Research Journal of Mathematical and Statistical Sciences ________________________________ISSN 2320-6047
Vol. **14(2),** 1-9, May **(2026)** *Res. J. Mathematical and Statistical Sci.*

Intuitionistic Fuzzy Data Envelopment Analysis Model with Variable Returns to Scale using Possibility Mean

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Available online at: www.isca.in , www.isca.me
Received 15th March 2026, revised 12th April 2026, accepted 2nd May 2026

Abstract

Evaluating efficiency in the banking sector is essential, as banks are key drivers of economic growth and must utilize resources effectively to remain competitive. While Data Envelopment Analysis (DEA) offers a framework for assessing efficiency without rigid functional assumptions, its application is constrained when data is uncertain, imprecise, or subjective. To address these challenges, this study develops a triangular intuitionistic fuzzy DEA model integrated with a weighted possibility mean technique for BCC model, incorporating membership, non-membership, and hesitation degrees simultaneously. This approach more accurately reflects ambiguity and data vagueness, providing a realistic assessment of bank performance. A case study of Indian banks demonstrates that the model delivers reliable, interpretable, and actionable efficiency results. By extending existing intuitionistic fuzzy DEA methodologies, this framework presents a robust and practical tool for performance evaluation under complex and uncertain conditions, facilitating informed decision-making and effective resource allocation.

Keywords: DEA, Efficiency, Defuzzification, Weighted possibility mean, Fuzzy numbers, Triangular intuitionistic fuzzy numbers.

Introduction

The banking industry plays a crucial role in economic growth by enabling the circulation of funds, fostering a culture of saving, and directing investments toward productive uses. In an era marked by intense competition and increasingly strict regulatory requirements, it has become vital for banks to employ more accurate and comprehensive techniques for evaluating their performance.

Charnes et al. introduced Data Envelopment Analysis (DEA) method for benchmarking effectiveness of organizations that use numerous inputs to generate multiple outputs, is commonly used to manage this complexity[1]. Among the various DEA methodologies, the BCC model, introduced by Banker, Charnes, and Cooper, is notable for incorporating variable returns to scale (VRS)[2]. In contrast to models that assume constant returns to scale, the BCC model separates inefficiencies resulting from managerial decisions from those related to the scale of operations, providing a deeper insight into organizational performance. This adaptability has led to its widespread use in areas like banking, healthcare, and education, where the size of operations can have a major impact on efficiency. Measuring inputs and outputs is rarely simple in practice since data is frequently imprecise or contains ambiguity. When information is unclear, the normal BCC-DEA model may be less reliable since it assumes that all facts are precise. Efficiency evaluation gets more complicated when it must take into consideration both quantifiable financial metrics and qualitative elements that are challenging to measure precisely. Such presumptions of accurate data do not hold true in many real-world banking circumstances, and traditional DEA may not accurately reflect performance. This restriction is overcome by introducing fuzzy logic through Fuzzy Data Envelopment Analysis (FDEA), which permits the inclusion of ambiguous or imprecise data in efficiency assessments[3].

Expanding on this, Intuitionistic Fuzzy DEA (IFDEA) offers an even more sophisticated method by specifically incorporating hesitation into the analysis method. Compared to classical DEA or traditional FDEA, IFDEA provides a more thorough representation of uncertainty by capturing membership, non-membership, and hesitation degrees. It draws on Atanassov's intuitionistic fuzzy set theory[4]. This is particularly valuable in the banking industry, where a number of performance metrics such as expenditure, revenue, number of employees, customer satisfaction, credit risk assessment, and managerial efficacy, are arbitrary and challenging to measure. IFDEA enhances detection of inefficiencies, improves benchmarking accuracy, and facilitates improved operational and strategic decision-making by effectively managing ambiguous data.

Building on these advantages, the present study introduces several novel contributions that extend the existing IFDEA and BCCDEA frameworks:

Novelty of the research: i. Proposed BCCDEA model in triangular intuitionistic fuzzy form. ii. Applied weighted possibility mean method of defuzzification for BCC model that captures membership, non-membership, and hesitation degrees for better uncertainty handling. iii. Applied to Indian banking sector, providing practical efficiency insights.

The remainder of this paper is organized as: A thorough analysis of the pertinent literature is provided in Section 2. The basic ideas of intuitionistic fuzzy set theory and defuzzification are presented in Section 3. Relying on earlier research, the building of an output-oriented Intuitionistic Fuzzy DEA model under the presumption of Variable Returns to Scale (VRS) is described in Section 4. Numerical analysis and results are discussed in section 5. The paper is finally concluded in Section 6, which summarizes the key findings and makes recommendations for further research.

