Research Journal of Mathematical and Statistical Sciences ______________________________ISSN 2320–6047
Vol. **1(3),** 23-30, April **(2013)** *Res. J. Mathematical & Statistical Sci.*

Review Paper

A review of literature relating to Balance Incomplete Block designs with Repeated Blocks

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Available online at: www.isca.in
Received 22nd November 2012, revised 19th January 2013, accepted 8th March 2013

Abstract

The concept of balance incomplete block designs with repeated blocks comes from experimental design. Many statisticians were thoroughly studied the problem of construction of balance incomplete block designs with repeated blocks. In recent years there has been very rapid development in this area of experimental design. This paper presents a review of the available literature on balance incomplete block designs with repeated blocks.

Keywords: Incomplete block design, balance incomplete block design, balance incomplete block design with repeated blocks, variance balance design, efficiency balance design, neighbour balance block designs.

Introduction

The subject Design of Experiments in its present form owes its existence to the sound foundation laid by Sir R.A. Fisher, who formulated and developed the basic ideas of statistical designing in the period 1919-1930. Of Fisher's three principles of design of experiments i.e. randomization, replication and blocking; blocking is the most difficult because it places special constraints on experimental designs. The concept of blocking in statistically planned experiments has its origin in the agricultural field experiments, conducted at the Rothamsted Experimental Station during the tenure of Fisher as the Chief Statistician. The terms and terminology used in the design of experiments are borrowed from agricultural experiments e.g. the word 'yield' means scores obtained by a subject in a psychological experiments to which a certain type of design is used. Treatments are not exactly treatments of agricultural experiments when applied to problems in education and could be different methods of teaching.

It is well known that proper blocking reduces experimental error. Reduced error makes an experiment more sensitive in detecting significance of effects, so less experimentation may be necessary. Blocking of experimental units to eliminate heterogeneity is not restricted to agricultural experimentation alone. In the agricultural field experiments, experimental units lying at right angles to the fertility gradient generally form the blocks. Blocking of experimental units on a variety of physical, chemical, genetic, socio-economic, psychological or temporal characters have been adopted by various researchers. Discussion on blocking in actual situations may be found in Cochran and Cox[1], Cox[2], Kempthorne[3] and Box et. al[4].

Although a large number of block designs are available in literature. These designs have immense applications in almost all areas of scientific investigation. But there exist some situations where there are more sources of variation that can not be controlled by ordinary blocking.

When the number of treatments is very large and blocking is must, the Incomplete Block Designs are generally used. The origins of incomplete block designs go back to Yates[5] who introduced the concept of balanced incomplete block designs and their analysis utilizing both intra- and interblock information (Yates[6]). Other incomplete block designs were also proposed by Yates[5,6], who referred to these designs as quasi-factorial or lattice designs. Further contributions in the early history of incomplete block designs were made by Bose[7-9] and Fisher[10] concerning the structure and construction of balanced incomplete block designs. The notion of balanced incomplete block design was generalized to that of partially balanced incomplete block designs by Bose and Nair[11], which encompass some of the lattice designs introduced earlier by Yates. Further extensions of the balanced incomplete block designs and lattice designs were made by Youden[12] and Harshbarger[13], respectively, by introducing balanced incomplete block designs for eliminating heterogeneity in two directions (generalizing the concept of the Latin square design) and rectangular lattices some of which are more general designs than partially balanced incomplete block designs. After this there has been a very rapid development in this area of experimental design.

In order to eliminate heterogeneity; a concept of Balanced Incomplete Block Design was introduced, which reduce heterogeneity to a greater extent than is possible with randomized block design and latin square design.The history of BIB designs probably dates back to the 19th Century. The solution of the famous Kirkman's School girl problem (Kirkman[14]) has one-one correspondence with the solution of BIB design. In 1853, Steiner[15] proposed the problem of arranging 'n' objects in triplets such that every pair of objects appears in precisely one triplet. Such an arrangement is called a Steiner's triple system and is, infact, a BIB design.

The importance of BIB designs in statistical design of experiments for variental trials was, however, realized only in 1936 when Yates[5] discussed these designs in the context of biological experiments. F. Yates introduced these designs in his paper, "A new method of arranging variety trials, involving a large number of varieties", Journal Agr. Sci. 26, 424-455, 1936. Different methods of construction of balanced incomplete block designs have been given in literature, like, Agrawal and Prasad[16,17], Caliński[18], Alltop[19], Bose[7], Hanani[20], Majinder[21], Mills[22], Shrikhande and RagavaRao[23] etc.

