



On Model of Weight Gain of Farm Animals

E.L. Pankratov

Nizhny Novgorod State Agrotechnical University, 97 Gagarin Avenue, Nizhny Novgorod, 603950, Russia
elp2004@mail.ru

Available online at: www.isca.in, www.isca.me

Received 28th April 2026, revised 2nd May 2026, accepted 16th June 2026

Abstract

In this paper we introduce a model to estimate increasing of weight of farm animals in depends on several parameters. We introduce an analytical approach for analysis of the introduce model with account of changing of the above parameters in space and time, as well as taking into account the nonlinearity of the considered process. We consider the possibility to accelerate and decelerate of the fattening of farm animals.

Keywords: Fattening of farm animals; model of fattening; analytical approach for analysis.

Introduction

An analysis of the experience of livestock development shows that in the modern economy, its profitability and competitiveness can be achieved through the use of various technologies, feed and additives¹⁻⁶. One of the directions of development of the considered technologies may be associated with using of various stimulating additives. At the same time, it is of interest to forecast the socio-economic effect to take into account the maximum possible number of factors.

In this paper we introduce a model to estimate increasing of weight of farm animals depending on a several parameters. We introduce an analytical approach to analyze the introduced model with account changing of parameters of the model in space (inside the animal's body) and time (during the assimilation of food), as well as taking into account the nonlinearity of the considered. We consider the possibility to accelerate and decelerate of the fattening of farm animals.

Method of solution

In this section we consider a model to estimate and future analysis of spatio- temporal distribution of concentration of feed in an organism with account of possibility to take into account changing of conditions of assimilation of the feed. We determined the required of the spatio- temporal distribution of concentration of feed as solution of the second law of Fick in the following form

$$\frac{\partial C(x,t)}{\partial t} = \frac{\partial}{\partial x} \left[D(x,T) \frac{\partial C(x,t)}{\partial x} \right] - K(x,T) C(x,t), \quad (1)$$

Where: $C(x,t)$ is the spatio-temporal distribution of concentration of feed; $D(x,T)$ is the diffusion of coefficient of feed (the value of which depends on the condition of the tissues in the body); $K(x,T)$ is the parameter of feed digestibility in the organism of a farm animal; T is the temperature of organism. Initial and boundary conditions for concentration $C(x,t)$ could be written as

$$\left. \frac{\partial C(x,t)}{\partial x} \right|_{x=0} = 0; C(L,t)=0; C(x,0)=f_C(x). \quad (2)$$

We calculate solution of equation (1) with conditions (2) by method of averaging of function corrections⁷⁻⁹. First of all we transform equation (1) to the following integral form

$$C(x,t) = C(x,t) + \frac{1}{L^2} \left\{ \int_0^x \int_0^t D(v,\tau) C(v,\tau) dv d\tau - \int_0^x (x-v) C(v,\tau) \frac{\partial D(v,\tau)}{\partial v} dv d\tau + \int_0^x (x-v) f(v) dv - \int_0^x (x-v) K(v,\tau) C(v,\tau) dv d\tau - \int_0^L D(v,\tau) C(v,\tau) dv d\tau - \int_0^x (x-v) C(v,t) dv + \int_0^L (L-v) C(v,t) dv + \int_0^L (L-v) C(v,\tau) \frac{\partial D(v,\tau)}{\partial v} dv d\tau \right\} \quad (3)$$

In the framework of the method of averaging of function corrections we replace the required concentration on it's not yet known average value α_1 in the right side of the equation (3). The replacement gives a possibility to obtain the first-order approximation of the considered feed in the following form

$$C_1(x,t) = \alpha_1 + \frac{1}{L^2} \left\{ \int_0^x (x-v) f(v) dv - \alpha_1 \int_0^x (x-v) K(v,\tau) dv d\tau + \alpha_1 \frac{L^2 - x^2}{2} \right\} \quad (4)$$

Average value α_1 was calculated by the following standard relation⁷⁻⁹

$$\alpha_1 = \frac{1}{L\Theta} \int_0^L \int_0^\Theta C_1(x,t) dx dt. \quad (5)$$

Substitution of the relation (4) into relation (5) gives a possibility to obtain the appropriate relation to determine the average value α_1 in the final form

$$\alpha_1 = \frac{\Theta \int_0^L (L^2 - x^2) f(x) dx}{\int_0^\Theta (\Theta - t) \int_0^L (L^2 - x^2) K(x, t) dx dt - 2 \int_0^\Theta (\Theta - t) \int_0^L x(L - x) K(x, t) dx dt - \frac{2}{3} \Theta L^3} \quad (6)$$

The second-order approximation of the required concentration of feed in the framework of the method of averaging of function corrections was obtained by using standard replacing of the considered concentration in the right side of equation (3) on the following sum: $C(x, t) \rightarrow \alpha_2 + C_1(x, t)^{7-9}$.

