*Research Journal of Mathematical and Statistical Sciences* ISSN 2320–6047
Vol. **1(10),** 1-4, November **(2013)** *Res. J. Mathematical and Statistical Sci.*

# Angular Displacement in A Shaft Associated with the Aleph Function and Generalized Polynomials

**Shekhawat Ashok Singh and Garg Harshita**
Department of Mathematics, Arya College of Engineering and Information Technology, Jaipur, Rajasthan, INDIA

**Available online at: www.isca.in, www.isca.me**
Received 2nd October 2013, revised 24th October 2013, accepted 5th November 2013

## Abstract

*The main aim of the present paper is to find the application of certain products involving Aleph function (ℵ-function) and generalized polynomials in obtaining a solution of the partial differential equation,* $\frac{\partial^2 \phi}{\partial t^2} = k^2 \frac{\partial^2 \phi}{\partial x^2}$ *Concerning to a problem of angular displacement in a shaft.*

**Keywords**: Aleph Function, general class of polynomials, partial differential equation, Angular displacement.

## Introduction

Let the problem of determining the twist $\phi(x,t)$ in a shaft of circular section with its axis along the x-axis. Now the displacement $\phi(x,t)$ due to initial twist must satisfy the boundary value problem[1,2,3]. If we assume that both the ends $x = 0$ and $x = \mu$ of the shaft are free

$$\frac{\partial^2 \phi}{\partial t^2} = k^2 \frac{\partial^2 \phi}{\partial x^2} \tag{1}$$

Where k is a constant

$$\frac{\partial \phi}{\partial x}(0,t) = 0, \quad \frac{\partial \phi}{\partial x}(x,0) \quad \text{and} \quad \phi(x,0) = f(x) \tag{2}$$

Let

$$f(x) = \left(\sin\frac{\pi x}{2\mu}\right)^{2\delta-\lambda-1}\left(\cos\frac{\pi x}{2\mu}\right)^{\lambda-1} S_{n_1,\ldots,n_s}^{m_1,\ldots,m_s}\left[y_1\left(\tan\frac{\pi x}{2\mu}\right)^{2k_1},\ldots,y_s\left(\tan\frac{\pi x}{2\mu}\right)^{2k_s}\right] \aleph_{p_i,q_i,c_i;r}^{m,n}\left[z\left(\tan\frac{\pi x}{2\mu}\right)^{2h}\right] \tag{3}$$

The Aleph function introduced by Südland et al[4] is defined as Mellin-Barnes type contour integrals as following

$$\aleph(z) = \aleph_{p_i,q_i,c_i;r}^{m,n}\left[z \left| \begin{matrix} (a_j,A_j)_{1,n}, [c_i(a_{ji},A_{ji})]_{n+1,p_i;r} \\ (b_j,B_j)_{1,m}, [c_i(b_{ji},B_{ji})]_{m+1,q_i;r} \end{matrix} \right.\right]$$

$$= \frac{1}{2\pi i}\int_L \Omega_{p_i,q_i,c_i;r}^{m,n}(\xi)\, z^{-\xi}\, d\xi \tag{4}$$

For all $z \neq 0$, where $i = \sqrt{-1}$ and

$$\Omega_{p_i,q_i,c_i;r}^{m,n}(\xi) = \frac{\prod_{j=1}^{m}\Gamma(b_j + B_j\xi)\prod_{j=1}^{n}\Gamma(1-a_j-A_j\xi)}{\sum_{i=1}^{r} c_i \prod_{j=n+1}^{p_i}\Gamma(a_{ji}+A_{ji}\xi)\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}-B_{ji}\xi)} \tag{5}$$

The $L = L_{i\gamma\infty}$ is a suitable contour of the Mellin-Barnes type which runs from $\gamma - i\infty$ to $\gamma + i\infty$ with $\gamma \in R$, the integers m,n, p, q satisfy the inequality $0 \le n \le p_i$, $1 \le m \le q_i$, $c_i > 0$; $i = 1,\ldots,r$. The parameters $A_j$, $B_j$, $A_{ji}$, $B_{ji}$ are positive real numbers and $a_j$, $b_j$, $a_{ji}$, $b_{ji}$ are complex numbers, such that the poles of $\Gamma(b_j + B_j\xi)$, $j = 1,2,\ldots,m$ separating from those of $\Gamma(1-a_j-A_j\xi)$, $j = 1,\ldots,n$. All the poles of the integrand (4) are supposed to be easy and empty products are considered as unity. The existence conditions[5] for the Aleph function (4) are given below:

$$\psi_k > 0, |\arg(z)| < \frac{\pi}{2}\psi_k ; k = 1, ..., r, \tag{6}$$

$$\psi_k \geq 0, |\arg(z)| < \frac{\pi}{2}\psi_k \text{ and } R\{\Lambda_k\} + 1 < 0 \tag{7}$$

