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Characterization of cubic crystalline systems: a field theory uniting elasticity and electromagnetism

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Abstract

This paper examines the microscopy of cubic crystalline systems from Navier equation in perfect media. We researched potential solutions in terms of scalar and vector gauge fields from one Helmholtz theorem. To near a sign, it appeared two gauge relations for both field kinds. Vector field description is similar to Maxwell electromagnetic theory such as the translation is immediat to describe fields of electrons and holes. The elastic phenomena are then relatable to the electromagnetic ones, provided that the previous theory be completed. When examining different ways of gauge and field invariances, we found that: i. local fermions describe plane and central motions leading to conservation laws of energy and kinetic momentum. ii. These describe longitudinal spin waves originated by the free electrons of lattice and explain thermal radiations. iii. Linked electrons define four kinds of crystalline magnetism even in non-perfect media… To characterize cubic crystals, we determined the expressions of their scalar and vector fields at interfaces. iv. We found four different gauge couplings corresponding to four systems, i.e. the three primitive (cP, bcc, fcc) and another including all non-primitive systems. v. The two firsts are characterized by zero electric fields and transverse stationary waves; the two lasts by non-zero electric fields which are local and transverse. vi. There is no rotating charge at interfaces for the four systems. This field theory then describes elastic and electromagnetic phenomena in the same way and at quantum scale.

Keywords: Crystal magnetism, cubic system, elastic characterization, navier equation, spin wave, thermal radiation.

Introduction

The material characterization implies the description of particular behaviors or characteristics of these which make them different one from another. Here, this regards the distinction of cubic systems from their elastic field characteristics implying furthermore electronic charges. The wave propagation in media relies on the crystalline symmetries defining the elasticity or stiffness tensor. The case of homogeneous and isotropic media (so-called perfect) relies on cubic systems. In this case, there are only two propagating modes for each system. As with any other system, one can say that the modes are degenerated relatively to numerous systems involved. One can ask about the possibilities of breaking such a degenerency in wave propagation. The solution should come from the determination of wave microscopy hidden by the common symmetries. This is such as each system should correspond to a different field expression; otherwise, each should be characterized by a specific gauge or a set of gauges.

Gauge theories constitute one of the most important field of research in theoretical physics. These first began in modern physics and are since used in the study of material properties[1,2]. In linear elasticity, one often uses the Lagrangian formalism more general than the Newtonian one[3,4]. In both cases, one obtains however wave equations which constitute the common point. But as far as we are aware, a theory unifying all fields is not yet officially known in physics. Hence, in the line of our previous works concerning vacuum[5,6], we found opportune to export the theory originality toward cubic crystalline systems, which are comparable to elastic behaviors of vacuum. This initiated the field unification from wave field equations and reveals the existence of four fundamental fields. One can then expect to identify four main cubic systems. Here, we are then interested to explore the motion equation of cells in perfect media in order to characterize the related material systems. To our knowledge, this is the first study of the kind.

Therefore, after a brief recall on elastic bases yielding Navier equation in these media, we will search to solve this in the case of potential forces acting on the medium cells. This implies determining the gauges and related fields. Then we will examine these invariances before characterization from specific conditions. To end, we will discuss the theory relevance via some phenomena relative to the metallic link in crystals.

Materials and methods

We summarize here linear elasticity bases leading to the motion general equation in a *perfect medium*, i.e. infinite, homogeneous and isotropic. The first adjective remains valid while boundary conditions are not required. The condition defining isotropic and non-homogeneous materials will naturally appears. We will consequently solve that equation by applying one Helmholtz theorem. As the next depends on the kind of inertial forces, we consider furthermore the case where these are potential, i.e. keep the host medium in thermodynamic equilibrium. Then will follow the expression examinations depending on gauge fields appearing from that theorem.

