Research Journal of Mathematical and Statistical Sciences ______________________________ ISSN 2320-6047
Vol. **13(3),** 20-25, September **(2025)** *Res. J. Mathematical and Statistical Sci.*

On the bias reduction in the ratio method of estimation using coefficient of variation of the auxiliary variable

Archana Panigrahi[1], Amiya Ojha[1] and L.N. Sahoo[2*]
[1]Department of Statistics, Ravenshaw University, Cuttack 753003, India
[2]Department of Statistics, Utkal University, Bhubaneswar 751004, India
lnsahoostatuu@rediffmail.com

Available online at: www.isca.in , www.isca.me
Received 25th May 2025, revised 16th July 2025, accepted 30th August 2025

Abstract

In this paper, we focus attention on the construction of two bias reduced ratio estimators guided by a feasible and easily acceptable assumption that the coefficient of variation of the auxiliary variable is known prior to survey operation. Treating bias and mean square error as performance measures, superiority of the proposed estimators has been analyzed compared to the classical ratio and Tin's ratio estimators under (i) a finite population set-up, (ii) an infinite population set-up assuming bivariate normal distribution between the considered variables, and (iii) the assumption of a super-population model.

Keywords: Almost unbiased ratio estimator, Auxiliary variable, Bias, Efficiency, Super-population model.

Introduction

Let y and x denote the survey variable and an auxiliary variable taking values y_i and x_i respectively on the ith unit of a finite population of N units. Define $\overline{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$, $\overline{X} = \frac{1}{N}\sum_{i=1}^{N} x_i$ as the population means and $S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \overline{Y}\right)^2$, $S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \overline{X}\right)^2$ as the population variances of y and x, and $S_{yx} = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \overline{Y}\right)\left(x_i - \overline{X}\right)$ as the population covariance between y and x. Assume that a random sample of n units is drawn from the population according to simple random sampling without replacement (SRSWOR) to estimate unknown mean $\overline{Y}$ when $\overline{X}$ is known accurately. Let $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ and $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ be the sample means, $s_y^2 = \frac{1}{n-1}\sum_{i=1}^{y}(y_i - \bar{y})^2$ and $s_x^2 = \frac{1}{n-1}\sum_{i=1}^{y}(x_i - \bar{x})^2$ be the sample variances, and $s_{yx} = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})(x_i - \bar{x})$ be the sample covariance.

When the correlation coefficient between y and x $\left(\rho_{yx}\right)$ has a high positive value, the classical ratio estimator is defined by
$t_R = \frac{\bar{y}}{\bar{x}}\overline{X}$,

is strongly preferred to the direct estimator $\bar{y}$. Holistically, the estimator is biased with an asymptotic expression for the bias *i.e.*, bias to terms of O (n^{-1}) given by

$$B(t_R) = \theta_1\overline{Y}\left(C_x^2 - C_{yx}\right), \qquad (1)$$

where $\theta_1 = \frac{1}{n} - \frac{1}{N}$, $C_x^2 = \frac{S_x^2}{\overline{X}^2}$ and $C_{yx} = \frac{S_{yx}}{\overline{Y}\overline{X}}$.

Although the bias may be small for large samples, in small samples its effect may be a matter of great concern. In survey sampling literature, attention is focused on the estimation of asymptotic bias. This leads to the creation of asymptotic unbiased estimators, *i.e.*, estimators with reduced bias called almost unbiased ratio (AUR) estimators[1-10].

For the simplest case, direct estimation of C_x^2 and C_{yx} by their respective consistent estimators $c_x^2 = \frac{s_x^2}{\bar{x}^2}$ and $c_{yx} = \frac{s_{yx}}{\bar{y}\bar{x}}$, and then correction of the bias given in (1) gives rise to an AUR estimator defined by
$t_T = t_R\left[1 + \theta_1\left(c_{yx} - c_x^2\right)\right]$.