Where, $$\psi_k = \sum_{j=1}^{n} A_j + \sum_{j=1}^{m} B_j - C_k \left( \sum_{j=n+1}^{p_k} A_{jk} + \sum_{j=m+1}^{q_k} B_{jk} \right) \tag{8}$$

$$\Lambda_k = \sum_{j=1}^{m} b_j - \sum_{j=1}^{n} a_k + C_k \left( \sum_{j=1}^{q_k} b_{jk} - \sum_{j=n+1}^{p_k} a_{jk} \right) + \frac{1}{2}(p_k - q_k) \tag{9}$$

The generalized polynomial defined by Srivastava[6] is as follows: $S_{n_1,...,n_s}^{m_1,...,m_s}[x_1, ...., x_s] = \sum_{\alpha_1=0}^{[n_1/m_1]} ... \sum_{\alpha_s=0}^{[n_s/m_s]} \frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!} ... \frac{(-n_s)_{m_s\alpha_s}}{\alpha_s!}$

$$B[n_1, \alpha_1; ...; n_s, \alpha_s] x_1^{\alpha_1} ... x_s^{\alpha_s} \tag{10}$$

Where $n_i = 0, 1, 2...$ $\forall i = (1, ..., s)$, $m_1, ..., m_s$ are arbitrary positive integers and the coefficients $[n_1, \alpha_1; ...; n_s, \alpha_s]$ are arbitrary constants, real or complex.

**The Main Result**: We derive the following result:

$$\int_0^{\alpha} \left(\cos\frac{\pi\delta x}{\mu}\right)\left(\sin\frac{\pi x}{2\mu}\right)^{2\delta-\lambda-1}\left(\cos\frac{\pi x}{2\mu}\right)^{\lambda-1} S_{n_1,...,n_s}^{m_1,...,m_s}\left[y_1\left(\tan\frac{\pi x}{2\mu}\right)^{2k_1}, .. \, y_s\left(\tan\frac{\pi x}{2\mu}\right)^{2k_s}\right] \aleph_{p_i,q_i,c_i;r}^{m,n}\left[z\left(\tan\frac{\pi x}{2\mu}\right)^{2h}\right] dx$$

$$= \frac{\mu \, 2^{2\delta-\lambda+2\sum_{i=1}^{s} k_i\alpha_i}}{\Gamma(2d)\sqrt{\pi}} \sum_{\alpha_1=0}^{[n_1/m_1]} ... \sum_{\alpha_s=0}^{[n_s/m_s]} \frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!} ... \frac{(-n_s)_{m_s\alpha_s}}{\alpha_s!}$$

$$B[n_1, \alpha_1; ...; n_s, \alpha_s] y_1^{\alpha_1} ... y_s^{\alpha_s} \aleph_{p_i+2,q_i+1,c_i;r}^{m+1,n+1}\left[\frac{1}{z\,4^h} \middle| \begin{matrix} \left(1-\delta+\frac{\lambda}{2}-\sum_{i=1}^{s} k_i\alpha_i, h; 1\right), (a_j, A_j)_{1,n}, [C_i(a_{ji}, A_{ji})]_{n+1,p_i;r}, \left(\lambda-\sum_{i=1}^{s} k_i\alpha_i, 2h\right) \\ \left(\frac{1}{2}-\delta+\frac{\lambda}{2}-\sum_{i=1}^{s} k_i\alpha_i, h\right), (b_j, B_j)_{1,m}, [C_i(b_{ji}, B_{ji})]_{m+1,q_i;r} \end{matrix}\right] \tag{11}$$

Where $k_i > 0\ (i = 1, ..., s)$, $h > 0$, $\operatorname{Re}\left(\lambda - 2k\frac{b_j}{B_j}\right) > 0\ (j = 1, ..., m)$, m is an arbitrary positive integer and the coefficient $B[n_1, \alpha_1; ...; n_s, \alpha_s]$ are arbitrary constants, real or complex.

**Evaluation of (11)**: The integral in (11) can be derived by using of the Aleph function in terms of Mellin-Barnes contour integral given by (4) and the definition of a generalized polynomials given by (10), then interchanging the order of summation and integration, find the inner integral by using a result given by Chaurasia and Gupta[7] and we get the desired result.