Waves propagation essential in linear elasticity One considers the elastic disturbance of a perfect medium. One classically describes the wave propagation by considering the set motion of oscillating parallelepipedic cells. If $\vec{u}$ is the displacement vector of one of these, its motion equation from Newton fundamental principle in rectangular coordinates $(Ox_1x_2x_3)$, is given in Eisnteinian notation by the relation

$$\rho\frac{\partial^2 u_i}{\partial^2 t}=\frac{\partial\sigma_{ij}}{\partial x_i}+f_i \quad (1)$$

where ρ is the medium density; σ_{ij} is the stress tensor and f_i represents the component-i of inertial force density on a cell. This stress tensor is related to the strain tensor (e_{kl}) through the generalized Hooke law, i.e. with the same notation $\sigma_{ij}=C_{ijkl}.e_{kl}$; where (C_{ijkl}) is the elasticity or stiffness tensor. One defines in linear elasticity e_{kl} by the expression

$$e_{kl}=\frac{1}{2}\left(\frac{\partial u_k}{\partial x_l}+\frac{\partial u_l}{\partial x_k}\right) \quad (2)$$

After substitutions, the initial equation writes under the form

$$\rho\frac{\partial^2 u_i}{\partial t^2}=C_{ijkl}\frac{\partial^2 u_k}{\partial x_j x_l}+f_i \quad (3)$$

Owing to the symmetry $(\sigma_{ij}=\sigma_{ji})$, the 81 elements of the elasticity tensor reduce to 21 independent. In isotropic media, one shows that $C_{ijkl}=\lambda\delta_{ij}\delta_{kl}+\mu(\delta_{ik}\delta_{jl}+\delta_{jk}\delta_{il})$; where λ and μ are Lamé constants $(\lambda>\mu)$. Substituting this in the previous equation, then doing some classical developments and transformations, one gets to one form of Navier equation[4]:

$$\frac{\partial^2\vec{u}}{\partial t^2}=-c_t^2\vec{\nabla}\times(\vec{\nabla}\times\vec{u})+c_l^2\vec{\nabla}(\vec{\nabla}\vec{u})+\frac{\vec{f}}{\rho} \quad (4)$$

where c_t and c_l are the respective celerities of the transverse (T) and the longitudinal (L) waves such as

$$c_t=\sqrt{\frac{\mu}{\rho}};\quad c_l=\sqrt{\frac{\lambda+2\mu}{\rho}} \quad (5)$$

That wave equation lets seeing two propagating modes whatever are the inertial forces. This solution is generally treated in particular cases. The general solution is unknown and represents a great challenge. We are going to solve this in the particular (but useful) case of potential inertial forces.

Helmholtz's theorem application According to one Helmholtz theorem, any field vanishing to infinite is expressable as a sum of both rotational and gradient terms. Applying this to the displacement field and taking into account both kinds of waves, we can write

$$\vec{u}=\underbrace{\vec{\nabla}\times\vec{A}_t}_{\vec{u}_t}\underbrace{-\frac{1}{c_l}\vec{\nabla}V_l}_{\vec{u}_l} \quad (6)$$

such as both displacements are orthogonal $(\vec{u}_t.\vec{u}_l=0)$. The quantities V_l and $\vec{A}_t$ are respectively the scalar and vector potentials originating the displacement vector. Their dimensions are such as $[V_l]=m^3s^{-1}$ and $[A_t]=m^2$. This definition suits with the existence of 4-potentials $(\vec{A}_t,iV_t/c_t)$ and $(\vec{A}_l,iV_l/c_l)$ such as V_l is the flow through the surface $\vec{A}_l$. Equivalently, V_t is the flow through the surface $\vec{A}_t$.. The wave equation then turns into

$$\rho\partial_t^2\vec{u}=\rho(c_t^2\Delta\vec{u}_t-c_l^2\Delta\vec{u}_l)+\vec{f} \quad (7)$$

or relatively to the potentials

$$c_t^2\vec{\nabla}\times\Box_t\vec{A}_t-c_l^2\vec{\nabla}\Box_l\frac{V_l}{c_l}=-\frac{\vec{f}}{\rho} \quad (8)$$

where both d'Alembertian operators are defined by

$$\Box_\alpha=\Delta-\frac{1}{c_\alpha^2}\partial_t^2 \text{ with } \partial_t=\frac{\partial}{\partial t};\ \alpha=l,t \quad (9)$$

Considering now the case of potential inertial forces, one can write by comparison

$$\vec{f}=\vec{\nabla}\times\vec{\psi}_t-\frac{1}{c_l}\vec{\nabla}\varphi_l \quad (10)$$

such as its substitution leads to the wave equations

$$\Box_t\vec{A}_t=-\frac{1}{c_t^2}\frac{\vec{\psi}_t}{\rho};\quad \Box_l V_l=-\frac{1}{c_l^2}\frac{\varphi_l}{\rho} \quad (11)$$

Thence, it becomes necessary to complete these with those of potentials V_t and $\vec{A}_l$. The next subsection shows the related procedure giving birth to the excepted concept of gauge-field.

