

# Ion Beam with General loss-cone Distribution Function on EMIC Waves in Multi-ions Magnetospheric Plasma

Ahirwar G.

School of Studies in Physics, Vikram University Ujjain, MP-456 010, INDIA

Available online at: [www.isca.in](http://www.isca.in)

Received 24<sup>th</sup> October 2012, revised 26<sup>th</sup> October 2012, accepted 29<sup>th</sup> October 2012

## Abstract

Electromagnetic ion-cyclotron (EMIC) waves have been studied by single particle approach. The effect of ion beam on EMIC instability in multi-ions is evaluated. The dispersion relation, growth rate of the electromagnetic ion-cyclotron waves in a low  $\beta$  (ratio of plasma pressure to magnetic pressure), homogeneous plasma have been obtained. The wave is assumed to propagate parallel to the static magnetic field. The effect of ion beam on EMIC waves in multi-ions is to enhance the growth rate of EMIC waves. The results are interpreted for the space plasma parameters appropriate to the auroral acceleration region of the earth's magnetospheric plasma.

**Key words:** Electromagnetic ion-cyclotron waves, Auroral acceleration region, solar plasma, ion beam, general loss-cone distribution function.

## Introduction

Electromagnetic ion cyclotron (EMIC) waves are considered to be one of the most important loss mechanisms. The theoretical studies of ion heating and acceleration perpendicular to the magnetic field are a common feature in the auroral region. In the present study, the beam effect in multi-ions  $H^+$  (hydrogen),  $He^+$  (helium),  $O^+$  (oxygen) has been studied for electromagnetic ion cyclotron waves<sup>1</sup>. The beam plasma interactions are present in several space environments and in laboratory plasma. These interactions are a non thermal feature that can trigger plasma instabilities<sup>2</sup>. We consider charge neutral plasma with a uniform magnetic field  $\hat{z}B$  and no ambient electric field. We also consider a two species, three component plasma, denoting the thermal ion component by the subscript ( $H^+$ ,  $He^+$  and  $O^+$ ) for hydrogen, helium and oxygen ions respectively also the subscripts  $\parallel$  and  $\perp$  denote parallel and perpendicular to magnetic field  $B^3$ . In past explain the  $O^+$  and  $H^+$  electromagnetic ion cyclotron waves excited by the energetic electron beam might explain the low frequency electric and magnetic field noise (100 Hz) in the auroral zone. Ionosphere electrons trapped or reflected by EMIC waves are accelerated to energies of several keV as described<sup>4,5</sup>. In areas of upward current, large amplitude electromagnetic ion-cyclotron waves with frequencies within 5% of the local proton gyro frequency  $\Omega_p$  and its harmonics are often observed where an up streaming ion beams exist<sup>6</sup>. The main aim of this study is to investigate the generation of EMIC waves in the magnetosphere and see the effect of beam in magnetospheric plasma. The parallel magnetic field plays a vital role and explains the observational details of EMIC waves. In the present work we have the effect of beam velocity on EMIC instability in magnetospheric plasma. The detailed description and formulae for the dispersion relation and growth rate is determined in the next section.

## Methodology

**Dispersion Relation:** We consider the cold plasma dispersion relation for the EMIC wave for multi-component plasma<sup>1</sup> as:

$$\frac{c^2 k_{\parallel}^2}{\omega^2} = \left( \frac{\omega_{pH^+}^2}{\Omega_{H^+}^2} \right) \left( 1 - \frac{\omega}{\Omega_{H^+}} \right)^{-1} + \left( \frac{\omega_{pHe^+}^2}{\Omega_{He^+}^2} \right) \left( 1 - \frac{\omega}{\Omega_{He^+}} \right)^{-1} + \left( \frac{\omega_{pO^+}^2}{\Omega_{O^+}^2} \right) \left( 1 - \frac{\omega}{\Omega_{O^+}} \right)^{-1} \quad (1)$$

Where  $\omega_{p\alpha}^2 = \frac{4\pi N_{\alpha 0} e^2}{m_{\alpha}}$  is the plasma frequency for the ions.

$N_{\alpha 0}$  is the plasma density of particles.