This estimator was considered by Tin[2] who derived the following expressions for its bias and mean square (MSE) to terms of O(n^{-2}):

$$B(t_T) = -\overline{Y}\left[3\theta_1^2 C_{02}(C_{02} - C_{11}) - \left(2\theta_2 - \frac{3\theta_1}{N}\right)\mathcal{H}\right] \qquad (2)$$

$$M(t_T) = \overline{Y}^2[\theta_1(C_{20} + C_{02} - 2C_{11}) + \theta_1^2(2C_{02}^2 - 4C_{02}C_{11} + C_{11}^2 + C_{20}C_{02}) + \frac{2\theta_1}{N}(C_{03} - 2C_{12} + C_{21})], \qquad (3)$$

where $\theta_2 = \frac{1}{n^2} - \frac{1}{N^2}$, $\mathcal{H} = (C_{03} - C_{12})$ and $C_{rs} = \frac{K_{rs}}{\overline{Y}^r\overline{X}^s}$, K_{rs} being the $(r,s)^{th}$ cumulant in y and x[11].

Some authors considered various ratio-type estimators for the estimation of $\overline{Y}$ under the assumption that the coefficient of variation C_x of the auxiliary variable is known[12-21]. But here, concentration has been given on the bias reduction of the classical ratio estimator with a known C_x.

Our discussion in this paper presumes that C_x is a pre assigned constant, *i.e*., is known as *a priori* either from a pilot study or previous census or past data or experience, or even a value with very good approximation to its true value available from the expert guesses of the concerned field. In repetitive surveys this is essentially practicable as CV exhibits both temporal and cross-sectional stability so that good, guessed values are comfortably available. Despite these arguments, we can also point out some real-life situations in favor of known C_x. For example, as the CV of income distribution measures economic inequality or diversity, its computed values or good guessed values from the past studies or experience can be readily available from the official records or published results. This information may be useful to a socio-economic survey that considers income as an auxiliary variable and a variable like house rent paid, or expenditure on food items or income tax paid as study variable. On the other hand, in many agricultural or demographic surveys, it is a common practice to consider geographical area, cultivated area, population size etc. as auxiliary variables so that their CV values can be determined directly from the census data.

Bias reduction and the suggested estimators

Two different cases have been considered here for bias reduction in t_R. In the first case, assuming C_x as a known quantity, bias of t_R given in (1) is estimated by

$$b(t_R) = \theta_1 t_R\left(C_x^2 - c_{yx}\right). \tag{4}$$

Then subtracting this estimated bias from t_R, the following estimator is obtained for $\overline{Y}$:

$$t_{MT} = t_R - b(t_R) = t_R\left[1 + \theta_1\left(c_{yx} - C_x^2\right)\right].$$

This estimator may be viewed as a modified version of Tin's estimator.

For the second case, let us rewrite the bias expression in (1) as

$$B(t_R) = \theta_1\left(\overline{Y}C_x^2 - \frac{S_{yx}}{\overline{X}}\right). \tag{5}$$

Treating $\overline{X}$ and C_x as known quantities, estimation of bias needs estimations of $\overline{Y}$ and S_{yx}. Hence, estimating $\overline{Y}$ by t_R and S_{yx} by its unbiased estimator s_{yx}, an estimate of the bias is obtained as

$$b^*(t_R) = \theta_1\left(t_R C_x^2 - \frac{s_{yx}}{\overline{X}}\right).$$

Adjusting t_R for this estimated bias, another modified version of Tin's estimator for $\overline{Y}$ is defined by

$$t_{MT}^* = (1 - \theta_1 C_x^2)t_R + \theta_1\frac{s_{yx}}{\overline{X}}.$$

To scrutinize unbiasedness property of t_{MT} and t_{MT}^*, we derived their approximate bias expressions to terms of order n^{-2} using Taylor linearization method. Omitting details of derivations, we present below these expressions as

$$B(t_{MT}) = \overline{Y}\left[\theta_1^2 C_{02}(2C_{02} + C_{11}) - 2\theta_1^2 C_{12} - \left(\theta_2 - \frac{3\theta_1}{N}\right)\mathcal{H}\right] \tag{6}$$

$$B(t_{MT}^*) = \overline{Y}\left[2\theta_1^2 C_{02}(C_{02} - C_{11}) - \left(\theta_2 - \frac{3\theta_1}{N}\right)\mathcal{H}\right]. \tag{7}$$

Hence, the proposed estimators t_{MT} and t_{MT}^* are unbiased to $O(n^{-1})$ *i.e*., they are almost unbiased.

