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Method of Principal Component Factors Estimation of Optimal Number of Factors: An Information Criteria Approach

Nwosu F. Dozie[1], Onyeagu I. Sidney[2], Osuji A. George[2] and Ekezie Dan Dan[3]
[1]Department of Mathematics and Statistics Federal Polytechnic Nekede, Owerri Imo State, NIGERIA
[2]Department of Statistics Nnamdi Azikiwe University Awka, Anambra State NIGERIA
[3]Department of Statistics, Imo State University, Owerri, PMB 2000, Owerri NIGERIA

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Abstract

This paper try to x-ray the number of factors (k) to be retained in a factor analysis for different sample sizes using the method of Principal Component Factor estimation when the number of variables are ten (10). Stimulated data were used for ample sizes of 30,50 and 70 and the Akaike Information Criterion (AIC), the Schwarz Information Criterion (SIC) and the Hannan Quinne Information Criterion (HQIC) values were obtained when the number of factors(k) are two, three, and five (2,3 and 5). It was discovered that the optimal number of factors to retain using the method of Principal Component Factors method of estimation is two (2) from all the sample sizes and also for all the methods considered except for the AIC in which the best is when k=3 follows by k=2 and k=5 respectively of sample thirty (30). Hence, conclusion is drawn that for the three sample sizes considered, the optimal number of factors to retain is 2.

Keywords: Factor Analysis, Factor Rotation, Principal Component Factors Method, Akaike, Schwarz, and Hannan Quinne Information Criteria.

Introduction

Factor analysis is a collection of methods used to examine how underlying constructs influences the responses on a number of measured variables. Factor analysis is based on the common factor model illustrated in figure 1. This model proposes that each observed response (measure 1 through measure 5) is influenced partially by underlying common factors (factor 1 and factor 2) and partially by underlying unique factors (E1 through E5). The strength of the link between each factor and each measure varies, such that a given factor influences some measures more than others[1].

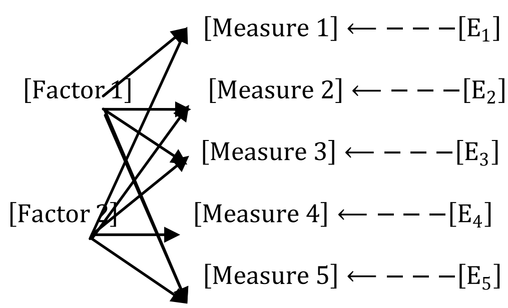

Figure-1
Factor analysis is based on the common factor model

Factor analysis is performed by examining the pattern of correlations (or covariance) between the observed measures. Measures that are highly correlated (either positively or negatively) are likely influenced by the same factors, while those that are not highly correlated are likely influenced by different factors.

In the factor analysis, we represent the variables $y_1, y_2, \dots, y_p$ as a linear combination of a few random variables $f_1, f_2, \dots, f_m$ ($m<p$) called factors. The factors are underlying constructs or latent variables that 'generate' the y's. Like the original variables, the factors vary from individual to individual; but unlike the variables, the factors cannot be measured or observed [2]. The existence of these hypothetical variables is therefore open to question. If the original variables $y_1, y_2, \dots, y_p$ are at least moderately correlated, the basic dimensionality of the system is less than *p*. The goal of factor analysis is to reduce the redundancy among

variables by using a smaller number of factors. Factor analysis can be viewed as an extension of principal components analysis. It related to the principal components analysis in that both seek a simpler structure in a set of variables but they differ in many respects. Both of them have the goal of reducing dimensionality. Because the objectives are similar, many authors discuss principal components analysis as another type of factor analysis.

The Mathematical Model for factor structure: Suppose that the multivariate system consist of a random sample $y_1, y_2, \ldots, y_n$ from a homogeneous population with mean μ and covariance matrix Σ. The factor analysis model expresses each variable as a linear combination of underlying common factors $f_1, f_2, \ldots, f_m$, with an accompanying error term to account for that part of the variable that is unique (not in common with the other variables).
For $y_1, y_2, \ldots, y_p$ in any observation vector y, the model is as follows

$$\begin{aligned} Y_1 - \mu_1 &= \lambda_{11} f_1 + \cdots + \lambda_{1m} f_m + \varepsilon_1 \\ Y_2 - \mu_2 &= \lambda_{21} f_1 + \cdots + \lambda_{2m} f_m + \varepsilon_2 \\ &\vdots \\ Y_p - \mu_p &= \lambda_{p1} f_1 + \cdots + \lambda_{pm} f_m + \varepsilon_p \end{aligned} \tag{1.1.1}$$

ideally, *m* should be substantially smaller than *p*.
f_i = j-th common-factor variates. ε_i = i-th specific factor variates.
The coefficients λ_{ii} are called loadings and serve as weights showing how each y_i individually depends on the *f's*. λ_{ij} indicates the important of j-th factor f_j to the i-th variable y_i and can be used in interpretation of f_j.

It is assumed that for j = 1,2, …, m
$E(f_j) = 0$, $Var(f_j) = 1$ and $Cov(f_j, f_k) = 0, j \neq k$.
The assumptions for ε_i, $i = 1, 2, \ldots, p$, are the same, except that we must allow each ε_i to have a different variance, since it shows the residual part of y_i that is not in common with the other variables. Hence, we assume that
$E(\varepsilon_i) = 0$, $Var(\varepsilon_i) = \psi_i$, and $Cov(\varepsilon_i, \varepsilon_k) = 0, i \neq k$.

Also,
$Cov(\varepsilon_i, f_j) = 0$ for all i and j
We refer to ψ_i as the specific variance. Since $E(y_i - \mu_i) = 0$, we need $E(f_j) = 0$, $j = 1,2,\ldots,m$.
The assumption $Cov(f_j, f_k) = 0$ is made for parsimony in expressing the y's as functions of few factors as possible.
The assumptions $Var(f_j) = 1$, $Var(\varepsilon_i) = \psi_i$, $Cov(f_j, f_k) = 0$, and $Cov(\varepsilon_i, f_j) = 0$ yield a simple expression for variance of y_i.

$$Var(y_i) = \lambda_{i1}^2 + \lambda_{i2}^2 + \cdots + \lambda_{im}^2 + \psi_i \tag{1.1.2}$$

Note that the assumption $Cov(\varepsilon_i, \varepsilon_k) = 0$ implies that the factors account for all the correlations among the y's, that is, all that the y's have in common. Thus the emphasis in factor analysis is on modelling the covariance or correlation among the y's[3].
For matrix version of the model in equation (1.1.1) above;

Let,
$f^I = (f_1, f_2, \ldots, f_m)$, $y^I = (y_1, y_2, \ldots, y_p)$
$\varepsilon^I = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_p)$, $\mu^I = (\mu_1, \mu_2, \ldots, \mu_p)$,
and $\Lambda = \begin{bmatrix} \lambda_{11} & \cdot\cdot & \lambda_{1m} \\ : & : & : \\ \lambda_{p1} & \cdot\cdot & \lambda_{pm} \end{bmatrix}$.