**Wave Energy:** The wave energy density  $W_w$  per unit wavelength is the sum of the pure field energy and the changes in the energy of the non-resonant particles i.e. the total energy per unit wavelength is given as

$$W_w = U + W_{\alpha} \quad (2)$$

Where  $U$  is the energy of electromagnetic wave as defined by the expression as,

$$U = \left( \frac{1}{16\pi} \right) \left[ \left( \frac{d}{d\omega} \right) (\omega \mathcal{E}_k) E_1^* E_k + |B|^2 \right] \quad (3)$$

Where  $\mathcal{E}_k$  is the dielectric tensor? After the calculation, the electromagnetic wave energy per unit wavelength is given by

$$U = \left( \frac{\lambda B^2}{8\pi} \right) \left[ \frac{(2\Omega_{H^+} - \omega)}{(\Omega_{H^+} - \omega)} + \frac{(2\Omega_{He^+} - \omega)}{(\Omega_{He^+} - \omega)} + \frac{(2\Omega_{O^+} - \omega)}{(\Omega_{O^+} - \omega)} \right] \quad (4)$$

$$\text{and } W_{\alpha} = \int_0^{\lambda} dz \int_0^{2\pi} d\psi \int_0^{\infty} V_{\perp\alpha} dV_{\perp\alpha} P \int_{-\infty}^{\infty} dV_{\parallel\alpha} \frac{m}{2} [(N + n_1)(V_{\alpha} + \delta V_{\alpha})^2] \quad (5)$$

The perpendicular (transverse) energy and the parallel resonant energy of the resonant ions are calculated with  $\mathcal{E} = 1$  as

### Perpendicular Resonant Energy

$$W_{r\perp\alpha} = \frac{\pi^{3/2} B^2}{C^2 K_{\Pi}^2 \omega k_{\Pi}} \left[ \frac{\omega_{pH^+}^2 \Omega_{H^+}^2}{V_{T\Pi c_{H^+}}} \left\{ (J+1) \frac{T_{\perp H^+}}{T_{\Pi H^+}} \left( \frac{\omega' - \Omega_{H^+}}{\Omega_{H^+}} \right) + 1 \right\} \exp \left[ -\frac{(\omega' - \Omega_{H^+})^2}{k_{\Pi}^2 V_{T\Pi c_{H^+}}^2} \right] + \right. \\ \left. \frac{\omega_{p_{He^+}}^2 \Omega_{He^+}^2}{V_{T\Pi c_{He^+}}} \left\{ (J+1) \frac{T_{\perp_{He^+}}}{T_{\Pi_{He^+}}} \left( \frac{\omega' - \Omega_{He^+}}{\Omega_{He^+}} \right) + 1 \right\} \exp \left[ -\frac{(\omega' - \Omega_{He^+})^2}{k_{\Pi}^2 V_{T\Pi c_{He^+}}^2} \right] + \right. \\ \left. \frac{\omega_{p_{o^+}}^2 \Omega_{o^+}^2}{V_{T\Pi c_{o^+}}} \left\{ (J+1) \frac{T_{\perp_{o^+}}}{T_{\Pi_{o^+}}} \left( \frac{\omega' - \Omega_{o^+}}{\Omega_{o^+}} \right) + 1 \right\} \exp \left[ -\frac{(\omega' - \Omega_{o^+})^2}{k_{\Pi}^2 V_{T\Pi c_{o^+}}^2} \right] \right] \quad (6)$$

### Parallel Resonant Energy

$$W_{r\Pi\alpha} = \frac{\pi^{3/2} B^2}{C^2 K_{\Pi}^2} \left[ \frac{\omega_{pH^+}^2 \Omega_{H^+}^2}{\omega k_{\Pi}^2 V_{T\Pi c_{H^+}}} \left\{ (J+1) \frac{T_{\perp H^+}}{T_{\Pi H^+}} \left( \frac{\omega' - \Omega_{H^+}}{\Omega_{H^+}} \right)^2 \right\} \exp \left[ -\frac{(\omega' - \Omega_{H^+})^2}{k_{\Pi}^2 V_{T\Pi c_{H^+}}^2} \right] + \right. \\ \left. \frac{\omega_{p_{He^+}}^2 \Omega_{He^+}^2}{\omega k_{\Pi}^2 V_{T\Pi c_{He^+}}} \left\{ (J+1) \frac{T_{\perp_{He^+}}}{T_{\Pi_{He^+}}} \left( \frac{\omega' - \Omega_{He^+}}{\Omega_{He^+}} \right)^2 \right\} \exp \left[ -\frac{(\omega' - \Omega_{He^+})^2}{k_{\Pi}^2 V_{T\Pi c_{He^+}}^2} \right] + \right. \\ \left. \frac{\omega_{p_{o^+}}^2 \Omega_{o^+}^2}{\omega k_{\Pi}^2 V_{T\Pi c_{o^+}}} \left\{ (J+1) \frac{T_{\perp_{o^+}}}{T_{\Pi_{o^+}}} \left( \frac{\omega' - \Omega_{o^+}}{\Omega_{o^+}} \right)^2 \right\} \exp \left[ -\frac{(\omega' - \Omega_{o^+})^2}{k_{\Pi}^2 V_{T\Pi c_{o^+}}^2} \right] \right] \quad (7)$$