The replacement gives a possibility to obtain the following relation to determine the required approximation of the considered concentration

$$C_2(x, t) = \alpha_2 + C_1(x, t) + \frac{1}{L^2} \left\{ \int_0^t \int_0^x D(v, \tau) [\alpha_2 + C_1(v, \tau)] dv d\tau - \int_0^x (x-v) [\alpha_2 + C_1(v, \tau)] \times \frac{\partial D(v, \tau)}{\partial v} dv d\tau - \int_0^x (x-v) K(v, \tau) [\alpha_2 + C_1(v, \tau)] dv d\tau + \int_0^L (L-v) [\alpha_2 + C_1(v, \tau)] dv - \int_0^t \int_0^L D(v, \tau) [\alpha_2 + C_1(v, \tau)] dv d\tau + \int_0^t \int_0^L (L-v) [\alpha_2 + C_1(v, \tau)] \frac{\partial D(v, \tau)}{\partial v} dv d\tau + \int_0^x (x-v) f(v) dv - \int_0^x (x-v) [\alpha_2 + C_1(v, t)] dv \right\}. \quad (7)$$

Average value α_2 of the second-order approximation of the required concentration of the considered feed was determined by using the following standard relation⁷⁻⁹

$$\alpha_2 = \frac{1}{L\Theta} \int_0^\Theta \int_0^L [C_2(x, t) - C_1(x, t)] dx dt \quad (8)$$

Substitution of the relation (7) into the relation (8) gives a possibility to obtain the following relation to determine the required average value α_2

$$\alpha_2 = \left[\frac{1}{2} \int_0^\Theta (\Theta - t) \int_0^L (L+x)^2 C_1(x, t) \frac{\partial D(x, t)}{\partial x} dx dt - \int_0^\Theta (\Theta - t) \int_0^L (x-v) D(x, t) C_1(x, t) dx dt + \right. \\ \left. + 2 \int_0^\Theta (\Theta - t) \int_0^L x^2 C_1(x, t) \frac{\partial D(x, t)}{\partial x} dx dt - \frac{1}{2} \int_0^\Theta (\Theta - t) \int_0^L (L+x)^2 K(x, t) C_1(x, t) dx dt - \right. \\ \left. - 2 \int_0^\Theta (\Theta - t) \int_0^L x^2 K(x, t) C_1(x, t) dx dt - \frac{1}{2} \int_0^\Theta (\Theta - t) \int_0^L (L^2 - x^2) C_1(x, t) \frac{\partial D(x, t)}{\partial x} dx dt - \right. \\ \left. - L \int_0^\Theta (\Theta - t) \int_0^L (L-x) C_1(x, t) dx dt - \int_0^\Theta (\Theta - t) \int_0^L x(L-x) f(x) dx dt + L \int_0^\Theta (\Theta - t) \int_0^L (L-x) D(x, t) C_1(x, t) dx dt + \right. \\ \left. + \frac{1}{2} \int_0^\Theta (\Theta - t) \int_0^L (L^2 + x^2) D(x, t) C_1(x, t) dx dt + 2 \int_0^\Theta (\Theta - t) \int_0^L x^2 C_1(x, t) D(x, t) dx dt \right] \times \\ \times \left[\int_0^\Theta (\Theta - t) \int_0^L (x-v) D(x, t) dx dt - \frac{1}{2} \int_0^\Theta (\Theta - t) \int_0^L (L+x)^2 \frac{\partial D(x, t)}{\partial x} dx dt + \frac{1}{2} \int_0^\Theta (\Theta - t) \times \right. \\ \left. \times \int_0^L (L+x)^2 K(x, t) dx dt + 2 \int_0^\Theta (\Theta - t) \int_0^L x^2 K(x, t) dx dt + 2 \int_0^\Theta (\Theta - t) \int_0^L x^2 \frac{\partial D(x, t)}{\partial x} dx dt \right. \\ \left. + \Theta^2 L^3 / 4 \right]^{-1} \quad (9)$$