We always need to set up a design in such a way that the variability in response due to uncontrolled variables (sometimes called experimental error) is not so great that it makes the effects of the controlled variables. We also want designs which are efficient, that is, designs where we can answer the questions of interest with a minimal amount of data because of the expense associated with data collection.

Though there have been balanced designs in various sense (see Puri and Nigam[24], Cali´nski[25], we will consider a balanced design of the following type. There are three main concepts of balancing in incomplete block designs, namely i. Variance Balanced, ii. Efficiency Balanced, iii. Neighbour Balanced.

Definitions

Let us consider v treatments arranged in b blocks, such that the j^{th} block contains k_j. experimental units and the i^{th} treatment appears r_i times in the entire design, i = 1,2,………,v ; j = 1,2,……….,b. For any block design there exist a incidence matrix N = $[n_{ij}]$ of order v x b, where n_{ij} denotes the number of experiment units in the j^{th} block getting the i^{th} treatment. When $n_{ij} = 1$ or 0 $\forall$ i and j, the design is said to be binary. Otherwise it is said to be nonbinary. The following additional notations are used k = $[k_1\ k_2 \ldots k_b]'$ is the column vector of block sizes, r = $[r_1\ r_2 \ldots r_v]'$ is the column vector of treatment replication, K_{bxb} = diag $[k_1\ k_2 \ldots k_b]$, R_{vxv} = diag $[r_1\ r_2 \ldots r_v]$, $\Sigma r_i = \Sigma k_j = n$ is the total number of experimental units , with this $N1_b = \underline{r}$ and $N'1_v = \underline{k}$,

Where 1_a is the a x 1 vector of ones.

The information matrix for treatment effects C defined below as

$$C = R - NK^{-1}N' \tag{1}$$

Where R= diag $(r_1, r_2 \ldots r_v)$, K = diag $(k_1, k_2 \ldots k_b)$

A block design with incidence matrix having all elements equal to unity is called a randomized (complete) block design. It can be verified that such a design is necessarily "orthogonal" and also "variance balanced". Rao[26] gives a necessary and sufficient condition for a general block design to be variance balanced.

A block design is said to be balanced if every elementary contrast of treatment is estimated with the same variance[27]. In this sense this design is also called a variance balance design. It is well known that block design is a variance balanced if and only if it has

$$C = \eta \left(I_v - \frac{1}{v} 1_v 1_v' \right) \tag{2}$$

where η is the unique nonzero eigenvalue of the matrix C with the multiplicity $v - 1$, I_v is the $v \times v$ identity matrix. For binary block design[28]

$$\eta = \frac{\sum_{i=1}^{v} r_i - b}{v-1} \tag{3}$$

In particular case when block design is a balanced incomplete block design then $\eta = \frac{v\,r - b}{v-1}$.

The concept of Efficiency Balanced was introduced by Jones[29] and the nomenclature "Efficiency Balanced" is due to Puri and Nigam[24] and Williams[30].

A block design is called efficiency balanced if every contrast of treatment effects is estimated through the design with the same efficiency factor. Let us consider the matrix M_o given by Cali´nski[25]

$$M_o = R^{-1}NK^{-1}N' - \frac{1}{n} 1_v r' \qquad (4)$$

$$M_o S = \mu S$$

Where $T = [T_1 \; T_2 \ldots\ldots T_v]'$ is the vector of treatment totals ; T_i is the total yield for the i^{th} treatment. μ is the unique non zero eigen value of M_o with multiplicity (ν-1) and M_o is given as (4).

Cali´nski[25] showed that for such designs every treatment contrast is estimated with the same efficiency (1-μ) and N is a EB block design if and only if

$$M_o = \mu \left(I_v - \frac{1}{n} 1_v r'\right) \qquad (5)$$

Kageyama[31] proved that for the EB block design N, eq^n (5) is fulfilled if and only if

$$C = (1-\mu)\left(R - \frac{1}{n} r r'\right) \qquad (6)$$

A block design is called proper if all its blocks are of equal size. Rees[32] in the year 1967 introduced the "neighbor designs" for use in serological experiment. According to these designs many virus preparations were arranged in circular plates so that every such preparation appears as a neighbor of every other preparation equally often. There are *v* types of virus preparations (treatments) to be arranged in *b* circular plates containing *k* treatments. Each treatment appears *r* times in the design (not necessarily on *r* distinct blocks) and is a neighbor of every other treatment exactly times.