Where  $\omega' = (\omega - K_{\Pi} V_{Di})$

$$C_J = \frac{2\pi}{V_{T\perp}^{2(J+1)} J!} \int_0^{\infty} dV_{\perp}^2 V_{\perp}^{2(J+1)} \exp \left( -\frac{V_{\perp}^2}{V_{T\perp}^2} \right)$$

$$D_J = \frac{2\pi}{V_{T\perp}^{2(J+1)} J!} \int_0^{\infty} dV_{\perp}^2 V_{\perp}^{2J} \exp \left( -\frac{V_{\perp}^2}{V_{T\perp}^2} \right)$$

and

$$f_r(V_r) = \left( \frac{m_{\alpha}}{2\pi T_{\Pi c\alpha}} \right)^{1/2} \exp \left[ -\frac{m_{\alpha}(\omega' - \Omega_{\alpha})^2}{2T_{\Pi c\alpha} k_{\Pi}^2} \right]$$

$$f'(V_r) = -2 \left( \frac{m_{\alpha}}{2\pi T_{\Pi c\alpha}} \right)^{1/2} \left( \frac{\omega' - \Omega_{\alpha}}{k_{\Pi} V_{T\Pi c\alpha}} \right) \exp \left[ -\left( \frac{\omega' - \Omega_{\alpha}}{k_{\Pi} V_{T\Pi c\alpha}} \right)^2 \right]$$

$$Z_n(\xi) = \frac{1}{\sqrt{\pi}} p \int_{-\infty}^{\infty} \frac{\exp(-x^2)}{(x - \xi)^{n+1}} dx$$

$$\xi = \frac{\omega - \Omega_{\alpha}}{k_{\Pi} V_{T\Pi c\alpha}}$$

**Growth Rate:** Using the law of conservation of energy

$$\frac{d}{dt}(W_r + W_w) = 0 \quad (8)$$

The growth / damping rate  $\gamma$  is derived<sup>1</sup> as.

$$\frac{\partial U}{\partial t} = 2\mathcal{U} \quad (9)$$

Where

$$\frac{dW_r}{dt} = -2 \frac{\partial U}{\partial t}$$

and

$$\frac{dW_\alpha}{dt} \sim \frac{\partial U}{\partial t}$$

Those particles with velocities near the phase velocity of the waves give up energy  $2U$  to the waves. Half of this goes to potential energy and the other half goes into kinetic energy of oscillation of the bulk of the particles.

Here it is noticed that the ion beam  $V_{bi}$  has affected the growth rate through plasma densities and change in the energy for the electromagnetic waves propagating parallel to the magnetic field. In the present analysis we have considered the thermal anisotropy due to the background plasma only.

## Results and Discussion

The characteristics of the EMIC waves were derived the dispersion relation and growth rate by using auroral acceleration parameters<sup>1</sup>.

