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I-Function and Boundary Value Problem in a Rectangular Plate

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Abstract

This paper will put an insight into an application of a solution of boundary value problem in a rectangular plate with the help of I-function of one variable.

Keywords: I-function, boundary value problem.

Introduction

The I-function of one variable is defined by Saxena[1] and we shall represent here in the following manner:

$$I[z]=I^{m,n}_{p_i,q_i;r}\left[z\left|\begin{matrix}[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1},p_i]\\ [(b_j,\beta_j)_{1,m}],[(b_{ji},\beta_{ji})_{m+1},q_i]\end{matrix}\right.\right]=\frac{1}{2\pi w}\int_L \theta(s)z^s\,ds \quad (1)$$

Where $\omega=\sqrt{(-1)},\ z(\neq 0)$ is a complex variable and

$$z^s=\exp[s\{\log|z|+w\arg z\}] \quad (2)$$

In which log |z| represent the natural logarithm of |z| and arg |z| is not necessarily the principle value. An empty product is interpreted as unity, Also,

$$\theta(s)=\frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j s)\prod_{j=1}^{n}\Gamma(1-a_j-\alpha_j s)}{\sum_{i=1}^{r}\left[\prod_{j=m+1}^{qi}\Gamma(1-b_{ji}-\beta_{ji}s)\prod_{j=n+1}^{pi}\Gamma(a_{ji}-\alpha_{ji}s)\right]} \quad (3)$$

m,n, and pi $\forall\ i \in (1,\ldots..r)$ are no –negative integers satisfying $0 \le n \le p_i$, $0 \le m \le q_i$, $\forall\ i \in (1,\ldots..r)$, α_{ji}, (j=1,….. p_i; I = 1,………r) and β_{ji} (j = 1,….q_i ; I = 1,….r) are assumed to be positive quantities for standardization purpose . Also a_{ji} (j = 1,…., p_i; I = 1,……,r) and b_{ji} (j = 1,…….., q_i; I = 1,…..,r) are complex numbers such that none of the points.

$$S=\{(bn+v)\,|\,\beta_h\,|\},\ h=1,\ldots\ldots,m;\ v=0,1,2,\ldots\ldots \quad (4)$$

Which are the poles of T (b_n – β_nS), h = 1,……m and the points.

$$S=\{(a_1-\eta-1)\,|\,\alpha_l\,|\,l=1,\ldots..,n;\ \eta=0,1,2,\ldots., \quad (5)$$

Which are the poles of $\Gamma(1-a_l+\alpha_l s)$ coincide with one another, i.e. with

$$\alpha_l(b_n+v)\neq b_n(a_l-\eta-1) \quad (6)$$

For n, h = 0, 1,2,….; h = 1,…., m; 1 = 1,…..,n

Further, the contour L runs from $-\omega_\infty$ to $+\omega_\infty$. Such that the poles of $\Gamma(b_n-\beta_n s)$, h = 1……, m; lie to the right of L and the poles $\Gamma(1-a_l+\alpha_l s)$, l = 1,….,n lie to the left of L . The integral (1.4.1) converges, if |arg =| < ½ B π (B>0), A ≤ 0, where

$$A=\sum_{j=1}^{pi}a_{ji}-\sum_{j=1}^{qi}\beta_{ji} \quad (7)$$

And

$$B=\sum_{j=1}^{n}\alpha_j-\sum_{j=n+1}^{pi}\alpha_{ji}+\sum_{j=1}^{m}\beta_j-\sum_{j=m+1}^{qi}\beta_{ji} \quad (8)$$

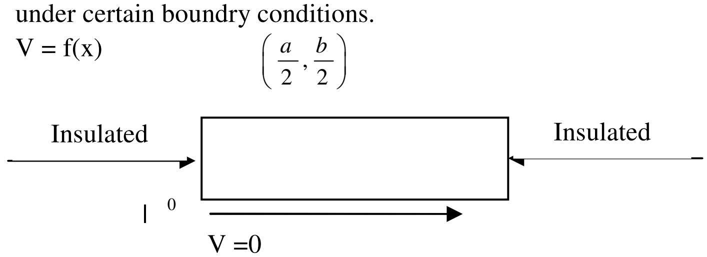
$\forall\ i \in (1,\ldots\ldots,r)$

And the second class of multivariable polynomials given by Srivastava[2] is defined as follows:

$$S^{m_1,\ldots,m_r}_{n_1,\ldots,n_r}(x_1,\ldots,x_r)=\sum_{K_1=0}^{[n_1/m_1]}\cdots\sum_{K_r=0}^{[n_r/m_r]}\frac{(-v_1)_{m_1k_1}}{k_1!}\cdots\frac{(-v_r)_{m_rk_r}}{k_r!}\,A[v_1,k_1;\ldots;v_r,k_r]\ x_1^{k_1}\ldots x_r^{k_r}$$

Boundary Value Problem in A Rectangular Plate

In this section we consider a problem in a rectangular plate under certain boundry conditions.