Literature Review

Data Envelopment Analysis (DEA), introduced by Charnes et al. provides a non-parametric, linear programming-based method to assess efficiency across DMUs and is widely used to rank and distinguish between efficient and inefficient units[1,5,6]. When inputs or outputs are uncertain or imprecise, fuzzy set theory, proposed by Zadeh in 1965, offers a way to quantify such ambiguity through membership functions[3]. However, highly uncertain or incomplete data may exceed the capabilities of traditional fuzzy sets. Intuitionistic fuzzy sets, developed by Atanassov, provide a more detailed representation by incorporating both membership and non-membership degrees[4,7].

To handle ambiguity and imprecision in performance evaluation, intuitionistic fuzzy data envelopment analysis, or IFDEA, has been used in a variety of fields. In order to assess the efficacy of 16 hospitals in Istanbul, Otay et al. used a hybrid technique that combined IFDEA with the intuitionistic fuzzy Analytic Hierarchy Process (AHP), encompassing various inputs and outputs to capture operational complexity[8]. An optimization framework that simultaneously handles constraint fulfilment and objective function rejection was introduced by Angelov in decision-making research[9]. This framework offers a comprehensive tool for managing uncertainty in complex scenarios. Addressing the computational aspect of intuitionistic fuzzy linear programming, Nagoorgani et al. developed a defuzzification method based on the division of two trapezoidal intuitionistic fuzzy numbers using α–β cuts, accompanied by newly proposed score and accuracy functions for these numbers[10]. Mohanta and Sharanappa proposed a general weighted possibility mean approach for defuzzifying intuitionistic fuzzy numbers with the CCR model[11]. To demonstrate the applicability of their method, they conducted a case study in the Indian agricultural sector, analyzing 28 states. By varying the risk level from 0 to 1 for both inputs and outputs, they calculated the efficiency scores of the states. In a related development, Gao et al. introduced an interactive approach for intuitionistic fuzzy DEA models, allowing decision-makers to incorporate their preferences directly into the evaluation process, and formulated algebraic operation rules for intuitionistic fuzzy numbers[12]. Similarly, Arteaga et al. developed a method to solve intuitionistic fuzzy DEA models by evaluating the performance of 12 decision-making units using two triangular intuitionistic fuzzy inputs and outputs[5]. They transformed the DEA model into a linear programming problem with an intuitionistic fuzzy objective function to facilitate computation.

Moradi and Meybodi proposed a novel DEA approach by introducing a triangular intuitionistic fuzzy series-parallel DEA model that integrates both optimistic and pessimistic perspectives[13]. The model was tested using synthetic data as well as a real-world case study involving U.S. electric power companies. Earlier, Moradi et al. developed a hybrid dynamic network DEA model for assessing sustainable supply chains[14]. Ardakani et al. presented a bounded two-stage DEA model within an intuitionistic fuzzy environment[6]. Arya and Yadav extended fuzzy DEA by incorporating intuitionistic fuzzy sets, allowing performance assessments to reflect hesitation and uncertainty in public health organizations[15]. Wen and Li introduced a fuzzy DEA approach with an associated ranking method, allowing organizations to evaluate efficiency when input and output data are uncertain or imprecise[16]. Babak applied a combination of DEA and intuitionistic fuzzy TOPSIS for performance assessment, illustrating how hesitation and fuzzy information can enhance the discrimination among units[17]. In a similar vein, Puri and Yadav used triangular intuitionistic fuzzy numbers to extend fuzzy DEA to intuitionistic fuzzy DEA to evaluate performance in the Indian banking sector[18]. Sonkariya and Yadav formulated intuitionistic fuzzy revenue-efficient DEA models, where an input-oriented framework was used, and output prices were represented by triangular intuitionistic fuzzy numbers; their methodology was applied to Indian public sector banks[19]. Additionally, Reig-Mullor and Brotons-Martinez evaluated the performance of six Spanish commercial banks for the period 2015–2017[20]. Kaur and Puri proposed a dynamic DEA model with parabolic fuzzy data to assess the efficiency of Indian banks, demonstrating its ability to handle uncertainty, enhance benchmarking, and aid more effective operational and strategic decision-making[21]. Table-1 outlines significant studies that have applied fuzzy and intuitionistic fuzzy approaches in DEA and related efficiency evaluation models. The table covers various defuzzification techniques, fuzzy types, and application areas, including healthcare, banking, agriculture, education, and energy.