Neighbour balanced block designs, wherein the allocation of treatments is such that every treatment occurs equally often with every other treatment as neighbours, are used for modeling and controlling interference effects between neighbouring plots. Neighbour balance is important if it is known or thought that the effect of a plot is influenced by its neighbouring plots, in such cases nearest neighbour analysis is considered to be more efficient than classical analysis methods, see Wilkinson[33] et al. The construction of nearest neighbour balanced designs in one-dimension has received much attention from several authors, examples are Kiefer and Wynn[34], Cheng[35]. Various types of two-dimensional nearest neighbour balanced designs are introduced and studied in Street and Street[36], Street[37], Freeman[38,39], Afsarinejad and Seeger[40], Morgan[41], Morgan and Uddin[42]. A one dimensional (block) design is defined to be nearest neighbour balanced (NNB) if each treatment has every other treatment as its neighbour on an adjacent plot an equal number of times. A two dimensional (row-column) design is said to be row neighbour balanced if each treatment has every other treatment as its nearest neighbour in rows an equal number of times, and similarly defined for column neighbor balance. Thus a two dimensional design is called NNB if it is row neighbor balanced and column neighbour balanced.

Related Work

Several authors discussed various properties of the balanced incomplete block design. From the point of view of application there is no reason to exclude the possibility that a BIB design would contain repeated blocks. Indeed, the statistical optimality of BIB designs is unaffected by the presence of repeated blocks. Consider a balanced incomplete block design with parameters: v, b, r, k and λ. Let the support block of balanced incomplete block design is the set of its distinct blocks and denote the cardinality of support block by b*. The question of whether , for a given v, b and k, there exist a balanced incomplete block design with repeated blocks, is interesting among the researchers in the area of experimental design.

Balance Incomplete Block Designs with repeated blocks were studied and constructed initially by van Lint and Ryser in 1972 and pursued by van Lint in 1973. The first published BIBDs with repeated blocks were those in series β_1 and series β_2 of Bose[9], but that paper was overlooked for some decades. To our knowledge, there has not been much study of necessary or sufficient conditions for the existence of a BIBD with repeated blocks and given parameters, nor on bounds for the multiplicity of a block in such a BIBD. For a (v, b, r, k, λ)-BIBD with m the maximum multiplicity of a block, Mann[43] proved in 1969 that $m \le b/v$. In 1972, van Lint and Ryser[44] proved that in addition, if m = b/v, then m divides gcd(b, r, λ). They also gave constructions for BIBDs with repeated blocks, usually with gcd(b, r, λ) > 1. In 1973, Van Lint[45] has discovered that, many of the BIB designs constructed by

Hanani[46] have repeated blocks. He considered tuples (v, b, r, k, λ) of positive integers satisfying $2 \leq k \leq v/2$, $\lambda (v - 1) = r(k - 1)$, $vr = bk$, $\lambda > 1$, gcd(b, r, λ) = 1 and $b > 2v$, and asked whether for each such tuple (v, b, r, k, λ) there exists a BIBD with repeated blocks. He showed that this is indeed the case when $k \leq 4$, except possibly when (v, k, λ) = (45, 4, 3). Additionally van Lint[45] tabulated all tuples (v, b, r, k, λ) satisfying his conditions with $v \leq 22$, and for many of these tuples constructed BIBDs with repeated blocks.

As Van Lint[45] has pointed out that many of the balanced incomplete block design constructed by Hanani[46] have repeated blocks. Parker[47] and Seiden[48] proved that there is no balanced incomplete block design with repeated blocks with parameters: v= 2x+2,b= 4x+2 and k= x+1. Parker[47] and Seiden[48] settled the case for general x not only for odd x. Stanton and Sprott[49] showed that if s blocks of a balanced incomplete block design are identical , then $bv \geq sv - (s-2)$. Mann[43] sharpened this result and showed that $b \geq sv$. Also the result of Parker[47] and Seiden[48] follows immediately from either of the inequalities.

Ho and Mendelsohn[50] gave the generalization of the Mann[43] inequality for t-design. Then Van Lint and Ryser[44] and Van Lint[45] thoroughly studied the problem of construction of balanced incomplete block design with repeated blocks. Their basic interest was in constructing a BIB design with repeated blocks with parameters v, b, r, k and λ, such that b,r and λ are relatively prime.