$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=a,\quad 0<x<\frac{a}{2},\quad 0<y<\frac{b}{2} \quad (10)$$

$$\frac{\partial u}{\partial x}\Big|_{x=0} = \frac{\partial u}{\partial x}\Big|_{x=\frac{a}{2}} = 0,\ 0 < y < \frac{b}{2}, \tag{11}$$

$$V(x,0) = 0,\quad 0 < x < \frac{a}{2}$$

$$V\left(x,\frac{b}{2}\right) = f(x) = \left[\cos\frac{\pi x}{a}\right] S^{m_1,\ldots\ldots,m_r}_{n_1,\ldots\ldots,n_r}$$

$$\left[y_1\left(\cos\frac{\pi x}{a}\right)^{2\rho}\right] \mathrm{I}^{m,n}_{p_i,q_i,r}\left[z\left(\cos\frac{\pi x}{a}\right)^{2\sigma}\Bigg|\begin{matrix}A*\\B*\end{matrix}\right]dx \tag{12}$$

Where, $0 < x < \frac{a}{2}$ provided that $\mathrm{Re}(\eta) > -1$, $\sigma > 0$

Main Integral

For the proof of main integral we use the following formula due to kumar[3] as,

$$\int_0^{a/2}\left(\cos\frac{\pi x}{a}\right)^{\eta}\cos\frac{2m\pi x}{a}dx = \frac{a\Gamma(\eta+1)}{2^{\eta+1}\left(\frac{\eta}{2}+m+1\right)\left(\frac{\eta}{2}-m+1\right)} \tag{13}$$

$$\int_0^{a/2}\left(\cos\frac{\pi x}{a}\right)^{\eta}\cos\frac{2m\pi x}{a}\ S^{m_1,\ldots,m_r}_{n_1,\ldots,n_r}\left[y\left(\cos\frac{\pi x}{a}\right)^{2\rho}\right]$$

$$\mathrm{I}^{m,n}_{p_i,q_i:r}\left[z\left(\cos\frac{\pi x}{z}\right)^{2\sigma}\right]dx$$

$$= \frac{a}{2^{\eta+1}}\sum_{k_1=0}^{[n_1/m_1]}\cdots\sum_{k_r=0}^{[n_r/m_r]}\frac{(-v)_{m_1k_1}}{k_1!}\cdots\frac{(-v)_{m_rk_r}}{k_r!} \tag{14}$$

$$A[v_1,k_1;\ldots v_r,k]\ \ \mathrm{I}(\theta)\left(\frac{y}{4\rho}\right)^{k_1}\cdots\left(\frac{y}{4\rho}\right)^{k_r}\ \cdots$$

Where,

$$I(\theta) = I^{m,n+1}_{p_i+1,q_i+2r}\left[\frac{z}{4^{\sigma}}\Bigg|\begin{matrix}(-\eta-2\rho k_1-\ldots-2\rho k_r)[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1},p_i]\\ b_j,\beta_j)_{1,m}],[(b_{ji},\beta_{ji})_{m+1},q_i]\left(\frac{-\eta}{2}-m-\rho k_1-\ldots-\rho k_r;\sigma,1\right)\left(\frac{-\eta}{2}-m-\rho k_1-\ldots-\rho k_r;\sigma,1\right)\end{matrix}\right] \tag{15}$$

Provided

$$\mathrm{Re}\left(\eta+\sigma\frac{b_j}{\beta_j}\right) > -1,\quad |arg\ z| \le \frac{1}{2}\pi B\,(B>0), A \le 0, where$$

$$A = \sum_{j=1}^{p_i}\alpha_{ji} - \sum_{j=1}^{q_i}\beta_{ji}$$

$$and\ B = \sum_{j=1}^{n}\alpha_j - \sum_{j=n+1}^{p_i}\alpha_{ji} + \sum_{j=1}^{m}\beta_j - \sum_{j=m+1}^{q_i}\beta_{ji}\ \forall i \in (1,\ldots,r)$$

We shall use the following notation:

$$A^* = [(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1},{}_{p_i}]$$

$$B^* = [(b_j,\beta_j)_{1,m}],[(b_{ji},\beta_{ji})_{m+1},{}_{q_i}]$$

Solution of the problem

Combining (10), (11) and (12) with the help of the method given Zill[4] as:

$$V(x,y) = A_o y + \sum_{p=1}^{\infty}A_p\sinh\frac{2p\pi y}{a}\cos\frac{2p\pi x}{a},\ 0<x<\frac{a}{2},\quad 0<y<\frac{a}{2} \tag{16}$$