Intuitionistic Fuzzy Sets

Definition 3.1 (Intuitionistic Fuzzy Set): Introduced by Atanassov, an intuitionistic fuzzy set can be defined on a finite universal set X. An intuitionistic fuzzy set $\tilde{A}$ on X is of the form $\tilde{A} = \{(y, \mu_{\tilde{A}}(y), \nu_{\tilde{A}}(y)) \mid y \in X\}$ where $\mu_{\tilde{A}}(y): X \to [0,1]$ and $\nu_{\tilde{A}}(y): X \to [0,1]$ define the degree of membership and the degree of non-membership of an element $y \in X$ such that $0 \le \mu_{\tilde{A}}(y) + \nu_{\tilde{A}}(y) \le 1, \forall y \in X$.

Table-1: Previous studies implementing DEA models incorporating intuitionistic fuzzy theory.

Reference	Fuzzy Approach	Data	Year	Defuzzification approach
8	Triangular intuitionistic fuzzy	16 hospitals in Istanbul	2017	Accuracy function approach
18	Triangular intuitionistic fuzzy	Bank sector in India	2015	Expected value approach
16	Trapezoidal fuzzy	Simulated data	2009	Credibility approach
10	Triangular intuitionistic fuzzy	Simulated data for linear programming	2012	Score and accuracy function for division approach
17	Triangular intuitionistic fuzzy TOPSIS	13 university departments	2011	-
11	Triangular intuitionistic fuzzy	Indian agriculture sector	2023	Weighted possibility mean for CCR model
15	Triangular intuitionistic fuzzy	Public health sector	2020	$\alpha - \beta$ cut with ranking method
5	Triangular intuitionistic fuzzy	Simulated data	2021	Accuracy function
13	Triangular intuitionistic fuzzy	Synthetic data and U.S. Electric Power Companies	2025	Expected value approach
6	Trapezoidal intuitionistic fuzzy	Simulated data	2022	Interval efficiency with $\alpha - \beta$ cuts
21	Parabolic fuzzy data	Indian bank sector data	2022	$\alpha -$ cut approach
19	Triangular intuitionistic fuzzy	Public sector banks in India	2023	$\alpha - \beta$ cut approach
21	Triangular intuitionistic fuzzy	Commercial banks in Spain	2021	Possibility mean
Proposed approach	Triangular intuitionistic fuzzy with fixed membership function	Indian bank sector	-----	Weighted possibility mean for BCC model

Definition 3.2 (Triangular Intuitionistic Fuzzy Number (TIFN)): An intuitionistic fuzzy number $\tilde{A} = \{(a_1, a_2, a_3; \mu_{\tilde{A}}(y)), (a_4, a_5, a_6; \nu_{\tilde{A}}(y))\}$ is defined on $\mathbb{R}$ and it is represented by the following functions:

$$\mu_{\tilde{A}}(y) = \begin{cases} \dfrac{(y - a_1)\mu_{\tilde{A}}}{a_2 - a_1} & a_1 \leq y < a_2 \\ \mu_{\tilde{A}} & y = a_2 \\ \dfrac{a_3 - y}{a_3 - a_2} & a_2 < y \leq a_3 \\ 0 & y < a_1 \text{ or } y > a_3 \end{cases},$$

$$\nu_{\tilde{A}}(y) = \begin{cases} \dfrac{a_5 y + \nu_{\tilde{A}}(y - a_4)}{a_5 - a_4} & a_4 \leq y \leq a_5 \\ \nu_{\tilde{A}} & y = a_5 \\ \dfrac{y a_5 + \nu_{\tilde{A}}(a_6 - y)}{a_6 - a_5} & a_5 \leq y \leq a_6 \\ 1 & y < a_4 \text{ or } y > a_6 \end{cases}$$

where $\mu_{\tilde{A}}(y)$ and $\nu_{\tilde{A}}(y)$ represents the maximum degree of membership and the minimum degree of nonmembership respectively such that $0 \leq \mu_{\tilde{A}}(y) \leq 1, 0 \leq \nu_{\tilde{A}}(y) \leq 1$, and $0 \leq \mu_{\tilde{A}}(y) + \nu_{\tilde{A}}(y) \leq 1$. The TIFN parameters satisfy the condition: $a_4 < a_1 < a_2 = a_5 < a_3 < a_6$.