Gauge and field definitions To establish the missing equations, the following procedure is necessary knowing that the nabla operator allows transforming a vector equation into a scalar one or conversely. Then one multiplies by $(c_t dt)$ and integrates with respect to time. One gets to the complementary wave equations provided that the mediations below be satisfy for each mode.

$$\Box_t V_t=-\frac{1}{c_t^2}\frac{\varphi_t}{\rho};\quad \Box_l\vec{A}_l=-\frac{1}{c_l^2}\frac{\vec{\psi}_l}{\rho} \quad (12)$$

Case of the T mode: the potentials satisfy the gauge relations below.

$$\vec{\nabla}\vec{A}_t \pm \frac{1}{c_t^2}\partial_t V_t = 0 \quad (13)$$

and the corresponding vector fields must be defined by

$$\vec{E}_\pm = -\vec{\nabla}V_t \mp \partial_t\vec{A}_t; \quad \vec{B} = \vec{\nabla}\times\vec{A}_t \quad (\equiv \vec{u}_t) \quad (14)$$

with $[E_\pm] = m^2.s^{-1}$ and $[B] = m$. Thence the wave equations can otherwise write

$$\begin{cases}\Box_t\vec{A}_t &= -\vec{\nabla}\times\vec{B} \pm \frac{1}{c_t^2}\partial_t\vec{E}_\pm \\ \Box_t V_t &= -\vec{\nabla}\vec{E}_\pm\end{cases} \quad (15)$$

These are similar to the classical Maxwell equations of electrodynamics. One can recognize the electric and magnetic like field definitions; Lorentz gauge as well (sign +) in the relations (13). One can therefore establish complete analogies with the related theory. However, the complementary expressions of gauge (sign -) and field ($\vec{E}_-$) are unusual. These are relatable to elastic charge defaults of any cell. In electromagnetism, these represent the so-called *holes* in complement to free electrons define by the sign (+). Defining, the four charge-current should achieve a complete analogy between tranverse-elastic and electromagnetic fields.

Case of the L mode: the potentials must also satisfy two gauge relations expressible into

$$\vec{\nabla}V_l \pm \partial_t\vec{A}_l = \vec{0} \quad (16)$$

That is, the vector potential is a gradient and must be orthogonal to the transverse one $\vec{A}_t$. The corresponding scalar fields are defined by

$$\Gamma_\pm = \pm\vec{\nabla}\vec{A}_l + \frac{1}{c_l^2}\partial_t V_l \quad (17)$$

with $[\Gamma_\pm] = m$ such as the wave equations write

$$\begin{cases}\Box_l\vec{A}_l &= \pm\vec{\nabla}\Gamma_\pm \\ \Box_l V_l &= -\partial_t\Gamma_\pm\end{cases} \quad (18)$$

This system expresses the scalar field of the previous vector field. One can also establish adequate analogies with Maxwell theory through mass densities. These having to show that the scalar fields originate masses.

Remark: each displacement component corresponds to a gauge field describing fermions and antifermions ½ as shown in our first cited article[5]. The simultaneous existence of both components constitutes phonons assuming the field propagation or virtual bosons in field theory language.

Gauge and field invariances The gauge relations directly define conservation laws when considering them directly and using charge-current representations in the equations second members; the d'alembertian of each yielding to charge conservations. Here, we show the existence of other laws and related phenomena. For the T mode, one can distinguish two invariance orders.

Conservation laws in T mode Both gauges are invariant in the following substitutions $\vec{A}_t \mapsto \vec{A}_t + \vec{f}_{ta}(t); V_t \mapsto V_t + f_{tv}(\vec{r})$. From the definitions (14) these let $\vec{B}$ invariant. For $\vec{E}_\pm$ also be invariant, the arbitrary functions must satisfy the relation

$$\vec{\nabla}f_{tv} \pm \partial_t\vec{f}_{ta} = \vec{0} \quad (19)$$

Owing to the meaning of 4-potentials, both relations express the *areolar velocity conservation* whose associated periodicity defines the wave pulsation. Each cell then describes *locally* a central plane motion around the gradient direction. Naturally, the cell spatial position depends on time. The gradient direction of $\vec{f}_{\text{ta}}(t)$ makes this orthoganal to the initial vector potential. That is, the gauges of T fields are invariant to near a gradient potential expliciting plane motion existence. Moreover, that law implies those of *kinetic momentum* and *mechanical energy* as known.

