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Resonance in Lattice Displacement in Magnetized Nondegenerate Semiconductor Plasmas

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Abstract

The Phenomenon of parametric interaction of coupled wave exhibits a vital role in nonlinear optics. Using the straight forward coupled mode theory, the lattice displacement plays the great role in the parametric amplification is analytically investigated in magnetized piezoelectric as well as non piezoelectric semiconductors. The origin of nonlinear interaction is taken to be in the second order optical susceptibility x^2 arising from the nonlinear induced current density and polarization through lattice displacement. The lattice displacement (u), effective non linear polarization (P_{EN}) and efficiency of crystal cell (β_o) are obtained for different situation of practical interest. The analytical investigations reveals that the large lattice displacement (order of 10^{-14}m) can be easily achieved in piezoelectric or both coupling and deformation potential coupling at scattering angle nearly $\theta = 34^0$ and $\theta = 146^0$ and $\theta = 36^0$ and $\theta = 148^0$ respectively. This typical resonance condition of scattering angle may be used to achieve high efficient nonlinear process. It is also found that wave number enhance the lattice displacement effectively. This study provides new means for construction of crystal cell and for diagnostics of semiconductors devices.

Keywords: Parametric Interaction, Lattice Displacement, Semiconductor Plasma.

Introduction

There are number of basic phenomena of nonlinear nature, out of which parametric interaction has an important place in nonlinear optics, especially in generating tunable laser light at a frequency not directly available from a laser source[1]. The parametric interactions in a nonlinear medium are responsible for the construction of parametric oscillators, amplifiers, optical phase conjugators, etc[2,3]. It is known that the origin of parametric interaction lies in the second order optical susceptibility of the medium through nonlinear induced current density or polarization.

It is observed that the crystalline nonlinear media offer the greatest device potential. This is because x^2 is nonzero for noncentrosymmetric crystals and secondly the birefringence of a crystalline medium could be used to phase match velocities of fundamental and harmonic radiations by compensating the material dispersion. However for nonlinear optical applications, nonlinear crystals should satisfy four basic criteria, namely adequate nonlinearity, optical transparency, proper birefringence for phase matching and sufficient resistance to optical damage by intense optical irradiation[4,5]. The doped semiconductors are found to be transparent to photons of energy bellow their energy band gap and there by prove to be advantages hosts. The properties of nonlinear material are better understood when discussed with reference to nonlinear devices and the theory of nonlinear interactions[6-8]. The parametric interaction of acoustic waves with microwave electric field in piezoelectric semiconductors was discovered by Economic and Sector[9]. The importance of the effect of a dc magnetic field on parametric action was very well emphasized by Cohen[10].

Motivated by the intense interest in the field of study of parametric interaction based on x^2. In the present paper the authors have made an attempt to investigate parametric amplification process originating from x^2 through lattice displacement, nonlinear current density or polarization, in a nondegenerate n-InSb crystal of noncentrosymmetric nature when a magneto static field is applied perpendicular to the direction of pump wave propagation. Here in the present paper we will confine to the study of lattice displacement (u), effective nonlinear polarization (P_{EN}), threshold field (E_{oth}) and crystal cell efficiency (β_0) in the presence of a magnetostatistic field and piezoelectric- deformation potential couplings.

Theoretical formulations: We consider the hydrodynamic model of homogeneous, nondegenerate n-type semiconductor plasma having both piezoelectric as well as deformation potential couplings and the medium is of infinite extent with electrons as carriers. This model restricts the validity of the analysis to the limit $kl << 1$, where k is the wave number an l is the mean free path of the electrons. In order to study parametric interaction processes originating from the effective nonlinear optical susceptibility (x_{EN}) the medium is subjected to the magnetic field B_0 (along z-axis) perpendicular to the propagation direction (x-axis) of spatially uniform high frequency pump electric field $E_0 \exp(-i\omega t)$. The scattered waves are propagating along a direction making an arbitrary angle θ with the pump wave propagation direction, i.e. propagating in x-z plane making an angle θ with x-axis. Thus θ is the scattering angle, i.e. the angle between k_0 and k_1. We apply the coupled mode theory to obtain a simplified expression for the acoustic waves via density perturbation.