Then, the factor model can be written as

$$(y - \mu) = \Lambda f + \varepsilon. \tag{1.1.3}$$

Hence,
$E(f_j) = 0$, $Var(f_j) = 1, i = 1,2,\ldots,m$ and $Cov(f_j, f_k) = 0, j \neq k$

become $Var(F) = I$.
$E(\varepsilon_i) = 0, i = 1,2,\ldots,p$ becomes $E(\varepsilon) = 0$; $Var(\varepsilon_i) = \psi_i, i = 1,2,\ldots,p$, and $Cov(\varepsilon_i, \varepsilon_k) = 0, i \neq k$ become

$$Cov(\varepsilon) = \psi = \begin{bmatrix} \psi_1 & 0 & \dots & 0 \\ 0 & \psi_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & \psi_p \end{bmatrix}$$

and

$\mathrm{Cov}(\varepsilon_i, f_j) = 0$, for i and j becomes $\mathrm{Cov}(\varepsilon, f) = 0$.

The notation $\mathrm{Cov}(\varepsilon, f)$ indicates a rectangular matrix containing the covariance of the f's with the ε's. $\mathrm{Cov}(\varepsilon, f) =$

$$\begin{bmatrix} \sigma f_1\varepsilon_1 & \sigma f_1\varepsilon_2 & \dots & \sigma f_1\varepsilon_p \\ \sigma f_2\varepsilon_1 & \sigma f_2\varepsilon_2 & \dots & \sigma f_2\varepsilon_p \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ \sigma f_m\varepsilon_1 & \sigma f_m\varepsilon_2 & \dots & \sigma f_m\varepsilon_p \end{bmatrix}$$

Since the emphasis in factor analysis is on modelling the covariance among the y's, we wish to express the $\frac{1}{2}p(p-1)$ covariance (and the p variances) of the variances $y_1, y_2, \dots, y_p$ in term of a simplified structure involving the *pm* loadings λ_{ij} and the p-specific variances ψ_i; that is, we wish to express Σ in terms of Λ and ψ. Since μ does not affect variances and covariances of y, we have

$\Sigma = \mathrm{Cov}(y) = \mathrm{Cov}(\Lambda f + \varepsilon)$

From $\mathrm{Cov}(\varepsilon, f) = 0$; Λf and ε are uncorrelated; therefore, the covariance matrix of their sum is the sum of their covariance matrices:

$\Sigma = \mathrm{Cov}(y) = \mathrm{Cov}(\Lambda f + \varepsilon) = \mathrm{Cov}(\Lambda f) + \mathrm{Cov}(\varepsilon)$

$= \Lambda \mathrm{Cov}(f)\Lambda^I + \psi$

$= \Lambda I \Lambda^I + \psi$

$= \Lambda\Lambda^I + \psi$ (1.1.4)

If Λ has only a few columns, say two or three, then $\Sigma = \Lambda\Lambda^I + \psi$ represents a simplified structure for Σ, in which the covariances are modelled by λ_{ij}'s alone since ψ is diagonal.

Also we can find the covariances of y's with the *f's* in terms of the λ's. the loading themselves represent covariances of the variables with the factors, it then implies that $\mathrm{Cov}(y_i, f_j) = \lambda_{ij}$; i = 1,2,…, p and j = 1,2,…, m. Since λ_{ij} is the (ij)th elements of Λ, we then have that

$\mathrm{Cov}(y, f) = \Lambda$ (1.1.5)

If standardized variables are used, equation (1.2.5) is replaced by $p_p = \Lambda\Lambda^I + \psi$, and the loadings become correlations:

$\mathrm{Cov}(y_i, f_j) = \lambda_{ij}$ (1.1.6)

Partitioning the variance of y_i into a component due to the common factors called the communality, and a component unique to y_i, called the specific variance[4].

Introducing communality, $Cov(Y_i) = \Lambda\Lambda^I + \Psi$ can be written as

$Var(Y_{ij}) = \Lambda_{i1}^2 + \dots + \Lambda_{im}^2 + \Psi_i^2$; $Cov(Y_{ij}, Y_{ik}) = \Lambda_{i1}\Lambda_{k1} + \dots + \Lambda_{jm}\Lambda_{km}$

and $Cov(Y_i, F_i) = \Lambda$ can be written as $Cov(Y_{ik}, X_{ik}) = \Lambda_{jk}$.

Communality is the portion of the variance of the variable contributed by the *m* common factors.

Suppose the ith communality is h_i^2, then

$\sigma_{ii} = \mathrm{Var}(y_i) = (\lambda_{i1}^2 + \lambda_{i2}^2 + \dots + \lambda_{im}^2) + \psi_i$

$= h_i^2 + \psi_i$

$=$ communality $+$ specific variance

where, communality $= h_i^2 = (\lambda_{i1}^2 + \lambda_{i2}^2 + \dots + \lambda_{im}^2)$, and specific variance $= \psi_i$.

The i-th communality is the sum of square of the loading of the ith variable on the m common factor. When the number of factors m > 1, there are multiple factor loadings that generate the same covariance matrix.

The loading in the model (1.1.3) can be multiply by an orthogonal matrix without impairing their ability to reproduce the covariance matrix in $\Sigma = \Lambda\Lambda^{I} + \psi$.
Let β be any mxm orthogonal matrix. If we let $\Lambda^{*} = \Lambda\beta$ and $f^{*} = \beta^{I} f$, then f^{*} has the same statistical properties as f since
$E(f^{*}) = E(\beta^{I} f) = \beta^{I} E(f) = 0$.
$Cov(f^{*}) = Cov(\beta^{I} f) = \beta^{I} Cov(f)\beta = \beta^{I}\beta = I_{mxm}$
Λ and Λ^{I} yield the same covariance because $\Lambda\Lambda^{I} = \Lambda^{*}\Lambda^{*I}$. The factor model $(y - \mu) = \Lambda f + \varepsilon = \Lambda\beta\beta^{I} f + \varepsilon$
$= \Lambda^{*} f^{*} + \varepsilon$, produces the same covariance matrix Σ,
since $\Sigma = \Lambda\Lambda^{I} + \psi = \Lambda\beta\beta^{I}\Lambda + \psi = \Lambda^{*}\Lambda^{*I} + \psi$.