Thermal-like radiations On the other hand, if we consider first the invariances of both vector fields (14) with the substitutions $\vec{A}_t \mapsto \vec{A}_t + \vec{\nabla}f_t$ and $V_t \mapsto V_t \mp \partial_t f_t$, provided that $\vec{A}_t.\vec{\nabla}f_t = 0$, the gauges (13) are invariant if

$$\Box_t f_t = 0 \quad (20)$$

This is interpretable as relative to the additional 4-potential $(\vec{\nabla}f_t, \mp i\,\partial_t f_t/c_t)$ such as $\vec{f}_{\text{ta}} = \vec{\nabla}f_t$ and $f_{tv} = \mp\partial_t f_t$. The existence of central motions at the cell scale suggests considering quantum solutions. One gets these from Schrödinger equation of a free particle. This is rather a fermion accordingly to 4-potential meanings. This has no mass in addition since that equation corresponds to a zero scalar gauge field according to the relations (16) and (18). In fact, many fermions having the same celerity c_t respect that equation of harmonic waves. Hence, if $E_n = \hbar\omega_n$ is the n-harmonic energy and $p_n = \hbar k_n$ is its impulse, one has $c_t = E_n/p_n$. Moreover, if **L** is a free fermion orbital angular momentum having the spin operator **S**, its global angular momentum reads **J=L+S**. Around a cell-*j*, any fermion at *n*-energy level and kinetic momentum ℓ, is describable in spherical coordinates $(r_j, \theta_j, \varphi_j)$ under both alternative forms below, taking count of both spin orientations[7].

$$\begin{pmatrix} f_{tn}^{\ell\,+}(\vec{r}_j,t) \\ f_{tn}^{\ell\,+}(\vec{r}_j,t) \end{pmatrix} = R_n^\ell(r_j)\begin{pmatrix} a_n^{\ell+}\sum_{m=-\ell-\frac{1}{2}}^{\ell-\frac{1}{2}} Y_\ell^{m+\frac{1}{2}}(\theta_j,\varphi_j) \\ a_n^{\ell-}\sum_{m=-\ell+\frac{1}{2}}^{\ell+\frac{1}{2}} Y_\ell^{m-\frac{1}{2}}(\theta_j,\varphi_j) \end{pmatrix} e^{\mp i\omega_n t} \quad (21)$$

where $a_n^{\ell+}$ and $a_n^{\ell-}$ are integration constants, R_n^l are the radial functions and $Y_l^{m\pm\frac{1}{2}}$ are the spherical harmonic functions. Note that the limit value n_{max} of n relies on the medium thermodynamic equilibrium such as one can finally write $\ell = 0, \ldots, n_{max} - 1$; the fundamental frequency $\omega_o$depending on the corresponding temperature. Hence, this situation corresponds to thermal radiations. In the simplest case, one can write $\omega_n = n\omega_o$; otherwise, the ratio ω_n/ω_o is not necessarily integral by definition.

Weak-like electronic field Note that relatively to the previous case, the substitutions $\vec{A}_t \mapsto \vec{A}_t + \vec{\nabla} f_t'$ and $V_t \mapsto V_t \pm \partial_t f_t'$ lead to another result of gauge invariance. One obtains instead the local equation

$$\bar{\Box}_t f_t' = 0 \text{ with } \bar{\Box}_t = \Delta + \frac{\partial^2}{c_t^2\, \partial t^2} \quad (22)$$

whose quantized solutions are similar to the preceding for imaginary time (i.e. $t \mapsto \mp it$). Both solutions being simultanously possible, one has to interpret these as related to transverse fermions endowed with local and long range fields. This is certainly the case of cells free electrons; these expressing the weak fundamental field. Thus, it seems obvious that the T mode definitions of cells are finally invariant to near electronic fields.

Conservation laws in L mode Both previous invariance orders are also considerable. The two gauges are invariant in the following substitutions $\vec{A}_l \mapsto \vec{A}_l + \vec{f}_{la}(\vec{r})$; $V_l \mapsto V_l + f_{lv}(t)$. In addition, the scalar fields (17) are invariant if the arbitrary functions respect the relations

$$\vec{\nabla} \vec{f}_{la} \pm \frac{1}{c_l^2} \partial_t f_{lv} = 0 \quad (23)$$

Owing to the meaning of 4-potentials, this would express a length conservation. Mutiplying however this equation by c_l shows the conservation of areolar velocity projection in the gradient direction. Since space and time are dependent parameters as before, this allows connecting those arbitrary functions to the preceding ones (19) by taking the gradient of that equation. One obtains then $f_{tv} \propto c_l \vec{\nabla} \vec{f}_{la}$ and $\vec{f}_{ta} \propto \vec{\nabla} f_{lv}/c_l$. That is, both invariance orders express the same conservation laws in the plane perpendicular to the radial direction.