The basic equations used are as follows:

$$\frac{\partial \mathbf{v}_o}{\partial t} + \upsilon \mathbf{v}_o = -\frac{e}{m}[\mathbf{E}_o + \mathbf{v}_o \times \mathbf{B}_o], \qquad (1)$$

$$\frac{\partial \mathbf{v}_1}{\partial t} + \upsilon \mathbf{v}_1 + (\mathbf{v}_o \frac{\partial}{\partial x})\mathbf{v}_1 = -\frac{e}{m}[\mathbf{E}_1 + \mathbf{v}_1 X \mathbf{B}_o] \;, \qquad (2)$$

$$\mathbf{v}_o \frac{\partial n_1}{\partial x} + n_o \frac{\partial \mathbf{v}_1}{\partial x} = -\frac{\partial n_1}{\partial t} \quad , \qquad (3)$$

$$\frac{\partial \mathbf{E}_s}{\partial x} + \frac{\beta}{\varepsilon}\frac{\partial^2 \mathbf{u}}{\partial x^2} - \frac{C_d}{e}\frac{\partial^3 \mathbf{u}}{\partial x^3} = -\frac{n_1 e}{\varepsilon}, \qquad (4)$$

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} + 2\gamma_s \rho \frac{\partial \mathbf{u}}{\partial t} + \beta \frac{\partial \mathbf{E}_s}{\partial x} + \frac{C_d \varepsilon}{e}\frac{\partial^2 \mathbf{E}_s}{\partial x^2} = C\frac{\partial^2 \mathbf{u}}{\partial x^2}, \qquad (5)$$

Equations (1) and (2) represent the zeroth and first-order momentum transfer equations, respectively, in which $\boldsymbol{v}_o$ and $\boldsymbol{v}_1$ are the zeroth and first order oscillatory fluid velocities having effective mass m and charge - *e* and *v* is the phenomenological electron collision frequency. Equation (3) represents the continuity equation for electrons, where n_o and n_1 are the equilibrium and perturbed electron densities, respectively. The Poisson equation (4) gives the space charge field E_s in which the second and the third terms on the left hand side give the piezoelectric and deformation potential contribution to polarization, respectively. ε, β and C_d are the scalar dielectric, piezoelectric and deformation potential constants of the semiconductor, respectively. Equation (5) describes the motion of the lattice in a crystal having piezoelectric and deformation potential couplings both. In this equation ρ, u, γ_s and C being the mass density of the crystal, displacement of the lattice, phenomenological damping parameter of acoustic mode and crystal elastic constant, respectively. In (2) we have neglected the effect due to v_o x B_1 by assuming that the shear acoustic wave is propagating along such a direction of the crystal that it produces a longitudinal electric field, e.g. in n-InSb, if k is taken along (011) and the lattice displacement **u** is along (100) the electric field induced by the wave is a longitudinal field[11].

In a highly doped semiconductor the low frequency acoustic wave (ω_s) as well as the pump electromagnetic wave (ω_o) produce density perturbations (n_1) at the respective frequencies in the medium which can be obtained by using the standard approach[12]. Considering the low frequency perturbations (n_s) to be proportional to exp[$i(k_s x - \omega_s t)$], while v_o varies as $\exp(-i\omega_o t)$ and neglecting the Doppler shift under the assumption the $\omega_o \rangle\rangle \upsilon \rangle \mathbf{k}\mathbf{v}_o$, we get from equations (1) to (4) as follows:

$$\frac{\partial^2 n_1}{\partial t^2} + \upsilon \frac{\partial n_1}{\partial t} + n_1 \overline{\omega_p}^2 + \frac{n_o e \beta}{m\varepsilon}\frac{\partial^2 \mathbf{u}}{\partial x^2} - \frac{n_o C_d}{m}\frac{\partial^3 \mathbf{u}}{\partial x^3} = -\overline{\mathbf{E}}\frac{\partial n_1}{\partial x}, \qquad (6)$$

where $\overline{\mathbf{E}} = -\left[(\frac{e}{m})\mathbf{E}_o + \omega_c v_{oy}\right]$, and $\overline{\omega_p}^2 = \frac{\omega_p^2 \upsilon^2}{(\upsilon^2 + \omega_c^2)}$, in which $\omega_p^2 = \frac{n_o e^2}{m\varepsilon}$ is the electron plasma, υ being the electron collision and $\omega_c = (\frac{eB_o}{m})$ being the cyclotron frequencies, respectively.

