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Dispersion Characteristics of Settleable and Dissolvable Pollutants in Waste Stabilization Ponds

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Abstract

Determination of the dispersion number or dispersion coefficient of a pollutant in a receiving stream or a treatment plant is a very important aspect of pollution control. A model describing the relationship between the dispersion number of a settleable solid (d_2) and that of a dissolvable tracer (d_1) was presented and verified with data collected from a laboratory channel. The model predicted results closer to experimental data than the existing model. The method applied in this research allows for in-situ determination of a pollutant settling velocity more realistically than both stokes equation and quiescent settling analysis. It was shown that using a dissolvable tracer instead of a setteable solid could lead to error. The implication of this in waste stabilization pond design was also discussed.

Keywords: Dispersion, settleable pollutants, waste stabilization ponds.

Introduction

A waste stabilization pond (WSP) is a basin dug on earth, usually rectangular or trapezoidal in shape and is used for wastewater treatment. Its numerous advantages over the conventional treatment systems are well documented in the literature[1].

Among all the models available for describing the process of waste stabilization in ponds, the dispersed flow model is acclaimed to be the best[2,3]. Its usefulness, however, depends on accurate determination of the dispersion number (d)[4]. Polprasert and Bhattarai[5] defined dispersion number as:

$$d = \frac{D}{UL} \tag{1}$$

Where U is the mean wastewater flow velocity (m/day), L is the pond length (m) and D is the longitudinal dispersion coefficient (m^2/day) characterizing the degree of back-mixing and spreading of pollutants during flow.

The dispersion number is often determined by using tracers (e.g. sodium chloride) which are not settleable. Since settling affects dispersion[6], using non-settleable tracers to determine the dispersion number of settleable pollutants may lead to error. Settling effects are significant in anaerobic and primary facultative ponds where up to 30% of pollutants are removed by sedimentation[7].

Although there are some models that describe the effect of settling on dispersion for contaminants discharged into rivers[8], they are not suitable for waste stabilization ponds. None of the existing models indicated how the setting velocity of the pollutant could be measured. The experimental work reported were based on spherical particles and Strokes equation[6,9] whereas wastewater particles are irregular in shape and have velocities far below those of spherical objects of equivalent sizes[10]. The actual settling velocities of wastewater depends on the nature of flow, boundary conditions and pollutant shapes which are not reflected in Stokes equation. Besides, the mean velocity was assumed equal to the discharge velocity. That these two are not equal has been pointed out previously[9]. The aim of this paper is to present a model that is applicable to waste stabilization pond, and devoid of the above shortcomings.

Mathematical Formulations: In ponds, determination of d is based on one-dimensional dispersion equation for a non-settleable substance, i.e.

$$\frac{\delta C}{\delta t} + U\frac{\delta C}{\delta x} = D_1 \frac{\delta^2 C}{\delta x^2} \tag{2}$$

In which C is the cross-sectional mean concentration; U is the mean velocity; D_1 is the dispersion coefficient; t is the time from tracer injection to sampling; and x is the co-ordinate in the direction of mean flow.

For an initial tracer distribution concentration in the plane x=0 at time t=0, the solution of equation (2) is:

$$C = \frac{M}{At}\exp\frac{-(x-Ut)^2}{4D_i t} \tag{3}$$

Where M and A are the total mass of tracer and total cross-sectional area of flow normal to x respectively.

As stated earlier, equation (2) does not adequately describe the dispersion process in ponds, especially in anaerobic and primary facultative ponds where settling effects are significant. In order to account for the settling of wastewater pollutants equation (2) is modified to:

$$\frac{\delta C}{\delta t} + U\frac{\delta^2 C}{\delta x} = D_2\frac{\delta^2 C}{\delta x^2} + \frac{V_s C}{h} \quad (4)$$

In which D_2 is the dispersion coefficient of the settleable pollutant; V_s is the pollutant settling velocity; and h is the pond or channel depth.

The solution of equation (4) is obtained under the same initial conditions.