$$B_0 = 4300 \text{ nT}, \quad \Omega_{p\alpha} = 412 \text{ sec}^{-2}, \quad V_{T11H^+} = 3 \times 10^9 \text{ cm/s},$$

$$V_{T11He^+} = 5 \times 10^8 \text{ cm/s}, \quad V_{T11O^+} = 5 \times 10^8 \text{ cm/s}, \quad \omega_{pH^+} = 1.552 \times 10^9 \text{ s}^{-2},$$

$$\omega_{pHe^+} = .216 \times 10^8 \text{ s}^{-2}, \quad \omega_{pO^+} = .05 \times 10^8 \text{ s}^{-2}$$

$$\Omega_{pH^+} = 412 \text{ sec}^{-2}, \quad \Omega_{pHe^+} = 102.5 \text{ sec}^{-2},$$

$$\Omega_{pO^+} = 25.625 \text{ sec}^{-2}$$

Hence; the growth rate of EMIC waves is obtained as:

$$\begin{aligned} \gamma = & \frac{\frac{\Omega_{H^+}}{k_\Pi V_{T\Pi H^+}} \left[ \frac{(\Omega_{H^+} - \omega')}{\Omega_{H^+}} \frac{(J+1)V_{T\perp H^+}^2}{V_{T\Pi H^+}^2} - 1 \right] \exp\left[-\frac{1}{V_{T\Pi H^+}^2} \left(\frac{\omega' - \Omega_{H^+}}{k_\Pi}\right)^2\right]}{\left(\frac{ck_\Pi}{\omega_{pH^+}}\right)^2 \left(\frac{2\Omega_{H^+} - \omega}{\Omega_{H^+} - \omega}\right) + \frac{1}{2} \left(\frac{2\Omega_{H^+} - \omega'}{\Omega_{H^+} - \omega'}\right)^2} + \\ & \frac{\frac{\Omega_{He^+}}{k_\Pi V_{T\Pi He^+}} \left[ \frac{(\Omega_{He^+} - \omega')}{\Omega_{He^+}} \frac{(J+1)V_{T\perp He^+}^2}{V_{T\Pi He^+}^2} - 1 \right] \exp\left[-\frac{1}{V_{T\Pi He^+}^2} \left(\frac{\omega' - \Omega_{He^+}}{k_\Pi}\right)^2\right]}{\left(\frac{ck_\Pi}{\omega_{pHe^+}}\right)^2 \left(\frac{2\Omega_{He^+} - \omega}{\Omega_{He^+} - \omega}\right) + \frac{1}{2} \left(\frac{2\Omega_{He^+} - \omega'}{\Omega_{He^+} - \omega'}\right)^2} + \\ & \frac{\frac{\Omega_{O^+}}{k_\Pi V_{T\Pi O^+}} \left[ \frac{(\Omega_{O^+} - \omega')}{\Omega_{O^+}} \frac{(J+1)V_{T\perp O^+}^2}{V_{T\Pi O^+}^2} - 1 \right] \exp\left[-\frac{1}{V_{T\Pi O^+}^2} \left(\frac{\omega' - \Omega_{O^+}}{k_\Pi}\right)^2\right]}{\left(\frac{ck_\Pi}{\omega_{pO^+}}\right)^2 \left(\frac{2\Omega_{O^+} - \omega}{\Omega_{O^+} - \omega}\right) + \frac{1}{2} \left(\frac{2\Omega_{O^+} - \omega'}{\Omega_{O^+} - \omega'}\right)^2} \end{aligned} \quad (10)$$

Where  $\alpha = i, H^+, He^+, O^+$ ,  $\omega' = (\omega - K_{11}V_{bi})$  and Using the value of

$$V_{T\perp\alpha}^2 = (J+1)^{-1} \frac{2T_{\perp\alpha}}{m_\alpha}, \quad V_{T\Pi\alpha}^2 = \frac{2T_{\parallel\alpha}}{m_\alpha} \text{ for multi-ions plasma.}$$

Figure 1 shows the variation of frequency of plasma ( $\omega$ ) in  $\text{sec}^{-1}$  versus wave vector ( $k_{11}$ ) in  $\text{cm}^{-1}$  for multi-ions magnetospheric plasma. It is found that the frequency ( $\omega$ ) is linearly increases with the increasing of the parallel wave vector ( $k_{11}$ )  $\text{cm}^{-1}$  and the variation shows by the straight line.