Definition 3.3 (Weighted Possibility Mean): The weighted possibility mean for TIFN $\tilde{A}$ is:

$$\Pi(\tilde{A}) = \lambda \left(\frac{a_1 + 4a_2 + a_3}{6}\right) \mu_{\tilde{N}}^2 + (1 - \lambda) \left(\frac{2(a_4 + a_5 + a_6) - (a_4 - 2a_5 + a_6)\nu_{\tilde{N}} + (a_4 + 4a_5 + a_6)\nu_{\tilde{N}}^2}{6}\right) \quad (1)$$

Where λ∈[0,1] indicates how the decision maker perceives or responds to risk.

Mathematical Model: Indices: i. N - number of DMUs, $j = 1,2, \ldots, N$. ii. P - number of inputs, $p = 1,2, \ldots, P$. iii. Q - number of outputs, $q = 1,2, \ldots, Q$. iv. t - target / focal DMU. v. λ - risk parameter, where $0 \leq \lambda \leq 1$.

Notations: i. x_{pj} $- p^{\text{th}}$ input variable for j^{th} DMU. ii. y_{qj} $- q^{\text{th}}$ output variable for j^{th} DMU. iii. $\tilde{x}_{pj}$ $- p^{\text{th}}$ TIFN input variable for j^{th} DMU. iv. $\tilde{y}_{qj}$ $- q^{\text{th}}$ TIFN output variable for j^{th} DMU. v. 1/δ- efficiency of target DMU. vi. ψ_j - weights for input and output variable for j^{th} DMU.

BCC DEA Model: A traditional output-oriented BCC DEA model by considering variable returns to scale, for estimating the relative efficiency is presented in Equation 2 as,

Max δ

$$\text{s.t.} \sum_{j=1}^{N} x_{pj}\,\psi_j \le x_{pt}, p = 1,2,\dots,P,$$

$$\sum_{j=1}^{N} y_{qj}\,\psi_j \ge \delta y_{qt}, q = 1,2,\dots,Q,$$

$$\sum_{j=1}^{N} \psi_j = 1,$$

$$\psi_j \ge 0, j = 1,2,\dots,N. \tag{2}$$

TIFDEA BCC Model: Given the competitive nature of the market, the inputs and/or outputs of Decision-Making Units (DMUs) are often imprecise or ambiguous. As the nation's central bank and regulatory body, the Reserve Bank of India (RBI) is responsible for monitoring and controlling the nation's banking sector. Approximately every two to three months, the RBI reassesses and adjusts major policy rates. These monetary policy choices affect a number of factors in the banking industry, including as income, capital structure, and workforce size. To account for such uncertainties, these variables are modeled using fuzzy numbers. Specifically, in this study, TIFNs are employed. Accordingly, Equation 2 is reformulated using TIFNs.

Max δ

$$\text{s.t.} \sum_{j=1}^{N} \tilde{X}_{pj}\,\psi_j \le \tilde{X}_{pt}, p = 1,2,\dots,P,$$

$$\textstyle\sum_{j=1}^{N} \tilde{Y}_{qj}\,\psi_j \ge \delta \tilde{Y}_{qt}, q = 1,2,\dots,Q, \tag{3}$$

$$\sum_{j=1}^{N} \psi_j = 1,$$

$$\psi_j \ge 0, j = 1,2,\dots,N,$$

Crisp Intuitionistic Fuzzy BCCDEA Model: By employing the weighted possibility mean defuzzification approach to the TIFN-based BCCDEA model given in Equation 3, the corresponding crisp output-oriented BCCDEA model is obtained, as presented in Equation 4.

Max δ

$$\begin{aligned}
&\text{s.t.}\textstyle\sum_{j=1}^{N}[\,\lambda\big(x_{pj1} + 4x_{pj2} + 2x_{pj3}\big)\Big(\wedge_{\{j=1\}}^{\{N\}} \nu_{pj}\Big)^2 + \\
&(1 - \lambda)\,\{2\big(x_{pj4} + x_{pj5} + x_{pj6}\big) \\
&-\big(x_{pj4} - 2x_{pj5} - x_{pj6}\big)\Big(\vee_{\{j=1\}}^{\{N\}} \nu_{pj}\Big) \\
&\qquad - \big(x_{pj4} + 4x_{pj5} + x_{pj6}\big)\Big(\vee_{\{j=1\}}^{\{N\}} \nu_{pj}\Big)^2\}\,]\,\psi_j \\
&\le [\,\lambda\big(x_{pt1} + 4x_{pt2} + x_{pt3}\big)\big(\mu_{pt}\big)^2 + (1 - \lambda)\{2(x_{pt4} \\
&\qquad + x_{pt5} + x_{pt6}) \\
&-\big(x_{pt4} - 2x_{pt5} - x_{pt6}\big)\big(\nu_{pt}\big) \\
&\qquad - \big(x_{pt4} + 4x_{pt5} + x_{pt6}\big)\big(\nu_{pt}\big)^2\}\,]
\end{aligned}$$