The second term in the equation (1) describes mass of digested feed. To determine value of the mass we transform the equation (1) to the following integro-differential form

$$\int_0^x C(v, t) dv = \int_0^t D(x, T) \frac{\partial C(x, \tau)}{\partial x} d\tau - \int_0^t \int_0^x K(v, T) C(v, \tau) dv d\tau + \int_0^x f_C(v) dv \quad (10)$$

The limit passage $x \rightarrow L$ gives a possibility to obtain relation gives a possibility to obtain relation to determine value of different masses of feed: initial mass of feed, mass of digested feed and mass of not digested feed. Mass of not digested feed could be written as

$$M(t) = \int_0^L f_C(x) dx - \int_0^t \int_0^L K(x, T) C(x, \tau) dx d\tau \quad (11)$$

Substitution of relation (9) into relation (11) gives a possibility to obtain relation to determine of mass of not digested feed in the following final form

$$M_2(t) = \alpha_2 \int_0^t \int_0^L K(x, T) dx d\tau + \int_0^t \int_0^L K(x, T) C_1(x, \tau) dx d\tau + \frac{1}{L^2} \int_0^t \int_0^L (L-x) K(x, T) D(x, \tau) \times \\ \times [\alpha_2 + C_1(x, \tau)] dx (t-\tau) d\tau + \frac{1}{2L^2} \int_0^t \int_0^L K(x, T) (L^2 - x^2) [\alpha_2 + C_1(x, \tau)] \frac{\partial D(x, \tau)}{\partial x} dx \times \\ \times (t-\tau) d\tau - \frac{1}{L} \int_0^t (t-\tau) \int_0^L K(x, T) (L-x) [\alpha_2 + C_1(x, \tau)] \frac{\partial D(x, \tau)}{\partial x} dx d\tau - \frac{1}{2L^2} \int_0^t (t-\tau) \times \\ \times \int_0^L K(x, T) (L^2 - x^2) f(x) dx d\tau + \frac{1}{L} \int_0^t (t-\tau) \int_0^L (L-x) K(x, T) f(x) dx d\tau + \frac{1}{2L^2} \int_0^t (t-\tau) \times \\ \times \int_0^L K^2(x, T) (L^2 - x^2) [\alpha_2 + C_1(x, \tau)] dx d\tau - \int_0^t (t-\tau) \int_0^L (L-x) [\alpha_2 + C_1(x, \tau)] K^2(x, T) dx d\tau \times \\ \times \frac{1}{L} + \frac{1}{L^2} \int_0^t \int_0^L (L-x) [\alpha_2 + C_1(x, \tau)] dx \int_0^x K(v, T) dv d\tau - \int_0^t (t-\tau) \int_0^L [\alpha_2 + C_1(x, \tau)] D(x, \tau) dx \times \\ \times \frac{1}{L^2} \int_0^L K(x, T) dx d\tau + \frac{1}{L^2} \int_0^t (t-\tau) \int_0^L (L-x) [\alpha_2 + C_1(x, \tau)] \frac{\partial D(x, \tau)}{\partial x} dx \int_0^L K(x, T) dx d\tau - \\ - \frac{1}{L} \int_0^t \int_0^L (L-x) K(x, T) [\alpha_2 + C_1(x, t)] dx d\tau + \frac{1}{2L^2} \int_0^t \int_0^L K(x, T) [\alpha_2 + C_1(x, t)] dt \\ \times (L^2 - x^2) dx. \quad (12)$$

Mass of digested feed is other mass of feed. Part of the considered mass of feed is spent on the current needs of the considered organism. Another part of this mass of food is spent on increasing the body weight of the considered organism. Spatio-temporal distribution of concentration of the considered feed was analyzed analytically by using the second-order approximation in the framework of method of averaging of function corrections.

The approximation is usually enough good approximation for to make qualitative analysis and to obtain some quantitative results. All obtained results have been checked by comparison with results of numerical simulations.

Discussion

In this paper we analyzed increasing of weight of farm animal in depends on a several parameters.

Figure-1 and 2 show typical dependences of the considered growth of the weight of farm animals on mass of feed m , which was entered into organism of the considered animal. On the Figure-1 increasing of number of curve corresponds to increasing of diffusion coefficient of feed in farm animal organism. On the Figure-2 increasing of number of curve corresponds to increasing of parameter of feed digestibility in farm animal organism.