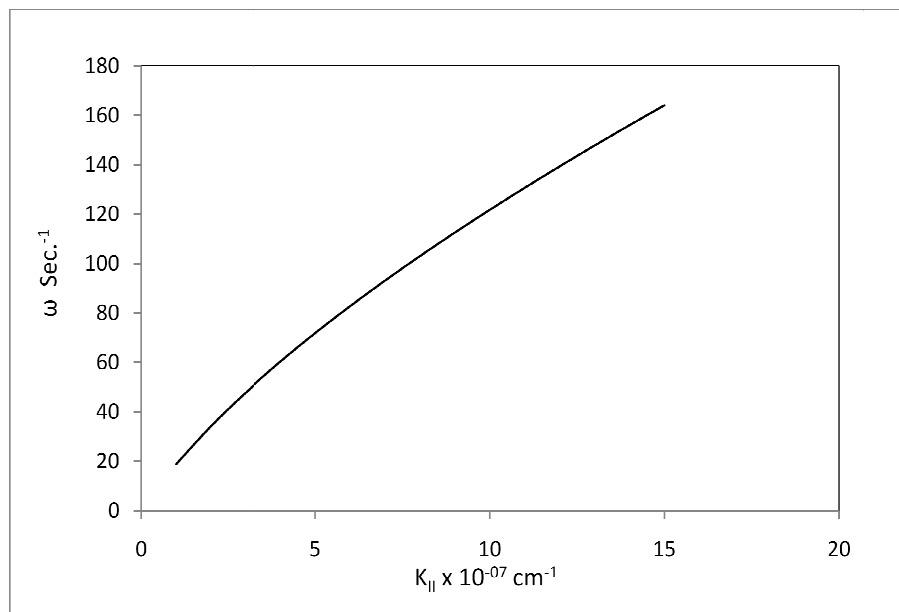
In the linear theory these waves have been studied both numerically and analytically<sup>7</sup> in multi-component plasmas. The findings of the investigations may be of importance to the coronal heating and acceleration of solar wind by EMIC waves<sup>8-11</sup>. It has been predicted that perpendicular heating of coronal ions is due to the dissipation of EMIC wave energy. However, the generation of EMIC waves has not been predicted yet on a firm basis. Since the mid-1990s, Solar and Heliospheric Observatory has predicted remarkable data and given impetus to studies on the ion-cyclotron resonance as the principal mechanism for heating the coronal holes, and ultimately driving the fast wind<sup>11</sup>. In the solar corona, the ions are reported with higher temperatures than the electrons. The differential ion heating could be due to large amplitude EMIC waves. The particle aspect analysis developed may be applicable to laboratory plasma as well as to estimate the heating rates, along with the study of emissions of EMIC waves.

Figure 2 predict the variation of the growth rate ( $\gamma$ ) with the wave vector ( $k_{11}$ )  $\text{cm}^{-1}$  for the different values of ion beam velocity  $V_{bi}$ . The ion beam velocity taking in our prediction is  $V_{bi} = -1 \times 10^7$ ,  $V_{bi} = -5 \times 10^7$  and  $V_{bi} = -1 \times 10^8$  cm/sec respectively for multi-component magnetospheric plasma. It is shows by this graph the peak of the growth rate which shifts towards the lower side of the parallel wave vector ( $k_{11}$ ) then, the growth rate is also decreases. The increasing of the growth rate