$$\begin{aligned}
&\sum_{j=1}^{N}[\,\lambda\big(y_{qj1} + 4y_{qj2} + y_{qj3}\big)\Big(\wedge_{\{j=1\}}^{\{N\}} \mu_{qj}\Big)^2 + (1 \\
&\qquad - \lambda)\{2(y_{qj4} + y_{qj5} + y_{qj6}) \\
&-(y_{qj4} - 2y_{qj5} + y_{qj6})(\vee_{\{j=1\}}^{\{N\}} \nu_{qj}) \\
&\qquad - \big(y_{qj4} + 4y_{qj5} + y_{qj6}\big)\Big(\vee_{\{j=1\}}^{\{N\}} \nu_{qj}\Big)^2\}\,]\,\psi_j \\
&\ge \delta[\lambda\big(y_{qt1} + 4y_{qt2} + y_{qt3}\big)\big(\mu_{qt}\big)^2 + (1 - \lambda)\{2(y_{qt4} \\
&\qquad + y_{qt5} + y_{qt6}) \\
&-(y_{qt4} - 2y_{qt5} + y_{qt6})(\nu_{qt}) - \big(y_{qt4} + 4y_{qt5} + \\
&y_{qt6})\big(\nu_{qt}\big)^2\}\,]
\end{aligned}$$

$$\sum_{j=1}^{N} \psi_j = 1,$$

$$\psi_j \ge 0,\; j = 1,2,\dots,N, p = 1,2,\dots,P,\; q = 1,2,\dots,Q \tag{4}$$

Data Description and Numerical Analysis

This study considers a sample of ten Indian banks for performance evaluation using a multi-input and multi-output framework. The dataset comprises two input variables (number of employees (X1), expenditure (X2) (Cr. INR), which represent the resources utilized by each bank, and two output variables (revenue (Y1) (Cr. INR) and gross NPA (Y2)), which reflect the corresponding service and financial outcomes generated.

The selected banks include CSB Bank, City Union Bank, DCB Bank, J&K Bank, Karnataka Bank, Karur Vysya Bank, South Indian Bank, Tamilnad Mercantile Bank, Bank of Maharashtra, and Punjab and Sind Bank. The input and output data of these banks shown in Table-2. The variation in input values across these institutions indicates differences in scale of operations and resource consumption. Similarly, the output measures differ significantly, showing variations in performance levels among the banks, with some banks generating comparatively higher outputs relative to their inputs. The descriptive statistics of the collected data shown in Table-3. Revenue (Y1) shows the highest value of 23.49 crore INR, while Gross NPA (Y2) has the lowest value of 0.58 crore INR, indicating variation across banks. The average number of employees is 9.06 ± 2.64 (in thousands), expenditure is 2.50 ± 1.12 crore INR, revenue is 9.20 ± 5.44 crore INR, and Gross NPA is 2.76 ± 1.39 crore INR. Slight skewness is observed in some variables, with revenue positively skewed and Gross NPA mildly negatively skewed. Due to data variability and uncertainty, inputs and outputs are converted into Triangular Intuitionistic Fuzzy Numbers (TIFNs) using ± 40% variation to better represent imprecision. The resulting fuzzy dataset is presented in Table-4 and 5.

For analytical purposes, the crisp values are obtained using the weighted possibility mean defuzzification approach. These defuzzified values are then used in the output-oriented BCC DEA model to evaluate the technical efficiency of the banks. The model is solved under a risk-neutral by setting the risk parameter $\delta = 0.5$, which represents a balanced attitude toward uncertainty.

Different banks are identified using suitable indicators, and their efficiency results are summarized in Table-6. The efficiency scores are obtained using the weighted possibility mean approach under a risk-neutral setting. Based on the results, out of the 10 banks considered, 5 banks are found to be fully efficient, while the remaining banks are relatively inefficient.

The ranking of banks based on efficiency scores is as follows: (B1 = B2 = B8 = B9 = B10 > B3 > B4 > B7 > B5 > B6). CSB Bank, City Union Bank, Tamilnad Mercantile Bank, Bank of Maharashtra, and Punjab and Sind Bank are the top performers, each achieving full efficiency (1.0) and ranked 1. These banks demonstrate that they are optimally utilizing their workforce and expenditure to generate high revenue and NPA.