Comparison of bias of the estimators

As t_T, t_{MT} and t_{MT}^* are bias reduced estimators over the classical ratio estimator t_R, a desirable task therefore is their bias comparison to gain idea on the reduced bias amount. Here we need expressions to terms of order $O(n^{-2})$ given in (2), (6) and (7) to compare their bias. But these expressions are complicated as they contain some second order cumulants (positive) and third order cumulants (either positive or negative), and the bias amount depends on the signs and magnitudes of the quantities $(C_{02} - C_{11})$, C_{12} and $\mathcal{H}$. Hence, direct dealing with these bias expressions will lead to no conclusive results. However, for simplicity, we carry out bias comparisons under two useful but practical situations that impose certain mild restrictions on the parent population as discussed in the following sub-sections. Further, noting that the bias of an estimator is either positive or negative, either its absolute value or square is taken into consideration for our purpose.

Assumption of bi-variate normality: Let us assume that the sample is drawn from an infinite population in which the joint distribution of y and x is bivariate normal. Then $C_{12} = C_{03} = 0$ and the following bias expressions for t_T, t_{MT} and t_{MT}^* to $O(n^{-2})$ are obtained:

$$B(t_T) = -\overline{Y}\frac{3}{n^2}C_x^4(1 - \emptyset) \tag{8}$$

$$B(t_{MT}) = \overline{Y}\frac{1}{n^2}C_x^4(2 + \emptyset) \tag{9}$$

$$B(t_{MT}^*) = \overline{Y}\frac{2}{n^2}C_x^4(1 - \emptyset) \tag{10}$$

where $\emptyset = \rho_{yx}\frac{C_y}{C_x}$.

From (8) and (9),

$$[B(t_{MT})]^2 - [B(t_T)]^2 = \frac{\overline{Y}}{n^4}C_x^6(5 - 2\emptyset)(4\emptyset - 1). \tag{11}$$

The right-hand side of (11) is positive if both $(5 - 2\emptyset)$ and $(4\emptyset - 1)$ are either positive or negative and is negative if one of the two factors is positive and other is negative. After solving respective simultaneous in equations, we see that t_{MT} is more biased than t_T *i.e.*, $|B(t_{MT})| > |B(t_T)|$ if

$$\frac{1}{4} < \emptyset < \frac{5}{2}, \tag{12}$$

and t_{MT} is less biased than t_T *i.e.*, $|B(t_{MT})| < |B(t_T)|$ if either $\emptyset < \frac{1}{4}$ or $\emptyset > \frac{5}{2}$. (13)

From (8) and (10) we directly have $|B(t^*_{MT})| < |B(t_T)|$ *i.e.*, t^*_{MT} is less biased than t_T. However, from (9) and (10)

$$[B(t_{MT})]^2 - [B(t^*_{MT})]^2 = \frac{\overline{Y}}{n^4} 3C_x^6 \emptyset(4-\emptyset). \quad (14)$$

It means that $|B(t_{MT})| >$ or $< |B(t^*_{MT})|$ according as $\emptyset < 4$ or $\emptyset > 4$. (15)

In the context of the sample survey, $\emptyset > 4$ is not an easily acceptable condition because in most of the situations the coefficient of variation ratio $\frac{C_y}{C_x}$ is not too far from unity[22]. Hence, t_{MT} is likely to be more biased than t^*_{MT}. However, compilation of foregoing derived results under the bivariate normality assumption leads to the following tentative conclusions:

Among the three AUR estimators $\mathrm{t_T}$, $\mathrm{t_{MT}}$ and $\mathrm{t^*_{MT}}$, $\mathrm{t^*_{MT}}$ emerges out as the least biased estimator with $|B(t^*_{MT})| < |B(t_T)| < |B(t_{MT})|$ if $\frac{1}{4} < \emptyset < \frac{5}{2}$, and $|B(t^*_{MT})| < |B(t_{MT})| < |B(t_T)|$ if either $\emptyset < \frac{1}{4}$ *or* $\emptyset > \frac{5}{2}$.