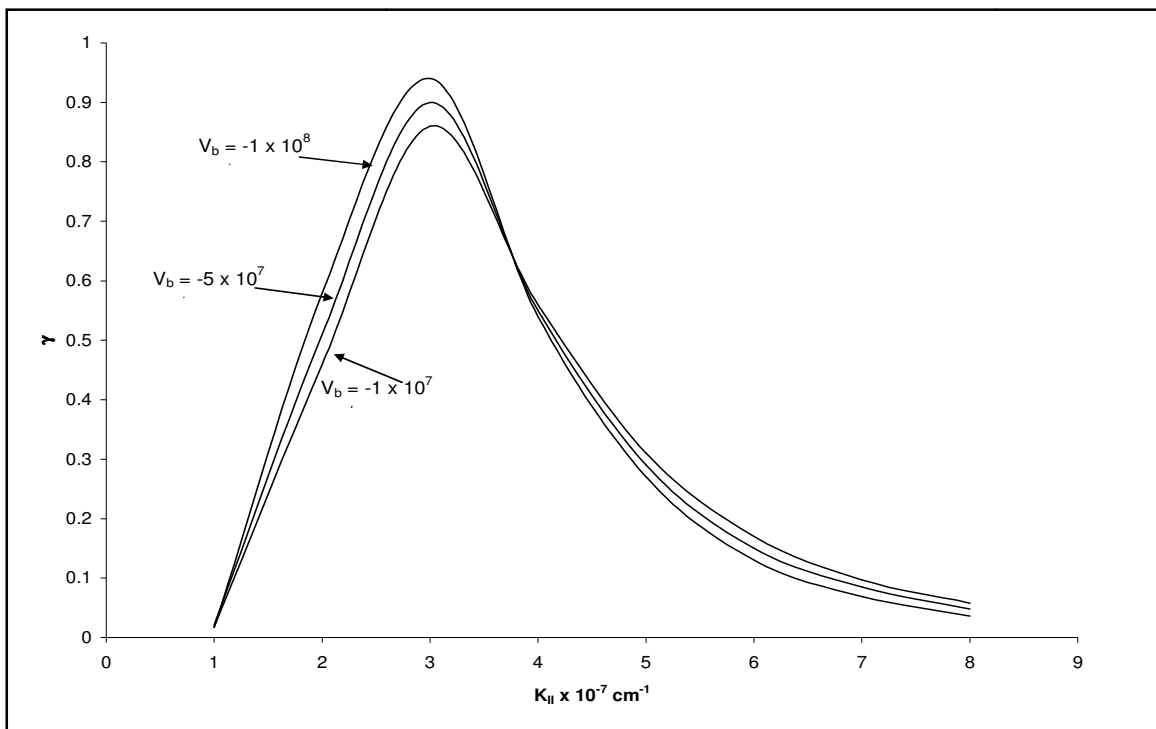
transferred in the presence of ion beam velocity  $V_{bi}$  for the EMIC waves in the magnetospheric plasma. We have not considered the contribution of heavy ions such as  $\text{H}_e^+$  and  $\text{O}^+$  which have the major contribution in space plasma, for example in their differential heating. The analysis can be extended including heavy ions in further investigations. Electromagnetic ion-cyclotron waves with ion beam plasma interactions take place in several space and astrophysics environments, as well as in laboratory plasmas.

Figure 3 predicts the variation of the perpendicular resonance energy ( $W_{r\perp}$ ) with the wave vector ( $k_{11}$ )  $\text{cm}^{-1}$  for the different values of ion beam velocity  $V_{bi}$ . The ion beam velocity taking in our prediction is  $V_{bi} = -1 \times 10^6$ ,  $V_{bi} = -3 \times 10^6$ ,  $V_{bi} = -5 \times 10^6$  and  $V_{bi} = -9 \times 10^6$  cm/sec respectively for multi-component magnetospheric plasma. It is shows by this graph the negative  $W_{r\perp}$  indicates that the particles energy is transferred to the waves. Thus, wave emissions occur by extracting energy from the ions moving perpendicular to the magnetic field. It is observed that the effect of increasing  $V_{bi}$  is to enhance the transfer of the particle's energy perpendicular to the magnetic field to the waves. Thus the perpendicular deceleration of ions is noticed through the EMIC wave by ion beam energy.