DCB Bank, with an efficiency of 0.897 and ranked 2, along with J&K Bank, with an efficiency of 0.863 and ranked 3, are considered strong performers.

South Indian Bank, with an efficiency of 0.836 and ranked 4, and Karnataka Bank, with an efficiency of 0.786 and ranked 5, are moderate performers.

Karur Vysya Bank, with an efficiency of 0.667 and ranked 6, could improve outputs by 33% through higher revenue generation and better NPA management.

Table-2: Input-Output Data: Banks in India.

Name of the bank	Inputs		Outputs	
	X_1	X_2	Y_1	Y_2
CSB Bank	7.863	1.281	3.512	3.611
City Union Bank	6.019	1.348	5.525	1.854
DCB Bank	11.268	1.538	5.836	1.353
J&K Bank	12.415	3.752	12.038	3.956
Karnataka Bank	8.652	2.454	8.213	2.578
Karur Vysya Bank	7.764	2.639	7.675	1.042
South Indian Bank	9.836	2.980	10.128	3.620
Tamilnad Mercantile Bank	4.505	1.314	4.710	0.575
Bank of Maharashtra	13.499	4.814	23.493	4.334
Punjab and Sind Bank	8.735	2.932	10.915	4.665

Table-3: Descriptive Statistics of Bank Data.

Variables	Mean	SD	Median	Min	Max
Number of employees (X1)	9.0556	2.6423	8.6935	4.505	13.499
Expenditure (X2) (Cr. Rs.)	2.5052	1.1183	2.5465	1.2807	4.8144
Revenue (Y1) (Cr. Rs.)	9.2045	5.4476	7.9441	3.5118	23.4925
Gross NPA (Y2)	2.7589	1.3985	3.0945	0.5751	4.6653

Table-4: Inputs in TIFNs for Banks.

Name of the Bank	Number of Employees						Expenditure					
	a_4	a_1	a_2	a_5	a_3	a_6	a_4	a_1	a_2	a_5	a_3	a_6
CSB Bank	4.88	6.45	8.02	8.02	8.96	9.99	0.79	1.05	1.31	1.31	1.46	1.63
City Union Bank	3.73	4.94	6.14	6.14	6.86	7.64	0.84	1.11	1.38	1.38	1.54	1.71
DCB Bank	6.99	9.24	11.49	11.49	12.85	14.31	0.95	1.26	1.57	1.57	1.75	1.95
J&K Bank	7.70	10.18	12.66	12.66	14.15	15.77	2.33	3.08	3.83	3.83	4.28	4.77
Karnataka Bank	5.36	7.09	8.83	8.83	9.86	10.99	1.52	2.01	2.50	2.50	2.80	3.12
Karur Vysya Bank	4.81	6.37	7.92	7.92	8.85	9.86	1.64	2.16	2.69	2.69	3.01	3.35
South Indian Bank	6.10	8.07	10.03	10.03	11.21	12.49	1.85	2.44	3.04	3.04	3.40	3.78
Tamilnad Mercantile Bank	2.79	3.69	4.60	4.60	5.14	5.72	0.81	1.08	1.34	1.34	1.50	1.67
Bank of Maharashtra	8.37	11.07	13.77	13.77	15.39	17.14	2.98	3.95	4.91	4.91	5.49	6.11
Punjab and Sind Bank	5.42	7.16	8.91	8.91	9.96	11.09	1.82	2.40	2.99	2.99	3.34	3.72

Table-5: Outputs in TIFNs for Banks.

Name of the Bank	Revenue (Y_1) (Cr. Rs.)						Gross NPA (Y_2) (Cr. Rs.)					
	a_4	a_1	a_2	a_5	a_3	a_6	a_4	a_1	a_2	a_5	a_3	a_6
CSB Bank	2.18	2.88	3.58	3.58	4.00	4.46	2.24	2.96	3.68	3.68	4.12	4.59
City Union Bank	3.43	4.53	5.64	5.64	6.30	7.02	1.15	1.52	1.89	1.89	2.11	2.36
DCB Bank	3.62	4.79	5.95	5.95	6.65	7.41	0.84	1.11	1.38	1.38	1.54	1.72
J&K Bank	7.46	9.87	12.28	12.28	13.72	15.29	2.45	3.24	4.04	4.04	4.51	5.02
Karnataka Bank	5.09	6.73	8.38	8.38	9.36	10.43	1.60	2.11	2.63	2.63	2.94	3.27
Karur Vysya Bank	4.76	6.29	7.83	7.83	8.75	9.75	0.65	0.85	1.06	1.06	1.19	1.32
South Indian Bank	6.28	8.31	10.33	10.33	11.55	12.86	2.24	2.97	3.69	3.69	4.13	4.60
Tamilnad Mercantile Bank	2.92	3.86	4.80	4.80	5.37	5.98	0.36	0.47	0.59	0.59	0.66	0.73
Bank of Maharashtra	14.57	19.26	23.96	23.96	26.78	29.84	2.69	3.55	4.42	4.42	4.94	5.50
Punjab and Sind Bank	6.77	8.95	11.13	11.13	12.44	13.86	2.89	3.83	4.76	4.76	5.32	5.92

Table-6: Efficiencies for δ=0.5 using output oriented Intuitionistic Fuzzy BCC model.