Assumption of a super-population model: Suppose that the population under consideration is a random sample from a super-population under the linear regression model $(\mathcal{M}_t)$

$$y_i = \alpha + \beta x_i + e_i,\ i = 1, 2, \ldots\ldots, N, \quad (16)$$

Where α and β are real constants called parameters or coefficients of the model, e_i's are uncorrelated random errors such that $E(e_i|x_i) = 0$ and $E(e_i^2|x_i) = \vartheta x_i^t$ for all i with $0 < \vartheta < \infty$ and $0 \le t \le 2$. Further, assume that $N \to \infty$ and x_i's are $i.i.d.$ gamma variates with a common parameter $h\ (> 0)$ taken equal to the mean $\overline{X}$.

By the direct substitution under the model, biases of the estimators to $\mathrm{O}(n^{-2})$ are derived as follows:

$$B(t_T) = \frac{\alpha}{n^2 h} \quad (17)$$

$$B(t_{MT}) = -\frac{\beta}{n^2 h} \quad (18)$$

$$B(t^*_{MT}) = 0 \quad (19)$$

It appears that to $\mathrm{O}(n^{-2})$ t^*_{MT} is unbiased whereas t_T is unbiased when $\alpha = 0$ and $|B(t_{MT})| < |B(t_T)|$ when $h < \left|\frac{\alpha}{\beta}\right|$. But, since the variables are positively correlated and β is the slope parameter, it cannot be negative. It is also quite possible that the intercept parameter α is non-negative. Because for $\alpha < 0, x_i, e_i = 0 \Rightarrow y_i < 0$ which is unrealistic in a sample survey situation as the measurements on y and x are positive. Hence under the model $\mathcal{M}_t$, t_{MT} is less or more biased than t_T according as $h \le$ or $\ge \frac{\alpha}{\beta}$.

Efficiency of t_{MT} and t^*_{MT} compared to t_T

To evaluate efficiencies of the proposed AUR estimators t_{MT} and t^*_{MT} compared to Tin's AUR estimator t_T, we derived expressions for their mean square errors (MSEs) to order n^{-2} as given below. Here we also follow the same notations and approximations used inTin[2] and Kendall et al.[11].

$$M(t_{MT}) = \overline{Y}^2[\theta_1(C_{20} + C_{02} - 2C_{11}) + \theta_1^2(6C_{02}^2 - 8C_{02}C_{11} + C_{11}^2 + C_{20}C_{02}) - 2\theta_1^2(C_{12} - C_{21}) - 2\left(\theta_2 - \frac{3\theta_1}{N}\right)(C_{03} - 2C_{12} + C_{21})] \quad (20)$$

$$M(t^*_{MT}) = \overline{Y}^2[\theta_1(C_{20} + C_{02} - 2C_{11}) + \theta_1^2(4C_{02}^2 - 10C_{02}C_{11} + 5C_{11}^2 + C_{20}C_{02}) - 2\theta_1^2(C_{12} - C_{21}) - 2\left(\theta_2 - \frac{3\theta_1}{N}\right)(C_{03} - 2C_{12} + C_{21})]. \quad (21)$$

From (3), (20) and (21), it is clearly understood that t_T, t_{MT} and t^*_{MT} are equally well to $\mathrm{O}(n^{-1})$ under MSE criteria. In view of this, comparing their MSEs considering terms up to $\mathrm{O}(n^{-2})$ the following results are obtained:

$$\frac{M(t_T) - M(t_{MT})}{2\theta_1^2 \overline{Y}^2} = 2C_x^4(\emptyset - 1) + \mathcal{H} \quad (22)$$

$$\frac{M(t_T) - M(t^*_{MT})}{2\theta_1^2 \overline{Y}^2} = C_x^4(\emptyset - 1)(1 - 2\emptyset) + \mathcal{H} \quad (23)$$

It seems that the efficiency difference between either t_T and t_{MT} or t_T and t^*_{MT} is strongly dependent on the range of $\emptyset$ as well as the sign and magnitude of the parametric function, $\mathcal{H}$. By expressing $\mathcal{H}$ as

$$\mathcal{H} = -\frac{1}{\overline{YX}^2} E\left[(y_i - Rx_i)(x_i - \overline{X})^2\right],$$