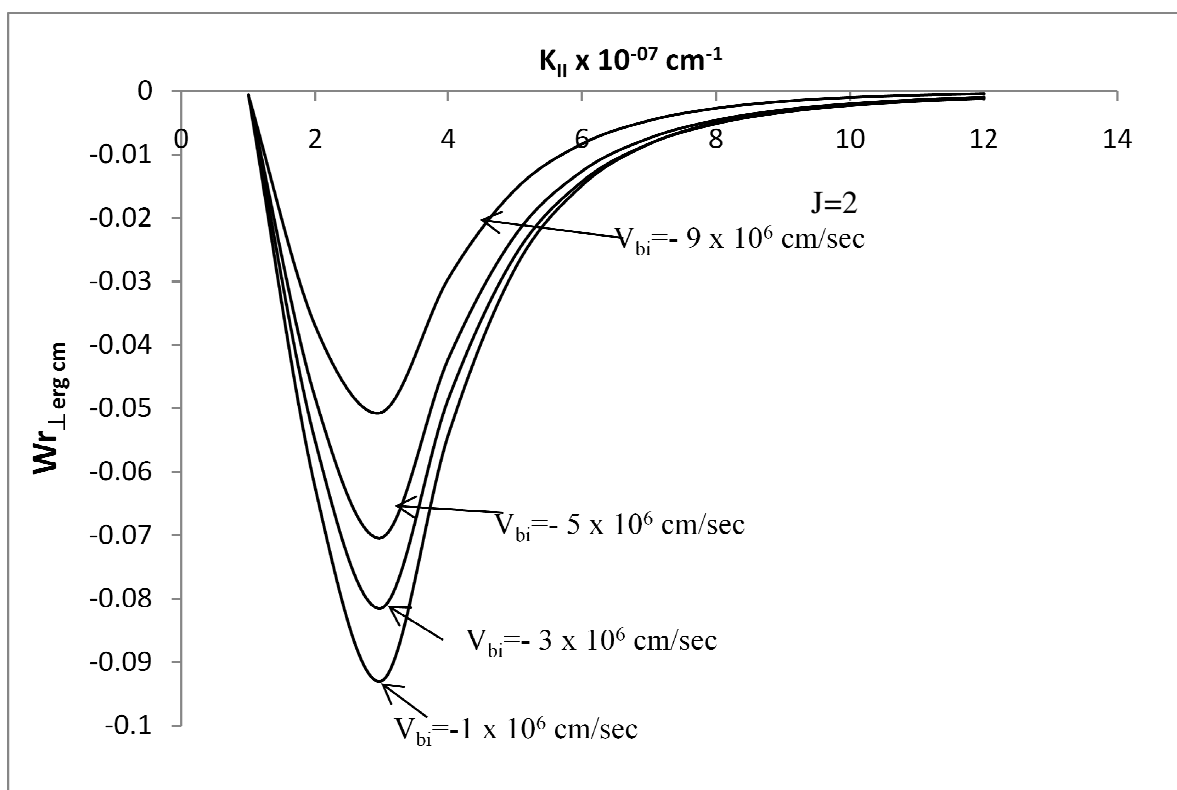
Figure 4 predicts the variation of the parallel resonance energy ( $W_{r\parallel}$ ) with the wave vector ( $k_{11}$ )  $\text{cm}^{-1}$  for the different values of ion beam velocity  $V_{bi}$ . The ion beam velocity taking in our prediction is  $V_{bi} = -1 \times 10^6$ ,  $V_{bi} = -3 \times 10^6$ ,  $V_{bi} = -5 \times 10^6$  and  $V_{bi} = -9 \times 10^6$  cm/sec respectively for multi-component magnetospheric plasma. It is seen that the effect of  $V_{bi}$  is to increase the parallel resonant energy. Thus, the ion beam may enhance the heating of resonant ions parallel to magnetic field.



**Figure-1**  
**Variation of wave frequency ( $\omega$ ) versus wave vector ( $K_{||}$ )**



**Figure-2**  
Variation of growth rate ( $\gamma$ ) versus wave vector  $K_{||} \text{ cm}^{-1}$  for different ion beam velocity



**Figure-3**  
Variation of perpendicular resonance energy ( $W_{r\perp}$ ) versus wave vector ( $K_{||}$ ) for different values of ion beam velocity  $V_{bi}$