The density perturbations associated with the phonon mode (viz, n_s) and the scattered electromagnetic waves (n_f) arising due to the three wave parametric interaction will propagate at the generated frequencies ω_s and $\omega_o \pm \omega_s$ respectively. For these modes the phase matching condition $\omega_o = \omega_1 + \omega_s$ and $k_o = k_1 + k_2$, i.e. the energy and momentum conservation relations should be satisfied. Now since θ is the angle between k_1 and k_0, thus in writing the conservation equations we have assumed $k_{1y} = 0$, i.e. the scattered wave to propagate in the x- z plane. It must be mentioned here that these conservation equations could be satisfied over a wide range of scattering angle. Now for spatially uniform laser irradiation $| k_0 | \approx 0$ and one obtains $| k_1 | = | k_s | = | k |$ (say). On resolving equation (6) into two components (fast and slow) by denoting $v = v_f + v_s$ and $n = n_f + n_s$ under rotating wave approximation (RWA) One obtains:

$$\frac{\partial^2 n_f}{\partial t^2} + \upsilon \frac{\partial n_f}{\partial t} + \overline{\omega_p}^2 n_f = -\overline{E}\frac{\partial n_s^*}{\partial x} \qquad (7a)$$

and

$$\frac{\partial^2 n_s}{\partial t^2} + \upsilon \frac{\partial n_s}{\partial t} + \overline{\omega_p}^2 n_s + \frac{n_o e \beta}{m\varepsilon}\frac{\partial^3 \mathbf{u}}{\partial x^3} = -\overline{E}\frac{\partial n_f^*}{\partial x}. \qquad (7b)$$

In the above analysis we have restricted ourselves only to the Stokes component $(\omega_o - \omega_s)$ of the scattered electromagnetic waves. One can easily infer from equation (7) that the slow and fast components of the density perturbations are coupled to each other via the pump electric field.

Thus the presence of the pump electric field is the fundamental necessity for the parametric interaction to occur. From equations (5), (7a) and (7b) one obtains the expression for n_s as:

$$n_s = \frac{ien_o k_x^3 E_s(\beta^2 + \frac{C_d^2 \varepsilon^2 k_x^2}{e^2})}{m\rho\varepsilon(\omega_s^2 - k_x^2 v_s^2 + 2i\gamma_s\omega_s)} \cdot \left[(\overline{\omega_p}^2 - \omega_s^2) - i\omega_s\upsilon - \frac{k_x^2 |\overline{E}|^2}{(\overline{\omega_p}^2 - \omega_1^2 + i\omega_1\upsilon)}\right]^{-1}, \qquad (8)$$

where k_x= k cosθ and $v_s^2 = (\frac{C}{\rho})$; v_s being the velocity of the acoustic wave. In the present report in order to study the effect of nonlinear current density on the induced polarization in a magnetized highly doped semiconductor, the effect of the transition dipole moment is neglected while analyzing parametric interaction in the crystal. It is evident from the above expression that n_s depends upon the various powers of pump intensity, $I=\frac{1}{2}\eta\varepsilon_o c_o\overline{|E_o|}^2$; η and c_0 being the background refractive index of the crystal and the velocity of light in vacuum, respectively. This produced density perturbation, thus affecting the propagation characteristics of the scattered waves, which can be studied by employing the electromagnetic wave equation:

$$\nabla\times\nabla\times\mathbf{E}_1=\frac{-1}{C_L^2}\frac{\partial^2\mathbf{E}_1}{\partial t^2}-\mu_o\frac{\partial\mathbf{J}_1}{\partial t}, \tag{9}$$

where $C_L = (\mu_o\varepsilon_o\varepsilon_L)^{-0.5}$ is the velocity of light in the medium and $\mathbf{J}_1$ is the perturbed current density and $\varepsilon_L=\frac{\varepsilon}{\varepsilon_o}$.

