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Short Communication

Solution of Multi-Objective Linear Programming Problem by using Relaxation Method

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Abstract

In this study, the relaxation method is presented for solving multi-objective linear programming problems. Generally, relaxation method is used to solve simultaneous linear equations in which coefficient matrix is a square matrix along with the condition that the elements of the principal diagonal dominate the other elements of that particular row. An illustrative example is given at the end to demonstrate the method. It is an easy method as compared to other methods to solve multi-objective linear programming problems available in the literature.

Keywords: Multi-objective linear programming problem, relaxation method, residual, square matrix, optimal solution.

Introduction

The general problem of Linear Programming was first developed and applied along with the Simplex method by George B. Dantzig[1] and their associates of the U.S. Department of the Air Force. Linear Programming is one of the most important techniques for optimization (Maximization or Minimization) developed in the field of Operations Research[2,3].

A business or industry concern has to function under various limitations of its available resources for maximum possible returns. Linear Programming is a technique which is very helpful in taking policy decisions in various practical situations. Linear Programming is a technique for resource utilization. It indicates how a manager can use his available resources under the given facilities in the most effective way[4-6].

Relaxation methods[7-11] were developed for solving large sparse linear systems, which arose as finite-difference discretization of differential equations. It is an iterative approach solution to find the solution of the system of linear equations. Relaxation method is highly useful for image processing. It provides preconditions for new methods. It is easily adoptable to computers. It can solve more than 100s of linear equations simultaneously.

The proposed work unfold as follows: The relaxation method is discussed in section 2. The formulation of the problem which is to be solved by the proposed relaxation methods is described in the section 3. An illustration is given in the section 4 to demonstrate the whole procedure. This concludes the proposed work.

The Relaxation Method

In this method, we improve the solution vector successively by reducing the largest residual at a particular iteration. The working procedure of relaxation method is as follows: i. Shift all the terms in one side i.e., either left side or right side of the equation and equate it to zero. ii. Suppose $x^{(i)}$ is an approximation to the solution of the linear system defined by $Ax = b$. iii. Let us suppose, after p[th] iteration the solution vector be $x^{(p)} = (x_1{}^{(p)}, x_2{}^{(p)}, \dots \dots \dots, x_n{}^{(p)})^T$ iv. Also, residuals $R_i{}^{(p)}$can be defined as $R_i{}^{(p)} = b_i - (a_{i1}x_1{}^{(p)} + a_{i2}x_2{}^{(p)} + \cdots \dots + a_{in}x_n{}^{(p)})$. By reducing the largest residual to zero at that iteration, one can improve the solution vector successively. v. If at any iteration, R_i comes out as largest residual in magnitude then one can give an increment to x_i as $dx_i = \frac{R_i}{a_{ii}}$. In other words, we change x_i to $x_i + dx_i$ to relax R_i i.e., to reduce R_i to zero. vi. Repeat the process until the values of all the residuals come out as zero. vii. Now, we have to find the sum of the residuals R_i . The values thus obtained denote the values of decision variables.

The Problem

Consider the multi-objectives linear programming problem of the type:

Max. $Z_1 = cx$
$Z_2 = dx$

Such that $Ax = b$ and $x \geq 0$

where it is assumed that the coefficient matrix A is a square matrix (necessarily) and the linear equations involved in the constraints are such that the principal diagonal elements dominate the other elements.

Numerical example

Find the solution of following multi-objective linear programming problem:

Max. $Z_1 = 2x + y$
Max. $Z_2 = 5x + 2y$

$$6x + 2y = 24$$
$$3x + 5y = 15$$
$$x, y \geq 0$$

Define residuals $R_1 = 24 - 6x - 2y$, $R_2 = 15 - 3x - 5y$

Table-1: Residuals Define.

	R_1	R_2
δx	-6	-3
δy	-2	-5

Solution steps are as follows: First Approximation:
$x = 0, y = 0$
$R_1 = 24 - 0 = 24,\ R_2 = 15 - 0 = 15$
Here, maximum residual is $R_1 = 24$ and $\delta x = \frac{24}{6} = 4$
Second Approximation:
$R_1 = 24 - 6 \times 4 = 0, \quad R_2 = 15 - 3 \times 4 = 3$

Here, maximum residual is $R_2 = 3$ and $\delta y = \frac{3}{5} = 0.6$
Proceeding in the similar fashion, one can approximate the values until the values of all the residuals come out to be near to zero or zero.

Here, the sum of the residual R_1 is 3.7501 which can be approximately written as 3.75; whereas the sum of the residual R_2 is 0.7498 which can be written approximately as 0.75.

Therefore, the value of the decision variables x and y are 3.75 and 0.75 respectively which can be proved by the graphical method or traditional simplex method available in the literature so far.

Hence, optimal solution of the given multi-objective linear programming problem comes out to be as:
x = 3.75, y = 0.75 and Max. Z_1 = 8.25 and Max. Z_2 = 20.25

Conclusion

We have shown in this research paper that one can easily solve anmulti-objective linear programming problem by the proposed relaxation method in short time and less calculations as compared to other methods.

Table-1: Maximum residual for first and second approximation.

Iteration	Operation	δx	δy	R_1	R_2
1	x=y=0	0	0	24	15
2	δx = 4	4	0	0	3
3	δy =0.6	0	0.6	-1.2	0
4	δx =-0.2	-0.2	0	0	0.6
5	δy =0.12	0	0.12	-0.24	0
6	δx =-.04	-.04	0	0	0.12
7	δy = .024	0	.024	-.048	0
8	δx = -.008	-.008	0	0	.024
9	δy = .0048	0	.0048	-.0096	0
10	δx = -.0016	-.0016	0	0	.0048
11	δy = .001	0	.001	-.0019	0
Total				3.7501	0.7498

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