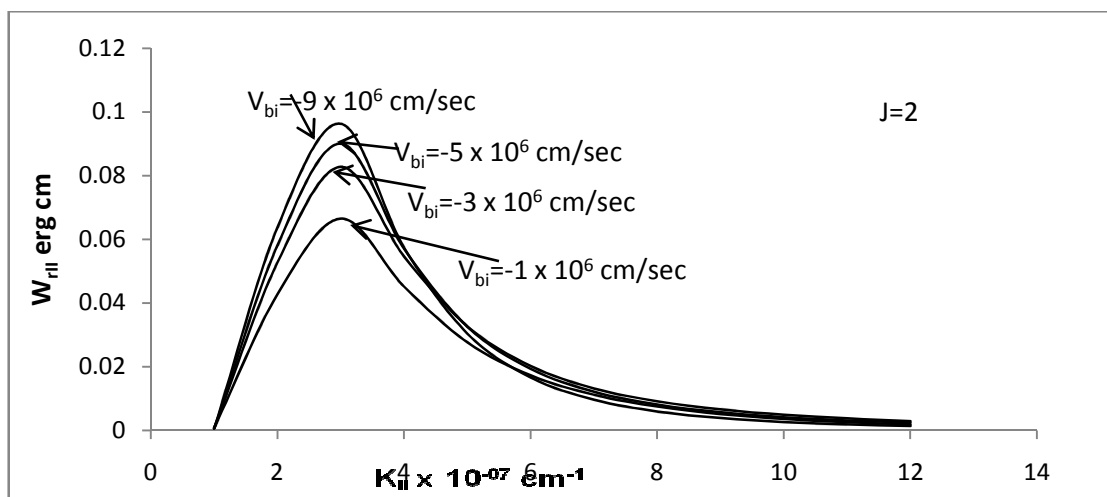


Figure-4

Variation of parallel resonance energy ( $W_{||}$ ) versus wave vector ( $K_{||}$ ) for different values of ion beam velocity  $V_{bi}$

## Conclusion

In the present work, we have study of an electromagnetic ion-cyclotron wave in multi-ions magnetospheric plasma. It is found that the growth rate enhance with effect of ion beam velocity at plasma densities in  $H^+$ ,  $He^+$  and  $O^+$  ions the auroral acceleration region to explain the waves emission.

The significance of this work is as follows: i. The effect of increasing ion beam velocity on electromagnetic ion cyclotron waves in  $H^+$ ,  $He^+$  and  $O^+$  ions enhance the growth rate, may be due a sub-storm phenomena. The growth rate increases with  $K_{||}$ , attains a peak and decrease again in all cases. ii. The behavior studied for the EMIC waves may be of importance in the electromagnetic emission in the auroral acceleration region. The result of the study is also applicable to the plasma devices that have the steep loss-cone distribution. iii. The effect of ion beam and plasma densities during the up flowing beam is to enhance the grow the wave emission and the energy of multi-ions  $H^+$ ,  $He^+$  and  $O^+$  the wave energy propagation outside and inside in plasma magnetosphere unidirectional and bidirectional. iv. The interpreted may be applicable to explain the ion heating in the solor wind as well as auroral acceleration region.

## Acknowledgement

Authors (GA) are thankful to DST for financial assistance.

## References

1. Ahirwar G., Varma P. and Tiwari M. S., Study of electromagnetic ion-cyclotron waves with general loss-cone distribution and multi-ions plasma-particle aspect approach, *Ind. J. of Pure & Appl. Phys.*, **48**, 334 (2010)
2. Hoyos J. and Gomberoff L., Influence of nonlinear circularly polarized waves on linear electromagnetic beam-plasma waves, *The Astrophys J.*, **630**, 1125 (2005)
3. Gary S.P., Los Alamos National Laboratory, Los Alamos, New Mexico, 87545 (1984)
4. Temerien M., McFadden J., Boehm M., Carlson C.W. and Lotko W., Production of flickering aurora and field-aligned electron fluxes by electromagnetic ion cyclotron waves, *J. Geophys. Res.*, **91**, 5769 (1986)
5. Ahirwar G., Varma P. and Tiwari M.S., Beam effect on electromagnetic ion-cyclotron waves with general loss-cone distribution function in an anisotropic plasma-particle aspect analysis, *Ann. Geophys.*, **25**, 557 (2007)
6. Chaston C.C., Bonnell J.W., McFadden J.P., Ergun R.E. and Carlson C.W., Electromagnetic ion cyclotron waves at proton cyclotron harmonics, *J. Geophys. Res.*, **107**(A11), 1351 (2002)
7. Gomberoff L. and Elgueta R., Resonant acceleration of alpha particles by ion cyclotron waves in the solar wind, *J. Geophys. Res.*, **96**, 9801 (1991)
8. Dusenbery P.B. and Hallweg J.V., Ion-cyclotron heating and acceleration of solar wind mirror ions, *J. Geophys. Res.*, **86**, 153 (1981)
9. Shukla P.K. and Stenflo L., Non-linear propagation of electromagnetic ion-cyclotron Alfvén waves, *Phys. Fluids*, **28**, 1576 (1985)
10. Crammer S.R. and Van-Ballegooijen A.A., Alfvénic turbulence in the extended solar corona: Kinetic effects and proton heating, *Astrophys. J.*, **594**, 573 (2003)
11. Hollweg J.V., Drives of the solar wind: then and now, *Phil. R. Soc. A.*, **364**, 505 (2006)