Indicator	Name of the bank	Efficiency	Rank	Performance Category
B1	CSB Bank	1	1	Efficient/ Top Performer
B2	City Union Bank	1	1	Efficient/Top Performer
B3	DCB Bank	0.897039	2	Inefficient/ Strong Performer
B4	J&K Bank	0.862791	3	Inefficient/Strong Performer
B5	Karnataka Bank	0.78608	5	Inefficient/Moderate Performer
B6	Karur Vysya Bank	0.666508	6	Inefficient/Needs Improvement
B7	South Indian Bank	0.835925	4	Inefficient/Moderate Performer
B8	Tamilnad Mercantile Bank	1	1	Efficient/ Top Performer
B9	Bank of Maharashtra	1	1	Efficient/Top Performer
B10	Punjab and Sind Bank	1	1	Efficient/Top Performer

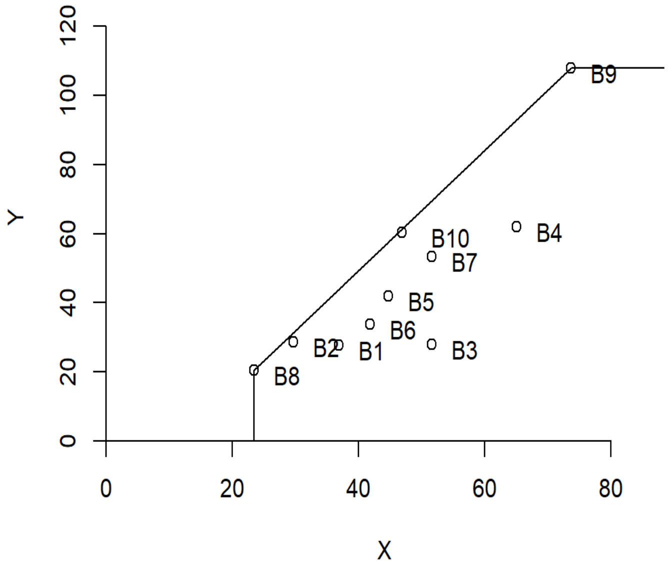

Figure-1: Efficiency Frontier of the Banks.

The Figure-1 presents the input–output distribution of the ten banks along with the efficiency frontier. Overall, the plot clearly differentiates efficient and inefficient banks and supports the DEA results. The line connecting B8, B2, B10, and B9 represents the efficient frontier. Banks on the frontier are fully efficient because, for their input level, no other bank produces higher output.In this plot, B8, B2, B10, and B9 appear to be efficient. Points B1, B3, B4, B5, B6, B7 are below the frontier, indicating inefficiency. These banks could potentially increase output (Y) without increasing input (X) to reach the frontier. The vertical distance from a bank to the frontier line indicates the gap to full efficiency. Banks on the frontier line are benchmarks for others.Inefficient banks can use DEA targets to see how much they can improve output with current inputs.

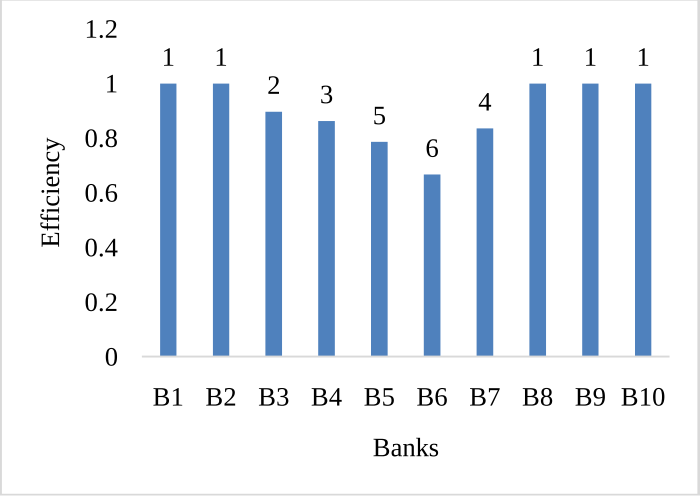

Figure-2: DEA Efficiency Scores.