Van Lint and Ryser[44] first gave a new proof of Mann's Inequality, stating that $e \leq b/v$ if a block is repeated, e times, Moreover, their methods allow them to conclude that equality is only possible if e/gcd (b, r, λ). They further show that the number 't' of distinct blocks satisfied $t \geq v$, with equality only if each block is repeated the same number of times, and also that $t \neq v+1$. Finally they constructed many examples of block designs with repeated blocks, including several infinite families.

Wynn[51] in the year 1975 constructed a BIB design with ν = 8, b = 56, k = 3 and b* = 24 with repeated blocks. In 1977, He also discussed the selection of a sample of k distinct elements from a set of ν elements (varieties). He was led in particular to consider balanced incomplete block designs in which some of the blocks are repeated. Wynn considered an example of a design for ν = 24, k = 5 with b = 56 and b* = 24. Peter W.M. John[52] shown that this design can be obtained from a hierarchic group divisible association scheme, and is one of a set of eight possible designs.

Foody and Hedayat[53] presented some potential applications of the balanced incomplete block designs with repeated blocks to experimental designs and controlled sampling. They also provided some necessary and sufficient conditions for the existence of these designs and some algorithms for their constructions. Bounds on b* have been obtained. A necessary and sufficient condition under which a set of blocks can be support of a BIB design were also found and a table of BIB designs with $22 \leq b^* \leq 56$ for v=8 and k=3 was included.

Designs with repeated blocks with the equireplications and with equal size of each block are discussed in the literature: Hedayat and Li[54], Hedayat and Hwang[55], Khosrovshahi and Mahmoodian[56].

Since BIB designs with repeated blocks, besides being optimal, have special applications in the design of experiments and controlled samplings. The construction of BIB (v, b, r, k, λ) designs with repeated blocks becomes complicated whenever the three parameters b, r and λ are relatively prime. BIB (8, 56, 21, 3, 6) designs are examples of such designs with the smallest number of varieties. BIB (10, 30, 9, 3, 2) designs are such designs with the smallest number of blocks. Hedayat and Hwang[55] made an interesting observation about BIB (8, 56, 21, 3, 6) designs and gave a table of such designs with 30 different support sizes. They proved by construction, that a BIB (10, 30, 9, 3, 2) design exists if and only if the support size belongs to {21, 23, 24, 25, 26, 27, 28, 29, 30}.

Khosrovshahi and Mahmoodian[56], In their paper studied the family of BIB designs with ν=9 and k=3 from the view of possible support sizes b*'s. They constructed a table of designs with support sizes belonging to {12, 18, 20, 21, 84}, for minimum possible 'b' in each case and for any larger admissible 'b'. In constructing this table the methods of tradeoff and composition of designs were utilized.

More recently different methods of constructing variance balanced and efficiency balanced block designs with repeated blocks have been given in the literature, like, Ghosh and Shrivastava[57], Ceranka and Graczyk[58-60].

Ghosh and Shrivastava[57] developed the methods of construction of BIB designs with repeated blocks so as to distinguish the usual BIBD with repeated blocks. Also, a class of BIB design with parameters ν=7, b=28, r=12, k=3, λ=4 has been constructed where, out of 15, 14 BIB designs have repeated blocks. Those 15 BIB designs, which have the same parameters, are compared on the basis of number of distinct blocks (d) and the multiplicities of variance of elementary contrasts of the block effect.

Ceranka and Graczyk[58] developed same new construction methods of the variance balanced block designs with repeated blocks. However from the practical point of view it may not possible to construct the design with equalize blocks accommodating the equireplication of each treatment in all the blocks. In this paper Ceranka and Graczyk consider a class of block designs called variance balanced block designs which can be made available in unequal block sizes and for varying replications.

In 2008 Ceranka and Graczyk[59] developed some new construction methods of the variance balanced block designs with repeated blocks, which are based on the specialized product of incidence matrices of the balanced incomplete block designs. From a practical point of view, it may not be possible to construct a design with equiblock sizes accommodating the equireplication of each treatment in all the blocks. In this paper researcher consider a class of block designs called variance balanced block designs which can be made available in unequal block sizes and for equal replications. Also Ceranka and Graczyk[60] presented some new construction schemes of Efficiency Balanced block designs with repeated blocks for ν treatments and some ways of admitting given design structures to construct new designs for other number of treatments.

Conclusion

Balance incomplete block designs with repeated blocks are useful in various problems experimental designs. In this study the research and literature review were organized according to the construction and subject.

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