We see that when the regression line of y on x confirms toa straight line through the origin which of course an ideal condition for the visibility of ratio method of estimation, $\mathcal{H}$ has a very small negative value. On the other hand, its value approaches to zero for a population where y is directly proportional to x *i.e.*, observations are tightly scattered around $y = Rx$. Thus, if the contribution of $\mathcal{H}$ in (22) and (23) is quite negligible, t_{MT} would be most likely to be more efficient than t_T if $\emptyset \ge 1$ whereas t^*_{MT} would be most likely to be more efficient than t_T if both $(\emptyset - 1)$ and $(1 - 2\emptyset)$ are either negative or positive *i.e.*, if either $\emptyset < 1$ and $\emptyset > \frac{1}{2}$ or $\emptyset > 1$ and $\emptyset < \frac{1}{2}$. Since the ratio method of estimation is normally used for $\emptyset \ge \frac{1}{2}, \frac{1}{2} \le \emptyset \le 1$ remains as an ideal condition in favor of t^*_{MT}. However, under the bi-variate normality assumption $\mathcal{H} \to 0$, and t_{MT} and t_{MT}^{*} are straightforwardly superior to t_T when $\emptyset \geq 1$ and $\frac{1}{2} \leq \emptyset \leq 1$ respectively.

$$\frac{M(t_{MT})-M(t_{MT}^{*})}{2\theta_1^2\bar{Y}^2} = C_x^4(1-\emptyset)(1+2\emptyset) \quad (24)$$

This means that $M(t_{MT}) >$ or $< M(t_{MT}^{*})$ according as $\emptyset \leq$ or ≥ 1 *i.e.*, for $\emptyset \leq 1$, t_{MT} is less efficient than t_{MT}^{*}.

After combining the forgoing results under the bi-variate normal assumption, we have the following conclusions:

$M(t_{MT}) < M(t_T) < M(t_{MT}^{*})$ when $\emptyset \geq 1$,
and $M(t_{MT}^{*}) < M(t_T) < M(t_{MT})$ when $\frac{1}{2} \leq \emptyset \leq 1$.

For many real-life situations, it is not so easy to check feasibility of the derived conditions to draw any meaningful conclusion as they depend on the survey situations, unknown population parameters, composition of population units, joint distribution of y and x, and many other constraints. This may mislead our efficiency comparison. However, this comparison clearly indicates that there is enough scope for using t_{MT} and t_{MT}^{*} over t_T.

To make our efficiency comparison more viable and to obtain an idea on the gain in efficiency, we computed numerical values of the percentage relative efficiencies (RE) of t_T, t_{MT} and t_{MT}^{*} compared to t_R under the assumption of bivariate normal distribution with $N \to \infty$. Here the computation is based on the MSE expressions to order n^{-2}.

RE in each case is calculated for some selected values of n, C_y, C_x and ρ_{yx} as shown in table 1. For a given value of C_y, the values of C_x and ρ_{yx} are chosen to satisfy the condition $\emptyset > 0.5$ to make the ratio method of estimation effective. Here the tabular results fully agreed with the analytical results in the sense that t_{MT}^{*} is superior to others if $\emptyset \in [0.5, 1.0]$ otherwise, *i.e.*, if $\emptyset \in [1.0, \infty)$ t_{MT} is superior. It is also noticed from the tabular results that the efficiency gains in t_{MT}^{*} decreases rapidity when $\emptyset$ increases gradually from 1.0 to ∞.