The Stokes component of the induced current density is given by

$$J_1 = -\, n_s^{*} e\,\mathbf{v}_o \tag{10a}$$

Using equations (8) and (10a) one gets:

$$\mathbf{J}_1=\frac{-iek_x^3 v_s^2\varepsilon\omega_p^2\omega_o E_s^{*}E_o(K^2+F^2k_x^2)}{2m\gamma_s\omega_s(\omega_c^2-\omega_o^2)}\left[\delta_i^2+i\omega_s\upsilon-\frac{k_x^2\left|\overline{E}\right|^2}{(\delta_2^2-i\omega_1\upsilon)}\right]^{-1} \tag{10b}$$

where $K^2=\frac{\beta^2}{\varepsilon C}$, $F^2=\frac{C_d^2\varepsilon}{e^2C}$, $\delta_1^2=\overline{\omega_p}^2-\omega_o^2$, $\delta_2^2=\overline{\omega_p}^2-\omega_1^2$ and $\omega_1=\omega_o-\omega_s$. In deriving equation (10) we have used the expression for the components of v_o (along x - and y-directions) which is the oscillatory electron fluid velocity in the presence of the pump and the magneto static fields. Using eq. (1), these expressions are obtained as:

$$v_{ox}=\frac{\overline{E}}{(\upsilon-i\omega_o)} \quad\text{and}\quad v_{oy}=-\frac{e}{m}\frac{\omega_c E_o}{(\omega_c^2-\omega_o^2)} \tag{11}$$

Lattice Displacement (u): The lattice displacement (**u**) in the coupled mode scheme obtained from equation. (5) as:

$$\mathbf{u}=\frac{ik_xE_s(\beta+\frac{iC_d\varepsilon k_x}{e})}{\rho\left[\omega_s^2-k_x^2v_s^2+2i\gamma_s\omega_s\right]} \tag{12a}$$

Or

$$|\mathbf{u}|=\frac{k_xE_s}{\rho}[\beta^2+F^2k_x^2]^{0.5}A \tag{12b}$$

Where $A=[(\omega_s^2-k_x^2v_s^2)^2+4\gamma_s^2\omega_s^2]^{-0.5}$

The different aspect of $|\mathbf{u}|$ for different situations for practical interest are obtained as :

For piezoelectric coupling ($\beta\neq 0, C_d=0$) $|\mathbf{u}|_p=\frac{\beta k_sE_s}{\rho}.A$ (13a)

For deformation potential coupling ($\beta=0, C_d\neq 0$)

$$|\mathbf{u}|_d=\frac{Fk_x^2E_s}{\rho}.A \tag{13b}$$

For both couplings

$$(\beta\neq 0, C_d\neq 0)\ |\mathbf{u}|_b=\frac{k_xE_s}{\rho}[\beta^2+F^2k_x^2]^{0.5}.A \tag{13c}$$

Effective nonlinear polarization and efficiency of crystal cell

It is well known that the induced polarization $\mathbf{P}_1$ as the time integral of the current density $\mathbf{J}_1$, one may write:

$$\mathbf{P}_1=\int\mathbf{J}_1dt \tag{14}$$

The effective nonlinear induced polarization $\mathbf{P}_{EN}(=-\mathbf{J}_1/i\omega_1)$ is obtained from equations (10) and (14) as:

$$\mathbf{P}_{EN}=\frac{e\varepsilon k_x^3v_s^2\omega_p^2\omega_o(K^2+F^2k_x^2)E_s^{*}E_o}{2m\gamma_s\omega_s\omega_1(\omega_c^2-\omega_o^2)}.\frac{[\overline{\omega_p}^4\omega_1^4\upsilon^4+(\omega_s\omega_1^2\upsilon^3-\omega_1\upsilon\overline{E}^2k_x^2]^{0.5}}{[(\omega_1\omega_s\upsilon^2-k_x^2\overline{E}^2)^2+\overline{\omega_p}^4\omega_1^2\upsilon^2]} \tag{15}$$