**Solution of the Problem posed**: The solution of the problem to be established is

$$\phi(x, t) = \frac{1}{2^{\lambda}\sqrt{\pi}} \sum_{\alpha_1=0}^{[n_1/m_1]} ... \sum_{\alpha_s=0}^{[n_s/m_s]} \frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!} ... \frac{(-n_s)_{m_s\alpha_s}}{\alpha_s!} B[n_1, \alpha_1; ...; n_s, \alpha_s] y_1^{\alpha_1} ... y_s^{\alpha_s}$$

$$= \frac{2^{2\tau+2\sum_{i=1}^{s} k_i\alpha_i}}{\Gamma(2\tau)} \aleph^{m+1,n+1}_{p_i+2,q_i+1,c_i;r}\left[\frac{1}{z\,4^h}\left|\begin{matrix}\left(1-\tau+\frac{\lambda}{2}-\sum_{i=1}^{s}k_i\alpha_i,h;1\right),(a_j,A_j)_{1,n},[C_i(a_{ji},A_{ji})]_{n+1,p_i;r},\left(\lambda-\sum_{i=1}^{s}k_i\alpha_i,2h\right)\\ \left(\frac{1}{2}-\tau+\frac{\lambda}{2}-\sum_{i=1}^{s}k_i\alpha_i,h\right),(b_j,B_j)_{1,m},[C_i(b_{ji},B_{ji})_{m+1,q_i;r}\end{matrix}\right.\right]$$

$$\left(\cos\frac{\pi\tau x}{\mu}\right)\left(\cos\frac{\pi\tau R\ t}{\mu}\right) \qquad (12)$$

Which are valid under the same conditions used for (11)

**Derivation of (12)**: The solution of the problem can be written as (by using Churchill[8])

$$\phi(x,\ t) = \frac{1}{2}a_0 + \sum_{\tau=1}^{\infty} a_\tau\left(\cos\frac{\pi x\,\tau}{\mu}\right)\left(\cos\frac{\pi\tau\,Rt}{\mu}\right) \qquad (13)$$

Where $a_\tau(\tau=0,1,2,...)$ are the coefficients in the Fourier Cosine Series for f(x) in the interval (0,μ), If t = 0, then by virtue of (1.3), we get

$$\left(\sin\frac{\pi x}{2\mu}\right)^{2\delta-\lambda-1}\left(\cos\frac{\pi x}{2\mu}\right)^{\lambda-1} S^{m_1,...,m_s}_{n_1,...,n_s}\left[y_1\left(\tan\frac{\pi x}{2\mu}\right)^{2k_1},...,\ y_s\left(\tan\frac{\pi x}{2\mu}\right)^{2k_s}\right]$$

$$\aleph^{m,n}_{p_i,q_i,c_i;r}\left[z\left(\tan\frac{\pi x}{2\mu}\right)^{2h}\right] = \frac{1}{2}a_0 + \sum_{\tau=1}^{\infty} a_\tau\left(\cos\frac{\pi\tau x}{\mu}\right) \qquad (14)$$

Now multiplying (14) both sides by $\left(\cos\frac{\pi\delta\ x}{\mu}\right)$ and integrate with respect to x from 0 toμ, we get

$$\int_0^{\mu}\left(\cos\frac{\pi\delta\ x}{\mu}\right)\left(\sin\frac{\pi x}{2\mu}\right)^{2\delta-\lambda-1}\left(\cos\frac{\pi x}{2\mu}\right)^{\lambda-1} S^{m_1,...,m_s}_{n_1,...,n_s}\left[y_1\left(\tan\frac{\pi x}{2\mu}\right)^{2k_1},...,\right.$$

$$\left.y_s\left(\tan\frac{\pi x}{2\mu}\right)^{2k_s}\right]\aleph^{m,n}_{p_i,q_i,c_i;r}\left[z\left(\tan\frac{\pi x}{2\mu}\right)^{2h}\right]$$

$$= \frac{1}{2}a_0\int_0^{\mu}\left(\cos\frac{\pi\delta\ x}{\mu}\right)dx + \sum_{\tau=1}^{\infty} a_\tau\left(\cos\frac{\pi\tau\ x}{\mu}\right)\left(\cos\frac{\pi\delta\ x}{\mu}\right)dx \qquad (15)$$