If C (x,t) = (x,t) exp (V_st/h) is substituted into equation (4), it reduces to:

$$\frac{\delta\phi}{\delta t} = D_2\frac{\delta^2\phi}{\delta x^2} - U\frac{\delta\phi}{\delta x} \quad (5)$$

Hence, its solution may be obtained in a form similar to equation (3) i.e.

$$\phi = \frac{M}{A\sqrt{4\pi D_2 t}}\exp\frac{-(x-Ut)^2}{4D_1 t} \quad (6)$$

Or in terms of the original concentration

$$C = \frac{M}{A\sqrt{4\pi D_2 t}}\exp\left[\frac{-(x-Ut)^2}{4D_1 t} - \frac{V_2 t}{h}\right] \quad (7)$$

Maximum Likelihood Method (MLM) of Parameter Estimation: Harris[11] derived the following MLM estimation formulas for the average flow velocity (U) and dispersion coefficient (D_1) for settleable pollutants from equation (3):

$$U = \frac{L}{n}\sum_{i=1}^{n}\frac{1}{t_1} \quad (8)$$

and

$$D = \frac{U}{2}(\bar{U}\,t - L) \quad (9)$$

Harris' method is preferred to the moment method because it involves only the first moment of the curve[12]. A similar approach is used to derive other formulas, which include the settling velocities based on equation (7). However, equation (7) must first fulfill the requirements of a probability density function (i.e. $\int (CV/M)dt = 1$.

Noting from Agunwamba[13] the relationship: $M^2L^2 = 4D_2t^2\left[\frac{U^2}{4D_2} - \frac{V_2}{h}\right]$, then

$$\int_0^\infty \frac{CV}{M}dt = \frac{1}{\sqrt{4\pi D_2}}\exp\frac{LU}{2D_2}\int_0^\infty t^{1/2}\exp - \frac{\sqrt[(1+M^2)L^2]{\left(\frac{U^2}{4D^2} - \frac{V_2}{h}\right)D}}{} \bullet \frac{LdM}{\sqrt{\left(\frac{U^2}{4D^2} - \frac{V_2}{h}\right)D_2}} \quad (10)$$

Evaluating from mathematical tables,

$$\int_0^\infty \frac{CV}{M}dt = \frac{1}{2}\exp\left[\frac{LU}{2d^2} - \frac{L}{D}\left(\frac{U^2}{4D_2} - \frac{V_2}{h}\right)D_2\right]^{1/2} \times \frac{1}{\left[\left(\frac{U^2}{4D_2} - \frac{V_2}{h}\right)D_2\right]^{1/2}} \quad (11)$$

Therefore, the function:

$$g(t) = \sqrt[2]{\alpha_1 D_2 \exp\left[\frac{L}{D_2}\sqrt{\alpha_1 D_2} - \frac{LU}{2D_2}\right]}\frac{1}{\sqrt{4\pi D_2}}\exp\left[\frac{(L-Ut)^2}{4D_2 t} - \frac{V_2 t}{h}\right] \quad (12)$$

Fulfills the requirements of probability density function where:

$$\alpha_1 = \frac{U^2}{4D_2} - \frac{V_2}{h} \quad (13)$$

In order to get the estimating equations for U, V_s and D_2 the method of maximum likelihood is used[14].

Let f (t; θ) be the density function of the random variable t, where θ = (θ_1 …, V_k) are parameters to be estimated. Suppose n observations are to be made on the variable t. Let t_1, …, t_n denote the random variables corresponding to n observations, then the function given by:

$$L(t_1 ..., t_n; \theta) = \prod_{i=l}^{n} f(t_1; \theta) \tag{14}$$

defines a function of the random sample values t_1, …, t_n and the parameters θ_1, …, θ_k and L is the likelihood function. It maximizes the probability of getting the observed samples. If the estimates of θ_1, …, θ_k exist, then the system of k likelihood equations;

$$\frac{\delta L(\theta, x)}{\delta \theta} = 0, i = 1, ..., k \tag{15}$$

must be satisfied for all x such that L has first order partial derivatives in θ. The most useful condition for asserting that solutions do correspond to maximal is concavity[14]. Since equation (3) has a maximal[15], equation (7) has also a maxima because exp (V_st/h) will not affect the shape of the curve.