The Principal Components Method

The first technique we consider is commonly called the principal component method. The covariance matrix Σ is represented by the spectral decomposition[5]. If Σ has eigenvalues λ_i with $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p \geq 0$ and corresponding eigenvector e_i, then,
$$\Sigma = \lambda_1 e_1 e_1^{I} + \lambda_2 e_2 e_2^{I} + \cdots + \lambda_p e_p e_p^{I} = \Lambda_{(pxp)}\Lambda_{(pxp)}^{I};$$

where $\Lambda_{(pxp)} = (\sqrt{\lambda_1} e_1, \sqrt{\lambda_2} e_2, \ldots, \sqrt{\lambda_p} e_p)$. Because this matrix product form represents the covariance matrix perfectly, there is no uniqueness. However, this for is not useful for factor analysis because the number of factors should be less than the number of variables. It is desirable to have a model that explain the covariance with small number of common factors and leave the differences between two as uniqueness. Suppose the number of variables is p and the number of factors is m. then we find m such that the last p-m eigenvalues are so small that we can ignore the contribution of $\Sigma = \lambda_{m+1} e_{m+1} e_{m+1}^{I} + \lambda_{m+2} e_{m+2} e_{m+2}^{I} + \cdots + \lambda_p e_p e_p^{I}$ to Σ. We then have,
$$\Sigma = \lambda_1 e_1 e_1^{I} + \lambda_2 e_2 e_2^{I} + \cdots + \lambda_m e_m e_m^{I} + diag(\Psi_1, \Psi_2, \ldots, \Psi_p)$$
$$= \Lambda_{(pxm)}\Lambda_{(pxm)}^{I} + \Psi;$$
where $\Lambda_{(pxp)} = (\sqrt{\lambda_1} e_1, \sqrt{\lambda_2} e_2, \ldots, \sqrt{\lambda_m} e_m)$ and $\Psi_1, \Psi_2, \ldots, \Psi_p$ are very small. This representation is principal components solution.

Because the units of the variables of the original data may be different, standardization is required for a factor model. That is, $Z_{ij} = \frac{(y_{ij} - \bar{y}_j)}{\sqrt{s_{jj}}}$; i = 1,…, m and j = 1, … , p, is required because some variables with large variances influences the determination of factor loading too much. In this case, the sample covariance matrix, S, becomes the sample correlation matrix, R. For the principal components method, the estimate of each factor loading is fixed independent of the number of factors.

If $\widetilde{\Lambda} = \sqrt{\hat{\lambda}_1}\hat{e}_1, \sqrt{\hat{\lambda}_2}\hat{e}_2, \ldots, \sqrt{\hat{\lambda}_q}\hat{e}_q)$ for m = q, and
$\widetilde{\Lambda} = \sqrt{\hat{\lambda}_1}\hat{e}_1, \sqrt{\hat{\lambda}_2}\hat{e}_2, \ldots, \sqrt{\hat{\lambda}_q}\hat{e}_q)$ for m = n, q < n,

then $\sqrt{\hat{\lambda}_1}\hat{e}_1, \sqrt{\hat{\lambda}_2}\hat{e}_2, \ldots, \sqrt{\hat{\lambda}_m}\hat{e}_m)$ are the same for both cases.

One way of determining the number of factors m is to consider the residual matrix $S - (\widetilde{\Lambda}\widetilde{\Lambda}^{I} + \Psi)$. If m's are chosen to ensure that the residual matrices are small enough, the least number of m among all m's that satisfy the small residual matrix condition is appropriate. The sum of squared entries of
$$S - \left(\widetilde{\Lambda}\widetilde{\Lambda}^{I} + \Psi\right) \leq \hat{\lambda}_{m+1}^2 + \hat{\lambda}_{m+2}^2 + \cdots + \hat{\lambda}_p^2.$$

This means that if the right hand side of the inequality is small, then the left side should also be small.

The contribution to the total sample variance from the k-th common factor is $\widetilde{\Lambda}_{1k}^2 + \widetilde{\Lambda}_{2k}^2 + \cdots + \widetilde{\Lambda}_{pk}^2 = \left(\sqrt{\hat{\lambda}_k} e_k\right)^{I}\left(\sqrt{\hat{\lambda}_k} e_k\right) = \hat{\lambda}_k$.

Therefore, the proportion of the total sample variance due to the k-th factors equal $\frac{\hat{\lambda}_k}{S_{11}+S_{22}+\cdots+S_{pp}}$, for a sample covariance matrix, S, and $\frac{\hat{\lambda}_k}{P}$ for a sample correlation matrix R. From this proportion, m is chosen to obtain the appropriate high proportion.

Information Criteria: The necessity of introducing the concept of model evaluation has been recognized as one of the important technical areas and the problem is posed on the choice of the best approximating model among a class of competing models by a suitable model evaluation criterion given a data set. Model evaluation criteria are figures of merit, or performance measures for competing models. Factor analysis can be characterized as multivariate technique for analyzing the internal relationship among a set of variables. Based on the usual factor analysis model, choosing a model with too few parameters can involve making unrealistically simple assumptions and lead to high bias, poor prediction, and missed opportunities for insight. Such models are not flexible enough to describe the sample or the population well. A model with too many parameters can fit the observed data very well, but be too closely tailored to it; such models may generalize poorly. Penalized-likelihood information criteria, such as Akaike's information criterion (AIC), the Schwarz's information criterion (SIC), the Hannan-Quinn information criterion (HQIC) and so on are widely used for model selection[6]. The comparison of the models using information criterion can be viewed as equivalent to a likelihood ratio test and understanding the differences among the criteria may make it easier to compare the results and to use them to make informed decisions.

AKAIKE'S information criterion is probably the most relevant and famous as for the comparison and selection between different models and is constructed on log likelihood
AIC = $-2logmax\ L + 2k$

where L denotes the likelihood function of the factor model and k is the number of the model's parameter/factors. $logmax\ L(k) = -\frac{1}{2}N\left|log|\Sigma_k| + tr\Sigma_k^{-1}S\right|$, where S denotes the sample covariance matrix of Y and $\Sigma_k = \Lambda_k\Lambda_k^{-1} + \psi^2$; Λ_k is the matrix factor of factor loading. The first term can be interpreted as a goodness-of-fit measurement, while the second gives a growing penalty to increasing numbers of parameters according to the parsimony principle. In the choice of model, a minimization rule is used to select the model with the minimum Akaike information criterion value.

Still in the likelihood based procedures, proposed the alternative information criterion given by
$SIC = -logmax\ L + \frac{1}{2}k\log N.$

Unlike the AIC. SIC considers the number of N of observations and is therefore less favourable to factors inclusion[7].

