initial blocks. (Assume q>1). The method of constriction of super-saturated design using cyclic resolvable design is presented below.

Let 'v' be the number of treatments and 'b' be the number of blocks of a cyclic resolvable incomplete block design developed through the 'q' initial blocks. (qk=v). Construct a design by considering each block is corresponding to a factor and each design point contains the factors with levels ±1, as, put +1 if the treatment presented in the block and put –1 for other factors/treatments, the resulting design is a supersaturated design in 'qv' factors with 'v' design points. This method is illustrated through the following example.

Example 3.1: Consider the cyclic resolvable design with parameters v=9, b=9, r=1 and k=3 generated through the initial blocks {1, 2} and {3, 6}. Put +1 if treatment presented in the block and Put –1 if treatment is absent. The generated initial vectors are (+1, +1, -1, -1, -1, -1,-1,-1,-1) and (-1, -1, +1, -1, -1,+1, -1, -1, -1). Permute these generated vectors to obtain the super-saturated design X in 9 design points with 18 treatments as

$$X=\left[\begin{array}{cccccccccccccccccc}
+1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & +1 & -1 & -1 & -1 & -1 & +1 & -1 & -1 & +1 & -1 \\
+1 & +1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & +1 & -1 & -1 & +1 \\
-1 & +1 & +1 & -1 & -1 & -1 & -1 & -1 & -1 & +1 & -1 & -1 & -1 & -1 & -1 & +1 & -1 & -1 \\
-1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 & -1 & -1 & +1 & -1 & -1 & -1 & -1 & -1 & +1 & -1 \\
-1 & -1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 & -1 & -1 & +1 & -1 & -1 & -1 & -1 & -1 & +1 \\
-1 & -1 & -1 & -1 & +1 & +1 & -1 & -1 & -1 & +1 & -1 & -1 & +1 & -1 & -1 & -1 & -1 & -1 \\
-1 & -1 & -1 & -1 & -1 & +1 & +1 & -1 & -1 & -1 & +1 & -1 & -1 & +1 & -1 & -1 & -1 & -1 \\
-1 & -1 & -1 & -1 & -1 & -1 & +1 & +1 & -1 & -1 & -1 & +1 & -1 & -1 & +1 & -1 & -1 & -1 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & +1 & +1 & -1 & -1 & -1 & +1 & -1 & -1 & +1 & -1 & -1
\end{array}\right]$$

The $E(s^2)$ optimal value for the design is 9.47.

Conclusion

In this paper we proposed two new construction methods from Row-Column and Cyclic Resolvable respectively. These designs are more efficient designs than any other existed designs.

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