The maximum likelihood of equation (13) is then obtained as:

$$L(t_1 ..., t_n; U, V, D) = \prod_{i=1}^{n} \left[\sqrt[2]{\alpha_1 D_2}.\exp\left(\frac{L}{D}\sqrt{\alpha_1 D_2} - \frac{LU}{2D_2} \right) \bullet \frac{1}{4\pi D_2} \times \exp\left(\frac{(L-Ut)^2}{4D_2 t} \right) \right] \tag{16}$$

Maximizing L is the same as maximizing log L[14]. Hence, if log of equation (16) is found and then differentiated with respect to U, V_s and D_2, the following equations are obtained:

$$\frac{\delta \log L}{\delta u} = \frac{nU}{4D_2\alpha_1} + \frac{nLU}{4D_2\sqrt{\alpha_1 D_2}} - \frac{Ln}{D_2} + \sum \frac{(L-Ut)}{2D_2} \tag{17}$$

$$\frac{\delta \log L}{\delta V_2} = \frac{n}{2h\alpha_1} - \frac{Ln}{\sqrt[2h]{\alpha_1 D_2}} + \frac{1}{2}\sum t \tag{18}$$

$$\frac{\delta \log L}{\delta D_2} = \frac{U^2 n}{8D_2^2\alpha_1} - \frac{nL}{D}\left(\frac{U^2}{8D_2^2}\sqrt{\frac{D_2}{\alpha_1}} + \frac{1}{2}\sqrt{\frac{\alpha_1}{D_2}} \right) + \frac{LUn}{2D_2^2} + \sum \frac{(L-Ut)^2}{4D_2^2 t} \tag{19}$$

In order to obtain the maximum likelihood estimates, equations (18) to (20) are set equal to zero. If the equations are solved simultaneously and simplified with the aid of equation (9), then:

$$\frac{U^2}{4D_2} - \frac{V_2}{h} = \alpha_1 = \frac{n}{\sum_{i=l}^{u} t_1} \tag{20}$$

and

$$D_2 = D_1 + LU - \frac{U^2}{n}\sum_{i=l}^{n} t_1 \tag{21}$$

Equations (17) and (18) can be shown to be the same. Hence, only two of the tree constants can be obtained. The average mean flow velocity is related to the settling velocity by the relationship:

$\frac{V_2}{h} = \frac{U}{L}$ Hence, we can determine V_s from:

$$V_2 = \frac{hL}{Ln}\sum \frac{1}{t_1} = \frac{h}{n}\sum \frac{1}{t_1} \tag{22}$$

$$\Rightarrow V_2 = \frac{h}{4D_2}(U^2 - U) \tag{23}$$

$$V_2 = \frac{hU^2}{4D_2}\left[1 - \left(\frac{D_2}{D_1} \right)^2 \right] \tag{24}$$

Through some mathematical manipulations of equations (20) and (24),

$$\frac{D_2}{D_1} = \sqrt{1 - \frac{4V_s D_2}{hU^2}} = \sqrt{1 - \frac{4V_s D_2 U}{hU^3}} \tag{25}$$

Equation (25) shows that for a given settleable contaminant with a known settling and flow velocities, it is possible to obtain its dispersion coefficient if that of a tracer (D_1) subjected to the same flow conditions is known.

For the sake of comparing the present work with the previous ones, two models are presented. Summer[8] derived an asymptotic relationship for a particle dispersion and computed D_2/hU_* for values of settling parameter ($\beta = V^1_s/4$) ranging from 0.1 to 0.6 in which:

$$\mu - \mu_s = -6k^{-2}[1 + \psi(1-\beta) - \psi(2)], \beta < 1 \tag{26}$$

Where μ and μ_s are respectively the dimensionless flow and particle velocity: k is Von Karman constant (= 0.42); and ψ is psi function. For neutrally buoyant particles, β = 0 and D_1/hU^* reduces to 5.52. Assuming that Stokes law applies and that particle flow velocity is equal to discharge velocity, Ojiako[6] obtained and empirical relationship for spherical objects as:

$$\frac{D_2}{D_1} = 1 - (-0.44 + 3.48U^{*1})V_2^1 \tag{27}$$

Where U^1 and V^1_s are the dimensionless shear and dimensionless settling velocities respectively. The two models above were compared with the new model based on the same values of D_2/hU_* obtained by Sumer[8] for different values of the settling velocity parameter (β). Equation (26) was evaluated with the aid of mathematical tables[16].

Material and Methods

Sieve and Settling Analysis: Saw-dust was used as the pollutant. Its specific gravity was found to be 1.0909[17], which is within the range of specific gravities of sewage solids[18]. Besides, sewage solids, like saw-dusts, are not spherical. The particle sizes of the saw-dust were determined by sieve analysis following the procedure described in Arora[17]. Sizes between 0.1cm and 0.005cm were used for further experimentation and analysis since this is the approximate range of solids found in wastewaters[19]. Computation of the terminal velocities were then made based on Stoke's equation for comparison with settling analysis[10].