Finally, the third criterion is the Hannan-Quinn information criterion (HQC); it is a criterion for model selection. It is an alternative to Akaike information criterion (AIC) and Bayesian information criterion (BIC). It is given as $HQIC = -logmax\ L + 2k\log\log N$
where k is the number of parameters, N is the number of observations.

Comparison of AIC and SIC after Rotation at different sample sizes and different retained number of factors (k)

For n=30, p =10 and k = 2

Table-1
Rotated Factor Loadings

Varimax Loadings		Equamax Loadings		Quartimax Loadings		Orthomax Loadings	
I	II	I	II	I	II	I	II
-0.1071	-0.1134	-0.1069	-0.1136	-0.1071	-0.1134	-0.1077	-0.1128
-0.0623	0.6989	-0.0632	0.6988	-0.0621	0.6989	-0.0585	0.6992
-0.4361	0.2037	-0.4364	0.2031	-0.4360	0.2038	-0.4350	0.2061
0.5881	0.1466	0.5879	0.1474	0.5882	0.1464	0.5889	0.1434
0.5502	-0.3825	0.5508	-0.3818	0.5501	-0.3827	0.5482	-0.3855
-0.7496	-0.2086	-0.7493	-0.2096	-0.7497	-0.2084	-0.7508	-0.2045
0.2759	0.6490	0.2750	0.6494	0.2761	0.6489	0.2794	0.6475
0.0533	-0.5654	0.0541	-0.5653	0.0531	-0.5654	0.0502	-0.5657
0.0847	0.3039	0.0842	0.3040	0.0848	0.3039	0.0863	0.3035
0.6170	-0.1309	0.6172	-0.1301	0.6170	-0.1311	0.6163	-0.1343

Table-2
Factor Rotation Matrix

Varimax

	Factor I	Factor II
Factor I	0.9998	0.0207
Factor II	-0.0207	0.9998

Equamax

	Factor I	Factor II
Factor I	0.9998	0.0220
Factor II	-0.0220	0.9998

Quartimax

	Factor I	Factor II
Factor I	0.9998	0.0204
Factor II	-0.0204	0.9998

Orthomax

	Factor I	Factor II
Factor I	0.9999	0.0152
Factor II	-0.0152	0.9999

Table-3
Information Criteria

Information Criteria	Values
Log Likelihood	-121.3815
Akaike	246.7630
Schwarz	122.8586
Hannan Quinne	122.1820

For n = 30 , p = 10 and k = 3

Table-4
Rotated Factor Loadings

Varimax Loadings			Equamax Loadings			Quartimax Loadings			Orthomax Loadings		
Factor I	Factor II	Factor III	Factor I	Factor II	Factor III	Factor I	Factor II	Factor III	Factor I	Factor II	Factor III
0.1884	-0.1966	0.6330	0.2074	-0.1990	0.6262	0.1833	-0.1959	0.6347	0.2250	-0.2095	0.6167
-0.1712	0.7448	-0.2466	-0.1780	0.7458	-0.2385	-0.1695	0.7445	-0.2487	-0.1800	0.7504	-0.2223
-0.1441	0.1444	0.6843	-0.1231	0.1420	0.6889	-0.1496	0.1451	0.6830	-0.1010	0.1327	0.6943
0.3710	0.1806	-0.5867	0.3531	0.1824	-0.5970	0.3756	0.1800	-0.5839	0.3360	0.1888	-0.6049
0.4865	-0.3997	-0.2358	0.4788	-0.3991	-0.2520	0.4885	-0.3998	-0.2315	0.4681	-0.3985	-0.2722
-0.7914	-0.1579	0.1001	-0.7881	-0.1577	0.1236	-0.7922	-0.1581	0.0939	-0.7851	-0.1544	0.1453
0.3644	0.6172	0.0802	0.3672	0.6166	0.0713	0.3636	0.6173	0.0824	0.3735	0.6131	0.0689
-0.0903	-0.5341	-0.2733	-0.0990	-0.5331	-0.2724	-0.0880	-0.5344	-0.2735	-0.1110	-0.5284	-0.2769
0.2452	0.2562	0.2904	0.2541	0.2550	0.2838	0.2428	0.2566	0.2921	0.2644	0.2492	0.2795
0.7108	-0.1947	0.0555	0.7120	-0.1954	0.0331	0.7104	-0.1944	0.0614	0.7113	-0.2005	0.0085

Table-5
Factor Rotation Matrix

Varimax

	Factor I	Factor II	Factor III
Factor I	0.8974	-0.0039	-0.4411
Factor II	0.0605	0.9916	0.1143
Factor III	0.4370	-0.1293	0.8901

Equamax

	Factor I	Factor II	Factor III
Factor I	0.8836	-0.0029	-0.4683
Factor II	0.0648	0.9911	0.1160
Factor III	0.4638	-0.1329	0.8759

Quartimax

	Factor I	Factor II	Factor III
Factor I	0.9009	-0.0041	-0.4340
Factor II	0.0593	0.9917	0.1137
Factor III	0.4299	-0.1282	0.8937

Orthomax

	Factor I	Factor II	Factor III
Factor I	0.8688	-0.0019	-0.4951
Factor II	0.0752	0.9889	0.1282
Factor III	0.4893	-0.1486	0.8593

Table-6
Information Criteria

Information Criteria	Values
Log Likelihood	-121.1655
Akaike	246.3310
Schwarz	123.3812
Hannan Quinne	122.1820

For n=30, p =10, and k = 5.