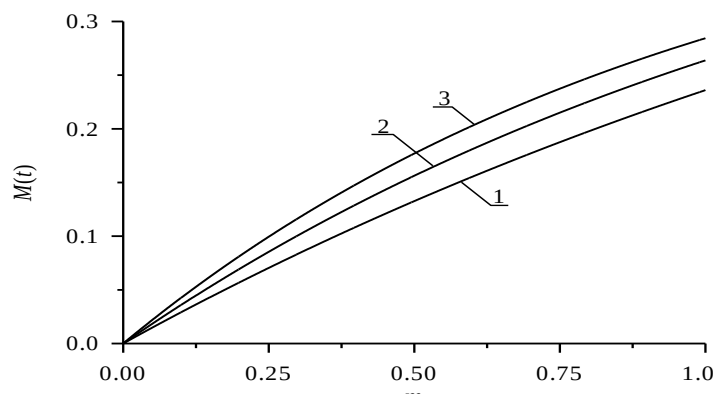


Figure-1: Typical dependences of the considered growth of the weight of farm animals on mass of feed m , which was entered into organism of the considered animal. Increasing of number of curve corresponds to increasing of diffusion coefficient of feed in farm animal organism.

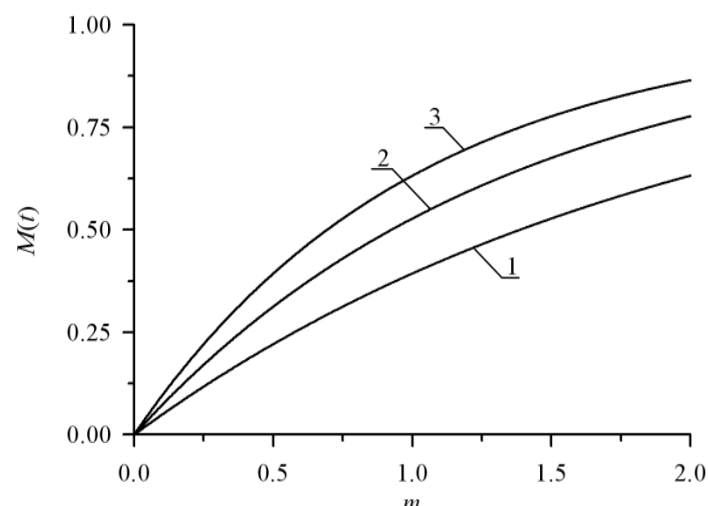


Figure-2: Typical dependences of the considered growth of the weight of farm animals on mass of feed m , which was entered into organism of the considered animal. Increasing of number of curve corresponds to increasing of parameter of feed digestibility in farm animal organism.

Conclusion

We introduced and tested a model to estimate increasing of weight of farm animals in depends on several native parameters. We introduce an analytical approach for analysis of the introduced model with account of changing of the above parameters in space and time, as well as taking into account the nonlinearity of the considered process. We formulate conditions to accelerate and decelerate of the fattening of farm animals.

References

1. Fainsilber, A. M. and Popova, G. V. (1987). Mathematical modeling of live weight gain of farm animals and optimization of their diets. *News of the Timiryazev Agricultural Academy*, (1), 170-173.
2. GG, C. (2002). Sistemno-kineticheskie printsipy i modeli v teorii pitaniya produktivnykh zhivotnykh (Systems-kinetic principles and models in the theory of productive animal nutrition). *Borovsk: VNIIFBiP Publ.*
3. Kutsenko, A. I. (2011). Modeling the dynamics of live weight gain and productivity of cattle. *Tekh Tekhnol APK*, 1, 45-49.
4. V.V. Kalyuga, V.I. Bazykin, E.A. Tikhonov and M.I. Zaitseva (2014). Graphic and mathematical modeling of five-phase stress-free technology for reproduction, growing and fattening of pigs. *Resources and technology*. 11(1). 66-76.
5. B.V. Lukyanov and P.B. Lukyanov (2017). Economic optimization of feeding farm animals: monograph. (Moscow: Rusayns Publishing House).
6. A.V. Trifanov, I.E. Plaksin, V.I. Bazykin and A.M. Sokolov (2022). Mathematical models of feed consumption and live weight gain of young rabbits when raised in a technological module. *Agricultural engineering*, 2, 156-167.
7. Yu. D. Sokolov (1955). *Applied mechanics*, 1(1), 23-35.
8. Pankratov, E. L. and Bulaeva, E. A. (2014). An approach to decrease dimensions of field-effect transistors without p-n-junctions. *International Journal of Modern Physics B*, 28(27), 1450190.
9. Pankratov, E. L. and Bulaeva, E. A. (2015). On optimization of regimes of epitaxy from gas phase. some analytical approaches to model physical processes in reactors for epitaxy from gas phase during growth films. *Reviews in theoretical science*, 3(4), 365-398.