Using (11) along with orthogonal property of the cosine functions, we get

$$a_\tau = \frac{2^{2\tau-\lambda+2\sum_{i=1}^{s}k_i\alpha_i+1}}{\sqrt{\pi}\ \Gamma(2\tau)}\sum_{\alpha_1=0}^{[n_1/m_1]}...\sum_{\alpha_s=0}^{[n_s/m_s]}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}...\frac{(-n_s)_{m_s\alpha_s}}{\alpha_s!}B[n_1,\alpha_1;...;n_s,\alpha_s]\,y_1^{\alpha_1}...y_s^{\alpha_s}$$

$$.\aleph^{m+1,n+1}_{p_i+2,q_i+1,c_i;r}\left[\frac{1}{z\,4^h}\left|\begin{matrix}\left(1-\tau+\frac{\lambda}{2}-\sum_{i=1}^{s}k_i\alpha_i,h;1\right),(a_j,A_j)_{1,n},[C_i(a_{ji},A_{ji})]_{n+1,p_i;r},\left(\frac{\lambda}{2}-\sum_{i=1}^{s}k_i\alpha_i,2h\right)\\ \left(\frac{1}{2}-\tau+\frac{\lambda}{2}-\sum_{i=1}^{s}k_i\alpha_i,h\right),(b_j,B_j)_{1,m},[C_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{matrix}\right.\right] \qquad (16)$$

Now by using (13) and (16), we get the desired solution in (12).

**Numerical Results**: i. Taking $C_i = 1$, i = 1,…,r in (4), the Aleph function coincide with the I-function given by Saxena[9,10].ii. Again for r = 1 and $C_1 = 1$, taking S = 2 and $k_i \to 0$ in (11), we find the known result concluded by Chaurasia and Godika[11]. iii. Taking $\overline{H}$ -function in place of Aleph-function in (11), we get the known result obtained by Chaurasia and Shekhawat[12].

## Conclusion

The result so established may be found useful in several interesting situation appearing in the literature on mathematical analysis, applied mathematics and mathematical physics.

## Acknowledgement

The authors are thankful to Professor H.M. Srivastava, University of Victoria, Canada for valuable comments and suggestions in the preparation of this paper.

## References

**1.** Dutta B.K., Arora L.K. and Borah J., on the solution of fractional kinetic equation, *Gen. Math. Notes,* **6(1),** 40-48 **(2011)**

**2.** Haubold H.J. and Mathai A.M., The fractional kinetic equation and thermonuclear functions, Astrophys. *Space Sci.,* **327**, 53-63 **(2000)**

**3.** Saxena R.K., Mathai A.M. and Haubold H.J., On generalized fractional kinetic equations, *Phys. A.,* **344**, 653-664 **(2004)**

**4.** Südland N., Baumann B. and Nonnenmacher T.F., Who knows about the Aleph (ℵ)-function? Fract, *Calc. Appl. Anal.,* **1(4)**, 401-402 **(1998)**

**5.** Saxena R.K. and Pogany T.K.: Mathieu-type series for the Aleph-function occurring in Fokker-Planck equation, *Eur. J. Pure Appl. Math.,* **3(6)**, 958-979 **(2010)**

**6.** Srivastava H.M., A multilinear generating function for the Konhauser sets of bi-orthogonal polynomials suggested by the Laguerre polynomials, *Pacific J. Math.,* **117**, 183-191 **(1985)**

**7.** Chaurasia V.B.L. and Gupta V.G., The H-function of several complex variables and angular displacement in a shaft – II, *Indian J. Pure Appl. Math.,* **14(5)**, 588-595 **(1983)**

**8.** Churchill R.V., Fourier series and boundary value problems, McGraw-Hill Book Co., *Inc. New York* **(1941)**

**9.** Saxena V.P., The I-function, Anamaya Publishers, *New Delhi* **(2008)**

**10.** Shukla Manoj Kumar, Pandey Sunil and Shrivastava Rajeev, On Some Integral Relations involving I-Function with Applications, *Res. J. Mathematical and Statistical Sci.,* **1(1),** 14-18 **(2013)**

**11.** Chaurasia V.B.L. and Godika A., A solution of the partial differential equation of angular displacement in shaft – II, *Acta Ciencia Indica,* **23M(1)**, 77-89 **(1997)**

**12.** Chaurasia V.B.L. and Shekhawat A.S., An application of $\overline{H}$-function and a generalized polynomials in the study of angular displacement in a shaft – II, *Appl. Sci. Period,* **8(1),** 37-46 **(2006)**