Other thermal-like radiations On the other side, if the scalar fields (17) are first considered invariant for the substitutions $\vec{A}_l \mapsto \vec{A}_l \mp \partial_t \vec{f}_l$ and $V_l \mapsto V_l + c_l^2 \vec{\nabla} \vec{f}_l$, provided that $\vec{A}_l . \partial_t \vec{f}_l = 0$, the gauge invariances yield

$$c_l^2 \vec{\nabla}(\vec{\nabla} \vec{f}_l) - \partial_t^2 \vec{f}_l = \vec{0} \quad (24)$$

Considering in addition the *invariance of the displacement* $\vec{u}_l$ (6), one must have $\vec{\nabla}(\vec{\nabla} \vec{f}_l) = \vec{0}$, i.e. $\partial_t^2 \vec{f}_l = \vec{0}$. This implies $\partial_t \vec{f}_l = \vec{f}_{la}(\vec{r})$. From those substitutions, one can now define new T vector fields from (14). One finds instead static fields given by

$$\vec{e}_\pm = \vec{0};\ \vec{b}_\mp = \mp \vec{\nabla} \times \vec{f}_{la}(\vec{r}) \quad (25)$$

Consequently, one deducts that the L mode definitions are invariant to near magnetic-like static fields. There is no electric like field due to that last invariance. This characterizes the perfect media. If the previous radiations are dependent of electrons, these should be due to other fermions executing longitudinal motions. In any cell ,these can only be the free electron complements, i.e. linked electrons of the crystalline structure. There exist then two kinds of such structures in this case.

Nuclear-like crystalline field As before, the choice of substitutions $\vec{A}_l \mapsto \vec{A}_l \pm \partial_t \vec{f}_l'$ and $V_l \mapsto V_l + c_l^2 \vec{\nabla} \vec{f}_l'$ lets invariant the gauges for local fields given by the relations

$$c_l^2 \vec{\nabla}(\vec{\nabla} \vec{f}_l') + \partial_t^2 \vec{f}_l' = \vec{0} \quad (26)$$

The *invariance of* $\vec{u}_l$ yields instead the equivalent magnetic-like static fields given by $\vec{b}_\pm' = \pm \vec{\nabla} \times \vec{f}_{la}'(\vec{r})$. These correspond then to two other kinds of crystal. The subsection below indicates how the different systems can be identified.

Characterization of cubic systems Note that the different equations describing the free electrons and crystals rely on the signs relatively to the gauges. Hence, it is necessary to examine the four gauge couplings originating these. We determined the different crystalline structures by considering the crystal frontiers such as *each cell mode respects both gauges of the related coupling*. Table-1 summarizes the results. The signs of both first columns identify the corresponding gauges; the first represents a scalar field gauge and the second a vector field gauge. The equations are similar to those given here (see also our second article[6]). One identified the structures with respect to their scalar field potentiallities; their electric-like fields as well. The sign (-) in Γ means that the crystal possesses at least one ion inside. That is the case of *bcc* and *cc* systems (for complex cubic). The last one includes all other kinds of crystals. The non-zero $\vec{E}$ means that the crystal possesses at least one ion on each face. This is the case of *fcc* and *cc* systems. The zero $\vec{B}$ means the non-existence of rotating charges on crystals six faces.

One better understands here the signs of gauge relations with respect to free electrons: *cP* system has both electron kinds on its faces contrarily to *bcc*; *fcc* system only has vector electrons while *cc* only has scalar electrons. Both first systems describe crystalline fields of long range while the two lasts describe crystalline nuclear fields. Note that the magnetism sign corresponds to the opposite sign of the scalar gauge as seen above. Two kinds of magnetism appear in cubic crystals. Both have opposite field orientations corresponding certainlty to both spin-1/2 orientations. By definition, $\vec{b}_+$ and $\vec{b}_+'$define the magnetism while $\vec{b}_-$ and $\vec{b}_-'$ define the antimagnetism by opposition.