The bar chart shows the DEA efficiency scores of the ten banks. Banks B1, B2, B8, B9, and B10 are fully efficient with a score of 1, while the remaining banks record scores below 1, indicating inefficiency. Among them, B6 has the lowest efficiency. Overall, the figure highlights clear differences in performance across the banks.

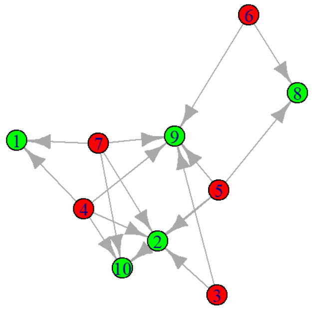

Figure-3: Network Representation of DEA Efficiency and Benchmarking among Banks (reference sets).

The DEA peer network provides a clear visualization of efficiency relationships among banks. Banks (1, 2, 8, 9, 10) identified as efficient, serve as benchmarks, while less efficient banks, are linked to their reference peers through directed arrows. Each arrow represents which efficient bank(s) an underperforming bank should emulate to reach the efficiency frontier. Central efficient banks, referenced by multiple inefficient banks, emerge as key role models and set standards for operational best practices. The network highlights both direct benchmarking paths and interdependencies among banks, helping to identify which banks need minor operational adjustments versus significant improvements.

The output-oriented DEA analysis highlights the potential for banks to enhance their performance using the resources they currently have. Table 7 shows the targets for each bank to become efficient. Banks like CSB Bank, City Union Bank, Tamilnad Mercantile Bank, Bank of Maharashtra, and Punjab and Sind Bank are already performing at their optimal level, as their actual revenue and gross NPA closely align with their DEA target values. In contrast, DCB Bank, J&K Bank, Karnataka Bank, Karur Vysya Bank, and South Indian Bank show noticeable gaps between actual and target revenues, indicating that they could increase income by several crores to reach the efficiency frontier. Likewise, the target adjustments for gross NPAs provide guidance on managing undesirable outputs effectively while scaling up revenue. Overall, these DEA targets serve as benchmarks, helping banks identify where to focus efforts to improve revenue and maintain or reduce NPA levels without additional resources.

Table-7: Targets for Outputs.

Name of the bank	Revenue Y1	target Revenue Y1	Gross NPA Y2	target Gross NPA Y2
CSB Bank	13.60488	13.6049	14.0451	14.0451
City Union Bank	21.40276	21.4028	7.213424	7.2134
DCB Bank	22.60983	25.20492	5.264773	7.740236
J&K Bank	46.63476	54.05112	15.38892	17.83619
Karnataka Bank	31.81654	40.47487	10.02962	12.759
Karur Vysya Bank	29.73495	44.61309	4.051806	7.535059
South Indian Bank	39.23729	46.93876	14.08252	16.8466
Tamilnad Mercantile Bank	18.24719	18.2472	2.236888	2.2369
Bank of Maharashtra	91.0105	91.0105	16.85854	16.8585
Punjab and Sind Bank	42.2866	42.2866	18.14744	18.1474

Conclusion

This study assesses the efficiency of selected Indian banks using a multi-input, multi-output DEA framework, enhanced with Triangular Intuitionistic Fuzzy Numbers (TIFN) and the weighted mean possibility approach. This hybrid methodology effectively addresses the uncertainty, vagueness, and imprecision inherent in banking data, providing a more nuanced and reliable measure of efficiency than traditional DEA models. The findings reveal substantial variations in performance: CSB Bank, City Union Bank, Tamilnad Mercantile Bank, Bank of Maharashtra, and Punjab and Sind Bank operate at full efficiency, while DCB Bank, J&K Bank, Karnataka Bank, Karur Vysya Bank, and South Indian Bank demonstrate opportunities for improvement in revenue generation and NPA management.

The incorporation of weighted mean possibility with TIFN allows the model to account for both membership and non-membership values, reflecting the degree of certainty and hesitation in the data, which leads to more robust efficiency scores. Efficient banks serve as benchmarks, offering guidance for less efficient banks to improve operational strategies, optimize resource utilization, and enhance output performance. Overall, the study demonstrates that fuzzy hybrid DEA approaches provide a practical and effective tool for bank managers and policymakers to identify best practices, address inefficiencies, and strengthen competitiveness in a dynamic and uncertain financial environment.

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