Table-1: Relative efficiencies of the estimators w.r.t. t_R (in %).

n	C_y	C_x	ρ_{yx}	$\emptyset$	t_T	t_{MT}	t_{MT}^{*}
10	0.50	0.45	0.5	0.55	105.76	102.50	105.93
		0.40	0.6	0.75	101.92	102.10	103.99
		0.35	0.7	1.00	100.25	100.25	100.25
		0.30	0.8	1.33	100.30	100.33	100.13
		0.25	0.9	1.80	102.67	104.13	100.81
20	0.75	0.49	0.4	0.61	102.11	101.23	102.21
		0.47	0.5	0.80	102.30	101.84	102.44
		0.45	0.6	1.00	102.00	102.00	102.00
		0.43	0.7	1.22	101.97	102.49	101.66
		0.41	0.8	1.46	102.29	103.40	101.24
30	1.0	1.0	0.6	0.60	109.62	103.00	110.33
		0.9	0.7	0.78	106.18	102.69	107.20
		0.8	0.8	1.00	104.18	104.18	104.18
		0.7	0.9	1.28	104.53	108.68	101.50
		0.6	1.0	1.67	108.29	116.50	100.00

Efficiency comparison under the super-population model $\mathcal{M}_t$: From equations (3), (20) and (21), we directly get the following model-based MSE expressions up to $O(n^{-2})$:

$$M(t_T) = \frac{\alpha^2}{nh}\left(1+\frac{2}{nh}\right)+\frac{\vartheta}{n}\frac{\Gamma(t+h)}{\Gamma(h)}\left(1+\frac{1}{nh}\right) \quad (25)$$

$$M(t_{MT}) = \frac{\alpha^2}{nh}\left(1+\frac{2}{nh}\right)+\frac{\vartheta}{n}\frac{\Gamma(t+h)}{\Gamma(h)}\left(1+\frac{1}{nh}\right) \quad (26)$$

$$M(t_{MT}^*) = \frac{\alpha^2}{nh}\left(1+\frac{2}{nh}-\frac{4\beta}{n\alpha}\right)+\frac{\vartheta}{n}\frac{\Gamma(t+h)}{\Gamma(h)}\left(1+\frac{1}{nh}\right). \quad (27)$$

$M(t_{MT}^*)$ formula involves both parameters of the model where as $M(t_T)$ and $M(t_{MT})$ formulas involve only α. As $M(t_T) = M(t_{MT})$, it seems that the estimators are equally efficient under the model up to terms of order n^{-2}. However, the MSE formulas show that the three estimators perform equally well when the intercept parameter $\alpha = 0$, *i.e.*, the regression line of y on x passes through the origin.

Comparing (27) with (25) or (26) with $\alpha \geq 0,\ \beta > 0$, we report the following model-based results:

$M(t_{MT}^*) < M(t_T) = M(t_{MT})$,

implying that t_{MT}^* is preferable to both t_T and t_{MT} on the ground of MSE.

Conclusion

The present work focalizes on the construction of almost unbiased ratio (AUR) estimators guided by a reasonable and easily achievable situation that the CV of the auxiliary variable is known in advance. Under this motivation, bias estimation and bias adjustment of the classical ratio estimator in the usual way led to the development of two new AUR estimators called as modified versions of Tin's AUR estimator. Our investigation under certain mild restrictions imposed on the parent population has revealed that the new estimators outperform the Tin's estimator in terms of two standard performance measures viz., bias and efficiency. To put it differently, there is a room where the proposed estimators can work more satisfactorily than the Tin's estimator provided an accurate value of said CV is available. Additionally, from the computational point of view the former estimators are also more acceptable. Finally, it may be concluded that the new estimation mechanism formulated here has a greater scope in many real-life scenarios and for further development of a wide variety of estimators.

References

1. Beale, E.M.L. (1962). Some uses of computers in operational research. *Industrielle Organization*, 31, 51-52.

2. Tin, M. (1965). Comparison of some ratio estimators.*Journal of the American Statistical Association*, 60(309), 294-307.

3. Sahoo, L.N. (1983). On a method of bias reduction in ratio estimation. *Journal of Statistical Research*, 17, 1-6.

4. Sahoo, L.N. (1987). On a class of almost unbiased estimators for population ratio. *Statistics*, 18, 119-121.

5. Adebola, F.B &Oshungade, I.O. (2012). On a new method of bias reduction: alternative to approximately unbiased ratio estimators.*Mathematical Theory and Modelling*, 8(2), 79-89. www.iiste.org.

6. Swain, A. K. P. C. (2019). On a generalized class of almost unbiased ratio estimators. *Model Assisted Statistics and Applications*, 14(3), 269-278.https:// doi.org/ 10.3233/MAS-190466.