Using the expression for effective induced polarization deduced for an infinite medium, one can calculate the electric field amplitude (E_T) in a crystal cell of length L, with the assumption that the sample length is of magnitude about two order than the pump wavelength,

$$E_T=-\frac{ik_xL}{\varepsilon}|\mathbf{P}_{EN}| \tag{16}$$

Now equation. (16) can be employed to determine the transmitted intensity (I_T) as,

$$I_T=\frac{1}{2}\eta\varepsilon_oc_o|E_T|^2 \tag{17}$$

The efficiency of the crystal cell (β_o) is given by

$$\beta_o=\frac{I_T}{I_{in}} \tag{18}$$

Using equations (17) and (18) one gets,

$$\beta_o=[\frac{k_xL}{\varepsilon E_o}]^2\mathbf{P}_{EN} \tag{19}$$

Equations (13), (15) and (19) may be used to study the lattice displacement, effective polarization and efficiency of the crystal cell made of noncentrosymmetric, respectively, as a function of magnetic field, scattering angle, couplings constant, carrier concentration and wave number.

Results and Discussion

For numerical appreciation, this analysis is applied to a specific case of a noncentrosymmetric crystal, which is assumed to be irradiated by a 10.6 μ m CO_2 laser. The material constants are taken as: $E_s = 10^5 V/m$, $\beta = 0.054 cm^{-2}$, $c_d = 4.5\times1.6\times10^{-19} J$, $\rho = 5.8\times10^3 kg.m^{-3}$, $\gamma_s = 5\times10^{-10} Fm^{-1}$, $\nu = 3\times10^{11} s^{-1}$, $\varepsilon = 15.8\times8.85\times10^{-12}$, $\varepsilon_o = 8.85\times10^{-12}$, $e = 1.6\times10^{-19} C$, $v_s = 4\times10^3 m.s^{-1}$, $\omega_0 = 1.78\times10^{14} s^{-1}$, $\omega_s = 2\times10^{11} s^{-1}$, $k = 5\times10^5 m^{-1}$, and $\varepsilon_1 = 15.8$.

In the Present analytical investigation, the lattice displacement, effective nonlinear induced polarization and the efficiency of the crystal cell are deduced in a heavily doped magnetized noncentrosymmetric semiconductor crystal. We now focus our attention on the factors, that affects the lattice displacement (u_b -both coupling, u_p - piezoelectric coupling and u_d - deformation potential coupling) in different coupling modes.

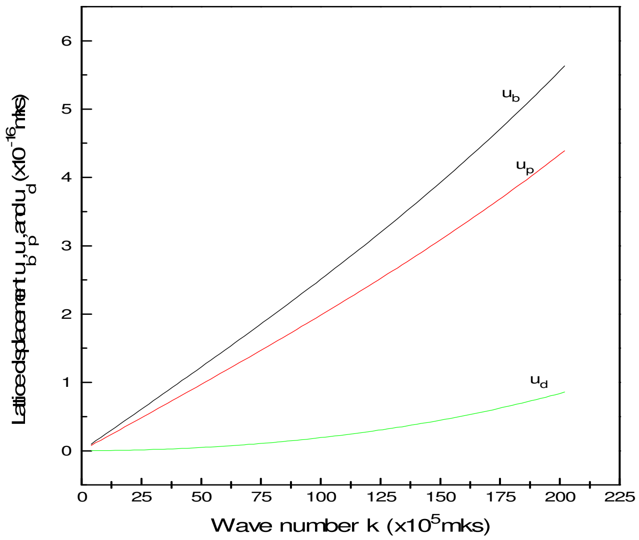

Figure-1
Variation of lattice displacement (u_b-Both coupling, u_p - piezoelectric coupling and u_d- deformation potential coupling) with wave number k at $\theta = 34^0$

It is found that lattice displacement increases with the wave number (k) as shown in figure 1 and this is quite obvious as the certain value of input wave number, maximum (resonance state) displacement will be at the scattering angle θ = 34 and 146 degree. The lattice displacement increases linearly with wave number in piezoelectric coupling only, while in deformation potential coupling and both the couplings, displacement increases gradually, but on the higher values of wave number (k= $2\times10^7 m^{-1}$) it attains maximum values as $u_b = 5.6\times10^{-16} m$, $u_p = 4.3\times10^{-16} m$ and $u_d = 0.839\times10^{-16} m$. It is also shows that piezoelectric coupling has more effect in compare to deformation coupling.