Visual examination showed that the saw-dust particles were irregular in shape. However, lack of appropriate measuring facilities prevented the identification of their specific irregular shapes. Hence, there was no possibility of determining their settling velocities by modification of stokes equation. Therefore, it was found necessary to perform settling analysis experiment.

The settling analysis experiment took place in a settling column 2m long and 0.1m internal diameter (figure-1). The apparatus for settling was filled with water to which 200g sample of saw-dust was added. The column was then shaken gently to distribute the particles evenly over the full depth. The test started when the water samples came to rest. At that moment, and at 30 seconds interval thereafter, water samples were taken at different depths and analyzed for suspended solids.

Flow Measurements: Flow measurements made on a channel of 750cm x 40cm rectangular cross-section include velocity of flow, discharge depth, surface water slope and temperature. The discharge was measured by a graduated cylinder and a stopwatch while the flow velocity was obtained as the quotient of the discharge and the average cross-sectional area. Point gauges were used to measure the depths at the inlet and outlet of the channel. Dividing the difference between the inlet and outlet water depths by the channel length yielded the surface water slope. Temperature measurements were made during each experiment in order to determine the kinematic viscosities from a standard table[20]. With the channel cross-section, flow velocity and kinematics viscosity known, Reynolds number was calculated[20].

Pollutant and tracer dispersion numbers: Experiments on dispersion characteristics were performed in the channel described above using sodium chloride as the tracer and saw-dust as the organic solid particles (pollutants). The procedure involved getting the time concentration curve for sodium chloride first and then obtaining that of the saw-dust. In every case the sample was introduced into the channel at the inlet and the samples collected at the outlet at known times for analysis[2]. Effluent chloride concentrations were corrected by subtracting the background levels from the measured concentrations. Chloride concentrations were determined by chloride test[21] while the dispersion number in all cases were determined following Levenspiel and Smith's method[22].

The dispersion number obtained by the tracer and pollutants were used for verifying the models derived on the relationship between pollutants and tracer dispersive properties. Where it was possible these comparisons were extended to the work of other researchers.

Results and Discussion

Setting Velocities: The result of the sieve analysis and the computed terminal velocities are given in table 1. The velocities range from 1.257cm/s for particle size 0.053mm to 7.058mm/s for 1.67mm size. Table 2 summarizes the hydraulic conditions under which the tracer studies were conducted. In particular, it is notable that the Reynolds numbers lie mainly between the transition and turbulent regions.

Table-1
Sieve analysis and computed terminal velocities saw dust particles

Sieve No.	Particle Size (mm)	Wt. Retained (g)	% Retained	% Passing	Terminal Velocity (cm/s) V_{ss}
8	2.00	6	3	97	-
10	1.67	8	4	93	7.058
12	1.40	8	4	89	6.431
16	1.003	18	9	80	5.470
25	0.599	49	24.5	55.5	4.227
36	0.43	46	23	32.5	3.583
44	0.353	13	6.5	6	3.245
60	0.251	20	10	16	-
85	0.178	14	7	9	2.304
150	0.104	-10	5	4	1.761
300	0.053	3	1.5	2.5	1.257
Tray		5	2.5	-	-

Specific Gravity (Ss) = 1.0909 Total wt. used = 200g, Source: Agunwamba (1992)