Table-7
Rotated Factor Loadings

Varimax Loadings					**Equamax Loadings**				
1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
0.0130	-0.1314	0.0082	0.0204	0.8806	0.0033	-0.1506	0.1926	0.0135	0.8564
0.0217	-0.0207	0.6813	0.1222	-0.5070	-0.200	-0.1429	0.5348	0.0446	-0.6544
0.0996	-0.8240	0.1610	0.1049	0.0759	0.1045	-0.8403	0.0295	0.1033	0.0537
0.2359	0.6938	0.1385	0.0143	-0.1348	0.2117	0.6576	0.2530	-0.0177	-0.1817
0.2348	0.3869	-0.6962	0.2811	-0.0546	0.2726	0.5192	-0.5415	0.3544	0.0944
-0.8738	-0.1520	-0.1496	-0.0278	-0.0121	-0.8570	-0.1260	-0.2421	-0.0054	0.0312
0.2094	0.3071	0.6165	0.3372	0.3265	0.1514	0.1733	0.7736	0.2496	0.1578
0.0563	0.1216	-0.1389	-0.7811	0.1033	0.0686	0.1454	-0.1736	-0.7623	0.1448
-0.0232	0.0211	-0.1228	0.7523	0.1179	-0.0232	0.0412	0.0059	0.7597	0.1284
0.8521	-0.1092	-0.2026	-0.0773	0.0108	0.8669	-0.0607	-0.1551	-0.0502	0.0587
Quartimax Loadings					Orthomax Loadings				
1	2	3	4	5	1	2	3	4	5
0.0134	-0.0059	-0.1360	0.0200	0.8799	0.0061	-0.1228	0.1794	0.0228	0.8634
0.0222	0.6896	-0.0153	0.1229	-0.4956	0.0043	-0.1060	0.5575	0.0920	-0.6376
0.0970	0.1631	-0.8240	0.1049	0.0744	0.1079	-0.8343	0.0859	0.1085	0.0683
0.2385	0.1374	0.6942	0.0142	-0.1292	0.2195	0.6744	0.1942	0.0024	-0.1840
0.2354	-0.6974	0.3834	0.2805	-0.0644	0.2463	0.4723	-0.6217	0.3093	0.0754
-0.8746	-0.1475	-0.1494	-0.0271	-0.0145	-0.8651	-0.1464	-0.1970	-0.0287	0.0225
0.2120	0.6091	0.3072	0.3373	0.3383	0.1767	0.2392	0.7233	0.3088	0.1776
0.0559	-0.1407	0.1203	-0.7813	0.1011	0.0668	0.1344	-0.1309	-0.7748	0.1341
-0.0225	-0.1253	0.0200	0.7522	0.1163	-0.0301	0.0410	-0.0600	0.7565	0.1318
0.8514	-0.2034	-0.1133	-0.0783	0.0060	0.8607	-0.0717	-0.1799	-0.0580	0.0594

Table-8
Factor Rotation Matrix

Varimax

	Factor 1	**Factor 2**	**Factor 3**	**Factor 4**	**Factor 5**
Factor 1	0.7775	0.6024	-0.1322	0.1139	-0.0473
Factor 2	0.0724	-0.0338	0.8097	0.5609	-0.1529
Factor 3	0.3688	-0.5222	-0.1608	0.3364	0.6725
Factor 4	-0.4536	0.3725	-0.4067	0.6938	0.0937
Factor 5	-0.2201	0.4739	0.3682	-0.2793	0.7165

Equamax

	Factor 1	**Factor 2**	**Factor 3**	**Factor 4**	**Factor 5**
Factor 1	0.7708	0.6234	0.0535	0.1149	-0.0350
Factor 2	0.0133	-0.1891	0.7936	0.4611	-0.3488
Factor 3	0.3787	-0.4930	-0.0264	0.3586	0.6958
Factor 4	-0.4394	0.4389	-0.2382	0.7281	0.1658
Factor 5	-0.2631	0.3741	0.5567	-0.3398	0.6045

Quartimax

	Factor 1	**Factor 2**	**Factor 3**	**Factor 4**	**Factor 5**
Factor 1	0.7796	-0.1351	0.5992	0.1131	-0.0471
Factor 2	0.0738	0.8118	-0.0298	0.5614	-0.1394
Factor 3	0.3676	-0.1704	-0.5277	0.3356	0.6669
Factor 4	-0.4521	-0.4097	0.3719	0.6939	0.0895
Factor 5	-0.2175	0.3547	0.4726	-0.2792	0.7250

Orthomax.

	Factor 1	**Factor 2**	**Factor 3**	**Factor 4**	**Factor 5**
Factor 1	0.7688	0.6257	-0.0369	0.1207	-0.0387
Factor 2	0.0423	-0.1276	0.7758	0.5257	-0.3222
Factor 3	0.3725	-0.4850	-0.0466	0.3539	0.7061
Factor 4	-0.4569	0.4168	-0.3179	0.7018	0.1545
Factor 5	-0.2443	0.4281	0.5418	-0.3021	0.6101

Table-9
Information Criteria

Information Criteria	**Values**
Log Likelihood	-126.6525
Akaike	263.3050
Schwarz	130.3453
Hannan Quinne	128.3467

For n =50 , p =10, and k =2.

Table-10
Rotated Factor Loadings

Varimax Loadings		**Equamax Loadings**		**Quartimax Loadings**		**Orthomax Loadings**	
I	**II**	**I**	**II**	**I**	**II**	**I**	**II**
0.5129	0.5339	0.5036	0.5427	0.5150	0.5318	0.4984	0.5474
-0.7218	0.0492	-0.7226	0.0367	-0.7216	0.0520	-0.7229	0.0298
-0.0903	0.5106	-0.0991	0.5090	-0.0882	0.5110	-0.1039	0.5080
-0.1462	-0.0546	-0.1452	-0.071	-0.1464	-0.0540	-0.1447	-0.0585
-0.1807	-0.3279	-0.1750	-0.3310	-0.1820	-0.3272	-0.1718	-0.3326
0.4581	-0.1704	0.4610	-0.1625	0.4574	-0.1723	0.4625	-0.1581
0.7001	0.0867	0.6985	0.0988	0.7004	0.0839	0.6975	0.1054
0.1276	-0.6377	0.1386	-0.6354	0.1251	-0.6382	0.1446	-0.6341
-0.4663	0.2947	-0.4713	0.2866	-0.4651	0.2966	-0.4740	0.2822
0.0592	0.5613	0.0495	0.5622	0.0615	0.5610	0.0442	0.5627

Table 11
Factor Rotation Matrix

Varimax

	Factor I	**Factor II**
Factor I	0.9816	0.1909
Factor II	-0.1909	0.9816

Equamax

	Factor I	**Factor II**
Factor I	0.9782	0.2079
Factor II	-0.2079	0.9782

Quartimax

	Factor I	**Factor II**
Factor I	0.9824	0.1870
Factor II	-0.1870	0.9824

Orthomax

	Factor I	**Factor II**
Factor I	0.9761	0.2171
Factor II	-0.2171	0.9761

Table-12
Information Criteria

Information Criteria	**Values**
Log Likelihood	-249.2975
Akaike	502.5950
Schwarz	250.9965
Hannan Quinne	250.2182

For n =50 , p =10, and k =3.