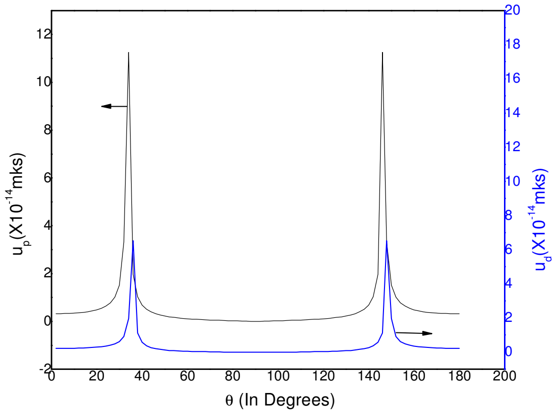

Figure-2
Variation of lattice displacement (u_p - piezoelectric coupling and u_d - deformation potential coupling) with scattering angle θ at k = $2\times10^5 m^{-1}$

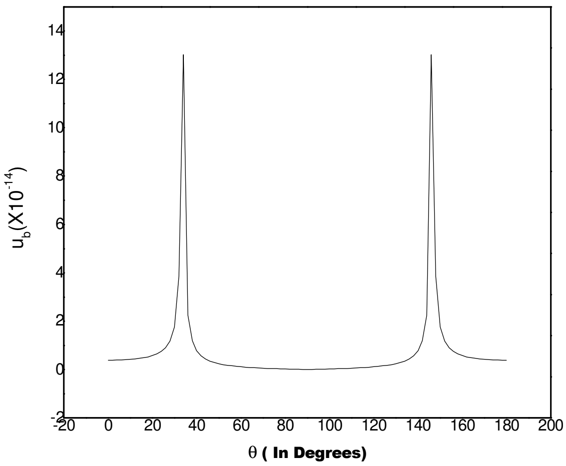

Figure-3
Variation of lattice displacement (u_b - both couplings) with scattering angle θ at k = $2\times10^5 m^{-1}$

Figures 2 and 3 exhibit, the variation of lattice displacement u_b, u_p and u_d with the scattering angle (θ) at constant wave number k = 1 $2\times10^5 m^{-1}$. It is obtained that u_b, u_p *and*, u_d increases sharply and attains their maximum value of about $u_b = 13.06\times10^{-14} m$, $u_p = 11.3\times10^{-14} m$ and $u_d = 6.52\times10^{-14} m$ at the scattering angle about θ = 34 or 146, 34 or 146, and 36 or 148 degree respectively. It is also inferred that in the scattering angle range ($\theta = 60-120$) and less than nearly 30^0 and more than nearly 144^0, the lattice displacement remains at minimum value in different couplings. Hence at the $\theta = 34^0$ and $\theta = 146^0$ for piezoelectric and both coupling and $\theta = 36^0$ and $\theta = 148^0$ for deformation coupling, the lattice displacement gets its maximum value, which gives the efficient polarization and other related parameters. This typical resonance condition of scattering angle may be used to achieve high efficient nonlinear process in magnetized semiconductor plasma.

One can be easily observed from equation. (15) that effective non linear induced polarization varies with carrier concentration of the medium via plasma frequency (ω_p), wave number, scattering angle and with magnetic field through cyclotron frequency (ω_c) and different coupling constants. By using material constants and adjusting depending parameters one can set the required condition which is useful in the fabrication of nonlinear devices.

A close look at equations (15) and (19) that efficiency of crystal cell is also a function of both the couplings, wave number, carrier concentration, external magnetic field, scattering angle and the length of the crystal cell. It is also observed that efficiency of crystal cell is strongly depends on the input pump intensity and magnetic field. Hence, in order to achieve maximum transited intensity and largest efficiency, it is always better to used higher pump intensities and dc magnetic field.

Conclusion

The above discussion reveals that the large lattice displacement, effective nonlinear polarization and efficiency of cell can be easily achieved in magnetized non Centro symmetric semiconductor crystal having both the piezoelectric and deformation potential couplings. The present theoretical study provides a model most appropriate for the finite laboratory solid state plasma and an experimental study based on this work would provide new means for construction of crystal cell and for characterization and diagnostics of semiconductors.

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