For $y = \frac{b}{2}$ we find that

$$V\left(x,\frac{b}{2}\right) = f(x) = \frac{A_0 b}{2} + \sum_{p=1}^{\infty}A_p\sinh\frac{p\pi b}{a} \tag{17}$$

$$\cos\frac{2p\pi x}{a}, 0 < x < \frac{a}{2}$$

Now we use (12) and (17) and interchanging the order of integration which is valid under the given conditions both sides with respect to x from 0 to a/2 we derive:

$$A_0 = \frac{2}{b\sqrt{\pi}}\sum_{k_1=0}^{[n_1/m_1]}\cdots\sum_{k_r=0}^{[n_r/m_r]}(-v_1)_{m_1k_1}\ldots(-v_r)_{m_rk_r} \tag{18}$$

$$A[v_1,k_1;\ldots;n_r,k_r]\quad \mathrm{I}(\theta)\frac{y^{k_1}}{k_1!}\cdots\frac{y^{k_r}}{k_r!}$$

Where

$$I(\theta) = I^{m,n+1}_{p_i+1,q_i+1:r}\left[z\Bigg|\begin{matrix}\left(\frac{-1}{2}-\frac{n}{2}-\rho k_1-\ldots-\rho k_r;\sigma,1\right)[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1},p_i]\\ [(b_j,\beta_j)_{1,m}],[(b_{ji},\beta_{ji})_{m+1},q_i]\left(-\frac{\eta}{2}-\rho k_1-\ldots-\rho k_r;\sigma,1\right)\end{matrix}\right] \tag{19}$$

Where all conditions of (12) , (13) and (15) are satisfied.

Making the use (12) and (17) and then we multiplying by $\cos\frac{2m\pi x}{a}$ both sides and we integrate that result from 0 to a/2 with respect to x we find :

$$A_m = \frac{1}{2^{\eta-1}\sinh\frac{p\pi b}{a}}\sum_{k_1=0}^{[n_1/m_1]}\cdots\sum_{k_r=0}^{[n_r/m_r]}\frac{(-n_1)_{m_1k_1}}{k_1!}\cdots\frac{(-n_r)_{m_rk_r}}{k_r!}$$

$$A[v_1,k_1;\ldots;v_r,k_r]\quad \mathrm{I}(\theta)\left(\frac{y}{4\rho}\right)^{k_1}\cdots\left(\frac{y}{4\rho}\right)^{k_r} \tag{20}$$

Provided that all conditions of (12), (13) and (15) are satisfied.

$$v(x,y) = \frac{2y}{b\sqrt{\pi}}\sum_{k_1=0}^{[n_1/m_1]}\cdots\sum_{k_r=0}^{[n_r/m_r]}\left[\prod_{j=1}^{r}\left((-v_j)_{m_jk_j}\frac{y^{k_j}}{k_j!}\right)\right]$$

$$A[v_1,k_1;..;v_r,k_r]+$$
$$\sum_{m=1}^{\infty}\frac{\sinh\frac{2m\pi y}{a}\cos\frac{2m\pi x}{a}}{2^{\eta-1}\sinh\frac{m\pi b}{a}}\sum_{k_1=0}^{[n_r/m_r]}\cdots\sum_{k_r=0}^{[n_r/m_r]}\left[\prod_{j=1}^{r}\left((-v_j)m_jk_j\left(\frac{y}{4\rho}\right)^{k_j}\frac{1}{k_j!}\right)\right]$$
$$A[v_1,k_1;\ldots;v_r,k_r]I(\theta) \qquad (21)$$

Where,
Provided that all conditions of (12), (13) and (15) are satisfied.

Expansion formula

With the aid of (12) and (21) and then setting y = b/2 we evaluate the expansion formula:

$$\left(\cos\frac{\pi x}{a}\right)^n S_{n_1.....n_r}^{m_1....m_r}\left[y\left(\cos\frac{\pi x}{a}\right)^{2\rho}\right]$$
$$I_{p_i,q_i:r}^{m,n}\left[z\left(\cos\frac{\pi x}{a}\right)^{2\sigma}\middle|\begin{matrix}A*\\B*\end{matrix}\right]$$
$$=\frac{1}{\sqrt{\pi}}\sum_{k_1=0}^{[n_1/m_1]}\cdots\cdots\sum_{k_r=0}^{[n_r/m_r]}\left[\prod_{j=1}^{r}\left((-v_j)_{m_jk_j}\frac{y^{k_j}}{k_j!}\right)\right]$$
$$A[v_1,k,;\ldots;v_r,k_r]\quad \mathrm{I}(\theta) \qquad (22)$$

$$=\sum_{m=1}^{\infty}\frac{\cos\frac{2m\pi x}{a}}{2^{\eta-1}}\sum_{k_1=0}^{[n_1/m_1]}\cdots\sum_{k_r=0}^{[n_r/m_r]}$$
$$\left[\prod_{j=1}^{r}\left|(-n_j)_{m_jk_j}\left(\frac{y}{4^{\rho}}\right)k_j\frac{1}{k_j!}\right|\right]\quad \mathrm{A}[v_1,k_1,....;v_r,k_r]\quad \mathrm{I}(\theta)$$
(23)

Where 0 < x < a/2
Provided the condition stated with (12) (13) and (15) are satiafied.

Conclusion

The I-function is a very general function and has for its particular cases a number of important special functions.

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