Table-13
Rotated Factor Loadings

Varimax Loadings			**Equamax Loadings**			**Quartimax Loadings**			**Orthomax Loadings**		
Factor I	**Factor II**	**Factor III**	**Factor I**	**Factor II**	**Factor III**	**Factor I**	**Factor II**	**Factor III**	**Factor I**	**Factor II**	**Factor III**
0.7793	0.2831	0.1743	0.7862	0.2798	0.1465	0.7770	0.2844	0.1821	0.7888	0.2826	0.1252
-0.7613	0.2713	0.1037	-0.7560	0.2719	0.1359	-0.7627	0.2708	0.0944	-0.7529	0.2653	0.1635
0.1297	0.4460	0.2557	0.1406	0.4432	0.2547	0.1264	0.4469	0.2558	0.1461	0.4406	0.2561
0.2092	-0.2878	0.6528	0.2337	-0.2945	0.6414	0.2021	-0.2856	0.6560	0.2557	-0.3016	0.6296
-0.1496	-0.3524	0.1878	-0.1431	0.3539	0.1901	-0.1514	-0.3520	0.1872	-0.1349	-0.3573	0.1897
0.2671	-0.1720	-0.3931	0.2511	-0.1688	-0.4049	0.2716	-0.1729	-0.3896	0.2391	-0.1618	-0.4149
0.5452	0.0338	-0.4638	0.5268	0.0372	-0.4843	0.5503	0.0330	-0.4577	0.5109	0.0469	-0.5002
0.0621	-0.7155	0.0304	0.0615	-0.7159	0.0211	0.0625	-0.7154	0.0333	0.0662	-0.7157	0.0094
-0.0835	0.1749	0.06855	-0.0564	0.1684	0.6899	-0.0914	0.1768	0.6840	-0.0353	0.1588	0.6936
0.1308	0.5753	-0.0469	0.1303	0.5754	-0.0464	0.1308	0.5753	-0.0472	0.1255	0.5768	-0.0427

Table-14
Factor Rotation Matrix

Varimax

	Factor I	**Factor II**	**Factor III**
Factor I	0.8690	0.0496	-0.4924
Factor II	0.1335	0.9346	0.3298
Factor III	0.4765	-0.3524	0.8055

Equamax

	Factor I	**Factor II**	**Factor III**
Factor I	0.8493	0.0526	-0.5253
Factor II	0.1486	0.9310	0.3333
Factor III	0.5066	-0.3612	0.7829

Quartimax

	Factor I	**Factor II**	**Factor III**
Factor I	0.8744	0.0490	-0.4828
Factor II	0.1289	0.9357	0.3285
Factor III	0.4678	-0.3494	0.8118

Orthomax

	Factor I	**Factor II**	**Factor III**
Factor I	0.8318	0.0648	-0.5513
Factor II	0.1537	0.9274	0.3411
Factor III	0.5334	-0.3684	0.7614

Table-15
Information Criteria

Information Criteria	**Values**
Log Likelihood	-250.6500
Akaike	507.3000
Schwarz	253.1985
Hannan Quinne	252.0311

For n=50, p =10, and k = 5.

Table-16
Rotated Factor Loadings

Varimax Loadings					**Equamax Loadings**				
1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
0.7725	0.3589	-0.0099	0.1997	-0.0544	0.7737	0.3583	-0.0267	0.1899	-0.0690
-0.8141	0.3238	0.0526	0.0628	-0.0731	-0.8130	0.3242	0.0684	0.0725	-0.0616
-0.0067	0.2653	0.1122	0.3080	-0.6098	-0.0105	0.2625	0.1112	0.3079	-0.6111
0.0500	-0.1042	-0.0054	0.8633	-0.0243	0.0598	-0.1035	-0.0095	0.8627	-0.0251
-0.0171	0.0566	0.0231	0.0798	0.8462	-0.0019	0.0612	0.0224	0.0804	0.8460
0.1127	0.1624	-0.8264	0.1100	0.0973	0.0989	0.1618	-0.8292	0.1055	0.0942
0.5227	-0.0854	-0.3227	-0.2938	-0.2353	0.5086	-0.0878	-0.3317	-0.3012	-0.2433
0.0584	-0.8830	0.0700	0.0908	0.0049	0.0604	-0.8828	0.0696	0.0913	0.0086
0.0119	0.2134	0.6254	0.3316	0.2042	0.0319	0.2157	0.6233	0.3335	0.2029
0.2532	0.4000	0.3425	-0.3577	-0.1310	0.2539	0.3932	0.3383	-0.3601	-0.1368
Quartimax Loadings					Orthomax Loadings				
1	2	3	4	5	1	2	3	4	5
0.7721	0.3590	-0.0055	0.2022	-0.0507	0.7696	0.3681	-0.0420	0.1753	0.0909
-0.8143	0.3238	0.0482	0.0604	-0.0760	-0.8159	0.3123	0.0811	0.0873	0.0483
-0.0058	0.2660	0.1124	0.3080	-0.694	-0.0198	0.2564	0.1083	0.3021	0.6168
0.0475	-0.1045	-0.0046	0.8634	-0.0241	0.0762	-0.1039	-0.0122	0.8610	0.0347
-0.0210	0.0554	0.0231	0.797	0.8462	0.0172	0.0694	0.0241	0.0894	-08442
0.1164	0.1624	-0.8257	0.1109	0.0980	0.0861	0.1609	-0.8314	0.1033	-0.0917
0.5264	-0.0848	-0.3200	-0.2919	-0.2332	0.4931	-0.0839	-0.3401	-0.3138	0.2490
0.0578	-0.8830	0.0702	0.0906	0.0039	0.0760	-0.8816	0.0715	0.0893	-0.0145
0.0065	0.2129	0.6257	0.3312	0.2045	0.0500	0.2200	0.6217	0.3364	-0.1947
0.2530	0.4003	0.3438	-0.3570	-0.1294	0.2444	0.4030	0.3329	-0.3649	0.1430

Table-17
Factor Rotation Matrix

Varimax

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5
Factor 1	0.8278	-0.0107	-0.4527	-0.2369	-0.2314
Factor 2	0.1520	0.7843	-0.4417	0.0475	-0.4055
Factor 3	0.4414	-0.2106	0.3021	0.7980	0.1807
Factor 4	-0.2747	-0.2233	-0.2875	0.3837	-0.8030
Factor 5	-0.1460	0.5391	-0.6527	0.3969	0.3234

Equamax.

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5
Factor 1	0.8117	-0.0134	-0.4683	-0.2486	-0.2448
Factor 2	0.1553	0.7826	0.4376	0.0462	-0.4120
Factor 3	0.4598	-0.2087	0.2903	0.7940	0.1743
Factor 4	-0.2891	-0.2273	-0.2824	0.3859	-0.7975
Factor 5	-0.1488	0.5404	-0.6521	0.3958	0.3223

Quartimax.