7. Dubey, Vyas, Rathore, J.B. & Dubey, U. (2019). An almost unbiased ratio estimator of population mean in stratified sampling. *International Journal of Statistics and Applied Mathematics*, 4(6), 61-64. http://www. mathsjournal.com.

8. Swain, A.K.P.C. & Panigrahi, P.K. (2021). A Note on bias reduction using almost unbiased estimators of population mean in finite population sampling. *Asian Journal of Statistical Sciences*, 1(2), 151-158. http://arfjournals. com/ajss.

9. Mehta, Nitu & Mandowara, V.L. (2022). Some efficient methods to remove bias in ratio and product type estimators in ranked set sampling. *International Journal of Bio-resource and Stress Management*, 13(3), 276-282. https://doi.org/10.23910/1.2022.2771a.

10. Singh, P., Singh, R. & Mishra, M. (2024). Almost unbiased optimum ratio-type estimator of population mean in stratified sampling in presence of non-response. *Brazilian Journal of Biometrics*, 42, 68-77.https://doi.org/10.28951/bjb.v42i1.656.

11. Kendall, M. G., Stuart, A. and Ord, J. K. (1983). The Advanced Theory of Statistics (Vol. III). Charles Griffin & Co. Limited, London & Wycombe, pp2441.

12. Sisodia, B. V. S. and Dwivedi, V. K. (1981). A modified ratio estimator using coefficient of variation of auxiliary variable. *Journal of the Indian Society of Agricultural Statistics*, 33, 13-18.

13. Kadilar, C. and Cingi, H. (2004). Ratio estimators in simple random sampling. *Applied Mathematics and Computation*, 151(3), 893–902. https://doi.org/10.1016/ S0096-3003(03)00803-8.

14. Kadılar, C. and Cıngı, H. (2006). An improvement in estimating the population means by using the correlation coefficient. *Hacettepe Journal of Mathematics and Statistics*. 35(1), 103-109.

15. Subramani, J. and Kumarapandiyan, G. (2014). Estimation of population means using known correlation coefficient and median. *Journal of Statistical Theory and Applications*, 13(4), 333-343.

16. Sangngam, P. (2014). Ratio estimators using coefficient of variation and coefficient of correlation. *Modern Applied*

Science, 8(5). Published by Canadian Center of Science and Education.http://dx.doi.org/10.5539/mas.v8n5p70.

17. Soponviwatkul, K and Lawson, N. (2017). New ratio estimators for estimating population mean in simple random sampling using a coefficient of variation, correlation coefficient and a regression coefficient. *Gazi University Journal of Science*, 30(4), 610-621. http://dergipark.gov.tr/gujs.

18. Kumar, S. and Chhaparwal, P. (2019). Ratio- and product-based estimators using known coefficient of variation of the auxiliary variable via modified maximum likelihood.*Life Cycle Reliability and Safety Engineering*, 8, 99-116. https://doi.org/10.1007/s41872-019-00077-0.

19. Onwuka, G.I., Babayemi, W.A., Audu, A. &Adejumobi, A. (2023). On modification of some ratio estimators using parameters of auxiliary variable for the estimation of the population mean. *Oriental Journal of Physical Sciences*, 8(1), 27-35. https://doi.org/10.13005/OJPS08.01.06.

20. Omiya, Maqbool, S., Ganie, A.H., Jeelani, M.I., Pukhta, M.S., Sultan, A. & Shaheen, F. (2024). Advancements in ratio estimators: new linear combinations for estimating population mean. *International Journal of Agriculture Extension and Social Development*, 7(11), 21-26. https://doi.org/10.33545/26180723.2024.v7.i11Sa.1301.

21. Zaman, Q., Haider, B., Shah, Q., Anwar, M., Kousar, S., Ali, F. & Ayub, G. (2024). A novel class of ratio estimators incorporating auxiliary variables in simple random sampling for estimating population mean: an application to real-life data. *Power System Technology*, 48(3), 609-621. https://doi.org/10.52783/pst.851.

22. Sarndal, C.E., Swensson, B. and Wretman, J. (1992). Model Assisted Survey Sampling. Springer-Verlag, pp250. ISBN: 0-387-40620-4.