Table-1: Gauge couplings and fundamental expressions at crystal interfaces (x=*l, t*)

Gauge coupling	Interface equation	Γ	$\vec{E}$	$\vec{B}$	System	Magnetism	Examples
$(+,+)$	$\square_t \lvert A_x^1\rangle = 0$	$\alpha_- \vec{\nabla}\vec{A}_x^1$	$\vec{0}$	$\vec{0}$	cP	$\vec{b}_-$	α-Po
$(-,-)$	$\square_t \lvert A_x^2\rangle = 0$	$-\alpha_- \vec{\nabla}\vec{A}_x^2$	$\vec{0}$	$\vec{0}$	bcc	$\vec{b}_+$	α-Fe, CsCl
$(-,+)$	$\bar{\square}_t \lvert A_x^3\rangle = 0$	$\alpha_+ \vec{\nabla}\vec{A}_x^3$	$-2\vec{\nabla}V_x$	$\vec{0}$	fcc	$\vec{b}'_+$	Al, Cu, Ni
$(+,-)$	$\bar{\square}_t \lvert A_x^4\rangle = 0$	$-\alpha_+ \vec{\nabla}\vec{A}_x^4$	$-2\vec{\nabla}V_x$	$\vec{0}$	cc	$\vec{b}'_-$	C(d), NaCl

With $\alpha_\pm = 1 \pm c_t^2/c_l^2$

Results and discussion

It appeared that the transverse elastic field description is comparable to that of electromagnetic field. A simple substitutions of suantities do for that; the celerity c_t becoming that of electromagnetic waves. However, the longitudinal mode has no equivalence. We showed from invariances that the cell fields are defined to near those of lattice free electrons and crystals linked electrons. These invariances express besides the conservations of energy and kinetic momentum of both kinds. Each crystal structure then behaves as a unique fermion-1/2 in each space-time direction defined by the L mode; the free electrons being defined by the T mode. One can then expect to have at the most four links between crystalline structures in ordinary espace-time. This characterizes then the metallic link in perfect media. This findings are very relevant to the best of our knowledge.

Moreover, it appeared that in cells, the free electrons are responsible of spin waves, which then explain the thermal radiations in electromagnetism. One already knows, from quantum theory, the existence of spin waves and their applications[8,9]. Their obtainment here illustrates this theory inclusion. In addition, cubic crystals can express four kinds of static magnetic fields whose origins are quantum due to invariance meanings (see Table 1 results). The known theory explains this through magnetic moments *in substances*[3,10]. Here, these are understandable from fermion fields. The absence of electric field fits with the known result in cubic crystals. However, when considering the non-invariance of the L displacement from relations (24) and (26), one obtains other kinds of magnetisms and electricity existence proper to non-perfect media. These should correspond to isotropic and inhomogeneous media accordingly to the theory; otherwise, the theory should be translated for each case of anisotropic medium.

Considering Table 1 solutions, the basic configurations surely originate the mechanical and electric properties of crystals, e.g. when crossed by an electric current. This is certainly the case of *fcc* system defining good conducting metals. The general solution of each crystalline field should however include those of less complex systems because of the number of lattice knots. That is, *bcc* system can also express *cP* field, *fcc* system can express *cP* field too and *cc* system can express all other fields; depending on a particular system. This also applies on magnetism. These findings are rather germane and unify phenomena of elasticity and electromagnetism at quantum scale.

Conclusion

To solve the motion equation for potential inertial forces, we defined the displacement vector from one Helmholtz theorem; this letting to introduce two kinds of 4-potentials. Then, we determined two gauges and two fields expressions for each through simple procedures. The vector fields are similar to those of Maxwell theory. This allowed establishing a complete analogy between transverse elasticity and electromagnetism. The scalar field, which independently exists but steadily linked to the previous, has no equivalence and this complets that classical theory. The consideration of gauge and field invariances in different ways showed the existence of special phenomena.

We found that: i. the free electrons of cells originate longitudinal spin waves which originate thermal radiations. ii. Each cubic structure is definable by a static magnetic field. This is either positive or negative as expected. Hence, the theory suits for explaining the different kinds of magnetism including non-perfect media to a great extend. iii. There are four kinds of cubic crystals accordingly to the number of possible gauge couples. iv. Each cubic system is characterized by particular expressions of transverse fields at interfaces in monocrystal lattices. This should help understanding, for instance, what happens with electrons in conducting materials. These are our findings.

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