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5
Factor 1	0.8319	-0.0101	-0.4484	-0.2340	-0.2280
Factor 2	0.1510	0.7848	0.4427	0.0480	-0.4038
Factor 3	0.4366	-0.2112	0.3050	0.7991	0.1823
Factor 4	-0.2710	-0.2223	-0.2888	0.3831	-0.8044
Factor 5	-0.1451	0.5386	-0.6531	0.3970	0.3236

Orthomax

	Factor 1	Factor 2	Factor 3	Factor 4	factor 5
Factor 1	0.7935	-0.0059	-0.4823	-0.2669	0.2579
Factor 2	0.1437	0.7823	0.4310	0.0408	0.4243
Factor 3	0.4856	-0.2003	0.2816	0.7875	-0.1569
Factor 4	-0.3006	-0.2406	-0.2793	0.3820	0.7924
Factor 5	-0.1536	0.5385	-0.6514	0.4013	-0.3178

Table-18
Information Criteria

Information Criteria	Values
Log Likelihood	-268.3175
Akaike	546.6350
Schwarz	272.5649
Hannan Quinne	270.6194

For n=70, p =10 and k = 2

Table-19
Rotated Factor Loadings

Varimax Loadings		**Equamax Loadings**		**Quartimax Loadings**		**Orthomax Loadings**	
I	**II**	**I**	**II**	**I**	**II**	**I**	**II**
0.4699	0.1452	0.4705	0.1433	0.4698	0.1456	0.4434	-0.2128
-0.2739	0.1140	-0.2735	0.1151	-0.2740	0.1138	-0.1235	0.2698
0.1437	-0.0249	0.1436	-0.0254	0.1437	-0.0248	0.0885	-0.1159
-0.0327	-0.3481	-0.0341	-0.3480	-0.0324	-0.3482	-0.2606	-0.2332
-0.4369	0.0296	-0.4368	0.0313	-0.4369	0.0292	-0.3005	0.3186
-0.1192	0.0560	-0.1189	0.0564	-0.1192	0.0559	-0.0494	0.1220
-0.0112	0.3385	-0.0098	0.3385	-0.0114	0.3385	0.2218	0.2559
0.2034	-0.1007	0.2030	-0.1015	0.2035	-0.1005	0.0808	-0.2121
0.0872	0.3954	0.0888	0.3950	0.0869	0.3955	0.3327	0.2308
0.2853	-0.1387	0.2847	-0.1399	0.2854	-0.1385	0.1150	-0.2956

Table-20
Factor Rotation Matrix

Varimax

	Factor I	**Factor II**
Factor I	0.9998	-0.0213
Factor II	0.0213	0.9998

Equamax

	Factor I	**Factor II**
Factor I	0.9997	-0.0243
Factor II	0.0253	0.997

Quartimax

	Factor I	**Factor II**
Factor I	0.9998	-0.0205
Factor II	0.0205	0.0277

Orthomax

	Factor I	**Factor II**
Factor I	0.7191	-0.6950
Factor II	0.6950	0.7191

Table-21
Information Criteria

Information Criteria	**Values**
Log Likelihood	-341.9745
Akaike	687.9690
Schwarz	343.8196
Hannan Quinne	343.0386

For n =70 , p =10, and k =3

Table-22
Rotated Factor Loadings

Varimax Loadings			**Equamax Loadings**			**Quartimax Loadings**			**Orthomax Loadings**		
Factor I	**Factor II**	**Factor III**	**Factor I**	**Factor II**	**Factor III**	**Factor I**	**Factor II**	**Factor III**	**Factor I**	**Factor II**	**Factor III**
0.5157	0.0368	-0.0268	0.5142	0.0452	0.0401	0.5145	0.0353	-0.0464	0.4769	0.1988	0.0326
-0.2715	0.1597	-0.0664	-0.2615	-0.0967	0.1609	-0.2735	0.1596	-0.0579	-0.2436	-0.1108	0.1789
0.0644	0.0974	0.2220	0.0327	0.2322	0.0890	0.0729	0.0995	0.2184	-0.0390	0.2477	0.0070
-0.0462	-0.3815	0.0461	-0.0483	0.0249	-0.3832	-0.0451	-0.3809	0.0517	-0.0100	-0.1118	-0.3704
-0.4095	0.0333	-0.1545	-0.3849	-0.2075	0.0375	-0.4149	0.0327	-0.1395	-0.3194	-0.2850	0.0972
-0.0771	0.0033	-0.1287	-0.0589	-0.1378	0.0079	-0.0819	0.0021	-0.1258	-0.0215	-0.1390	0.0523
-0.0272	0.4278	0.0172	-0.0334	0.0295	0.4267	-0.0257	0.4281	0.0138	-0.0874	0.1518	0.3917
0.1354	-0.0185	0.2108	0.1057	0.2264	-0.0261	0.1432	-0.0167	0.2058	0.0453	0.2268	-0.0981
0.2221	0.2129	-0.3325	0.2632	-0.2908	0.2267	0.2099	0.2090	-0.3427	0.3027	-0.1177	0.3158
0.1512	0.0466	0.3894	0.0964	0.4078	0.0320	0.1658	0.0502	0.3830	-0.0173	0.4068	-0.1041

Table-23
Factor Rotation Matrix

Varimax

	Factor I	**Factor II**	**Factor III**
Factor I	0.9246	0.0005	0.3809
Factor II	0.2148	0.8250	-0.5227
Factor III	-0.3145	0.5651	0.7627

Equamax

	Factor I	**Factor II**	**Factor III**
Factor I	0.8642	0.5031	-0.0102
Factor II	0.2759	-0.4568	0.8457
Factor III	-0.4208	0.7336	0.5336

Quartimax

	Factor I	**Factor II**	**Factor III**
Factor I	0.9382	0.0022	0.3460
Factor II	0.1968	0.8192	-0.5387
Factor III	-0.2846	0.5735	0.7681

Orthomax

	Factor I	**Factor II**	**Factor III**
Factor I	0.6990	0.6968	-0.1609
Factor II	0.2890	-0.0695	0.9548
Factor III	-0.6541	0.7139	0.2499

Table-24
Information Criteria

Information Criteria	**Values**
Log Likelihood	-349.9615
Akaike	705.9230
Schwarz	352.7291
Hannan Quinne	351.5576

For n=70, p =10, and k = 5.

Table-25
Rotated Factor Loadings

Varimax Loadings					**Equamax Loadings**				
1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
0.8480	0.0434	-0.0763	0.0003	-0.1500	0.8484	0.0419	-0.0701	-0.0113	-0.1507
-0.2272	0.4101	-0.3887	0.4412	0.0731	-0.2167	0.4135	-0.3921	0.4391	0.0796
-0.1121	0.1650	0.5811	-0.0283	-0.1516	-0.1171	0.1658	0.5791	-0.0247	-0.1549
-0.1100	-0.7003	-0.0253	0.0903	-0.0700	-0.1101	-0.6992	-0.0266	0.0975	-0.0707
-0.5837	0.1119	-0.3468	0.0205	-0.0295	-0.5802	0.1132	-0.3520	0.0266	-0.0261
-0.0847	-0.0169	-0.0040	-0.0023	0.9488	-0.0834	-0.0189	0.0014	-0.0068	0.9488
-0.0378	0.7481	0.0867	-0.0516	-0.1004	-0.0378	0.7480	0.0855	-0.0558	-0.0995
0.4047	0.0472	0.0599	0.7007	0.2211	0.4144	0.0512	0.0619	0.6934	0.2245
0.2920	0.2426	-0.1367	-0.6976	0.2619	0.2841	0.2359	-0.1300	-0.7056	0.2586
0.0201	0.0286	0.8060	0.1260	0.0466	0.0153	0.0299	0.8059	0.1284	0.0424
Quartimax Loadings					Orthomax Loadings				
1	2	3	4	5	1	2	3	4	5
0.8478	0.0438	-0.0779	0.0029	-0.1499	0.8432	0.0365	-0.0615	-0.0628	-0.1710
-0.2297	0.4093	-0.3878	0.4417	0.0716	-0.1812	0.4237	-0.4035	0.4319	0.0958
-0.1107	0.1648	0.5815	-0.0292	-0.1507	-0.1283	0.1682	0.5750	-0.0080	-0.1607
-0.1100	-0.7006	-0.0250	0.0885	-0.0698	-0.1083	-0.6960	-0.0286	0.1215	-0.0664
-0.5845	0.1116	-0.3455	0.0191	-0.0303	-0.5732	0.1157	-0.3606	0.0517	-0.0060
-0.0850	-0.0164	-0.0053	-0.0013	0.9487	-0.0609	-0.0190	0.0151	-0.0132	0.9504
-0.0378	0.7481	0.0870	-0.0505	-0.1007	-0.0413	0.7470	0.0820	-0.0689	-0.1004
0.4025	0.0462	0.0594	0.7023	0.2203	0.4605	0.0658	0.0569	0.6641	0.2210
0.2937	0.2442	-0.1384	-0.6957	0.2626	0.2495	0.2164	-0.1093	-0.7327	0.2453
0.0215	0.0282	0.8060	0.1253	0.0477	0.0146	0.0361	0.8038	0.1424	0.0315

Table-26
Factor Rotation Matrix

Varimax

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5
Factor 1	0.7668	-0.1114	0.6131	0.0032	-0.1538
Factor 2	0.2611	0.8640	-0.1397	-0.3919	0.1108
Factor 3	-0.4251	0.4686	0.5607	0.4893	-0.2142
Factor 4	0.3553	0.1229	-0.3129	0.7468	0.4505
Factor 5	-0,1917	-0.0803	0.4385	-0.2220	0.8457

Equamax

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5
Factor 1	0.7612	-0.1123	0.6185	-0.0033	-0.1591
Factor 2	0.2587	0.8600	-0.1358	-0.4034	0.1109
Factor 3	-0.4222	0.4741	0.5536	0.4951	-0.2130
Factor 4	0.3693	0.1266	-0.3100	0.7367	0.4568
Factor 5	-0.1975	-0.0833	0.4432	-0.2221	0.8416

Quartimax

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5
Factor 1	0.7683	-0.1113	0.6116	0.0046	-0.1526
Factor 2	0.2616	0.8649	-0.1407	-0.3891	0.1108
Factor 3	-0.4257	0.4672	0.5626	0.4879	-0.2144
Factor 4	0.3520	0.1221	-0.3136	0.7491	0.4490
Factor 5	-0.1903	-0.0796	0.4374	-0.2220	0.8467

Orthomax

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5
Factor 1	0.7477	-0.1142	0.6260	-0.0316	-0.1873
Factor 2	0.2416	0.8478	-0.1258	-0.4424	0.1019
Factor 3	-0.4009	0.4908	0.5333	0.5216	-0.2049
Factor 4	0.4285	0.1409	-0.3130	0.6976	0.4603
Factor 5	-0.1959	-0.0861	0.4581	-0.2091	0.8371

Table-27
Information Criteria

Information Criteria	Values
Log Likelihood	-367.6505
Akaike	745.3010
Schwarz	372.2632
Hannan Quinne	370.3197

Results and Discussion

When the sample size (n) considered is thirty (30), the values of Akaike's Information Criterion (AIC)(Akaike;1987), the Schwarz Information Criterion (SIC) and the Hannan Quinne Information Criterion (HQIC) for the different number of retained factors are as follows; for k = 2, the AIC, SIC, and HQIC values are 246.7630,122.8586 and 122.1820 respectively. When k = 3, AIC is 246.3310, SIC is 123.3812 and HQIC is 122.1820; and for k = 5, it shows that AIC =263.3050, SIC =130.3453 and HQIC = 128.3467.

When the sample size is increased to 50, the AIC, SIC, and HQIC are 502.5950,250.9965 and 250.2182; 507.3000,253.1985, and 252.0311; 546.6350, 272.5649 and 270.6194 for k = 2,3, and 5 respectively.

Finally, at the sample size of 70, the values are AIC = 687.9690, SIC = 343.8196 and HQIC = 343.0386 for k = 2. When k is 3, AIC = 705.9230, SIC = 352.7291 and HQIC = 351.5576; and finally for k = 5, the values are 745.3010, 372.2632, and 370.3197 for AIC, SIC and HQIC respectively.

When the sample sizes are 30, the SIC, and HQIC are smallest for k = 2 follows by k = 3 and highest for k =5 and the AIC value is smallest for k=3 follows by k =2, highest for k=5.But for the sample size of 50 and 70, the values are smallest for k =2 follows by for k =3 and highest for k =5.

Conclusion

Given the results from this research work (above), it shows that the optimal number of factors to retain using the method of Principal Component Factors method of estimation is two (2) from all the sample sizes and also for all the methods considered except for the AIC in which the best is when k=3 follows by k=2 and k=5 respectively of sample thirty (30). This conclusion is made based on the fact that in competing sets of models, the model with the smallest value of information criteria is chosen as the best model.

Also, from the vales of AIC, SIC, and HQIC obtained above, the Hannan Quinne information criterion performs best for all the three criteria considered. This is followed by the SIC and AIC respectively. Finally, observation was made that the higher the sample size, the higher the value of the information criteria.

The factor rotation matrix for all the sample sizes and the number of parameters retained as considered is almost the same for all the four methods of rotation considered here.

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