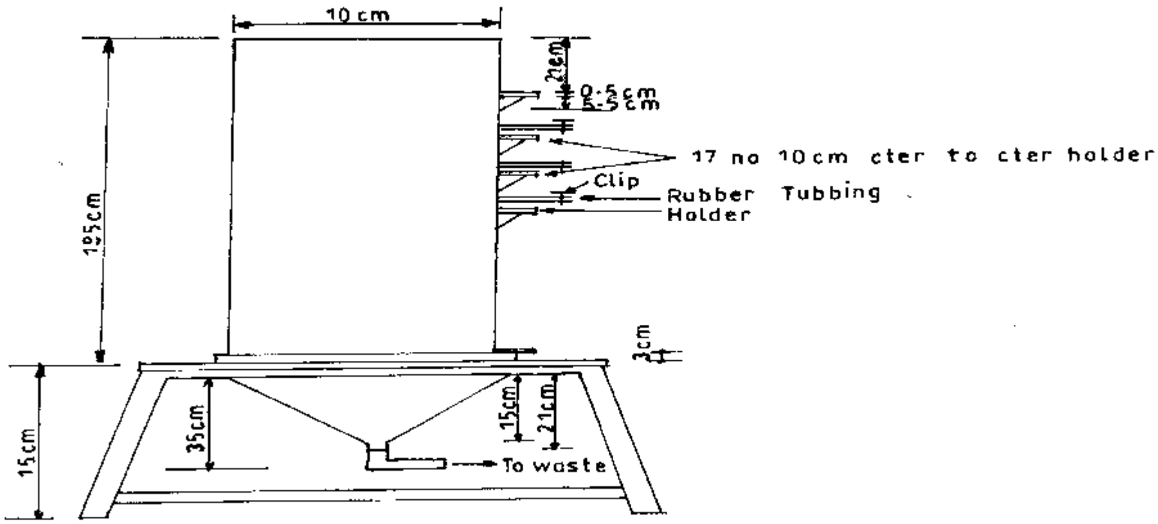

Figure-1
Apparatus for setting Analysis (vertical section)

Table-2
The hydraulic characteristics of experimental flows

Expt. No.	Particle size (mm)	Depth h cm	Mean flow velocity (U) cm/s	Viscosity x 10-2 (υ) cm^2/s	Hydraulic radius cm	Reynolds number x 10^3	Slope x 10^{-3}	Shear vel. U cm/s	Aspect ratio (w/h)
1	0.053	0.50	8.801	0.897	0.488	1.915	0.24	0.107	80
2	0.104	0.80	9.961	0.965	0.769	3.175	0.48	0.190	50
3	0.178	0.80	16.563	0.878	0.769	5.803	0.76	0.239	50
4	0.353	0.95	5.556	0.908	0.907	2.220	0.28	0.158	42
5	0.430	2.30	0.598	0.897	2.063	0.550	0.19	0.196	17
6	0.599	0.50	7.500	0.930	0.488	7.871	0.98	0.217	80
7	1.003	0.85	12.815	0.878	0.815	4.758	0.68	0.233	47
8	1.400	0.90	3.583	0.908	0.861	1.359	0.22	0.136	4
9	1.670	1.056	9.921	0.996	0.998	3.976	0.45	0.210	38

Source: Agunwamba (1992)

Table 3
The estimators and flow parameters computed from different formulae for L=210cm

Expt. No.	d1	Vs	Vss	UMLM	$U^1_8 = U^*/U$	$V^1_8 = V_{ss}/U_*$
1	0.192	.0014	.063	2.20	0.012	0.589
2	0.098	0.028	.125	2.46	0.019	0.658
3	0.089	.0020	.250	0.68	0.014	1.046
4	0.233	.0027	.600	2.20	0.028	3.444
5	0.217	.0063	.675	0.68	0.328	3.798
6	0.072	.0015	.900	2.20	0.006	4.178
7	0.158	.0025	1.063	4.74	0.018	4.562
8	0.67	.0026	1.125	3.68	0.038	8.272
9	0.136	.0026	1.225	3.68	0.021	5.833

Source: Agunwamba (1992)

Figure 2 shows the distribution of the settling velocities obtained from quiescent analysis (V_{sa}) while table 3 indicates that the settling velocities estimated by the present method (V_{sa}) are significantly lower than those estimated from Stokes equation (see table 1) and by settling analysis (V_{ss}) at 5% level of significance. Stokes equation is based on spherical objects but wastewater particles are irregular in shape[10]. By visual observation it is obvious that saw-dust particles are irregular. This irregularity in shape implies that a saw-dust particle having the same volume and weight as a given spherical particles will have a larger projected area in the direction of motion and higher value of the drag coefficient C_D under turbulent flow conditions. By both phenomena the settling velocities predicted by the empirical formula will be higher.

As for the quiescent settling analysis, it ignores the effect of the moving water since the experiment is normally performed in a column of standing water. It gives a certain settling velocity irrespective of the flowing velocity and Reynolds number of the moving water. It assumes equality of retention time of all particles and that all particles remain lying once they touch the bed whereas in actual channel measurements some particles settle, refloat and are scattered by turbulence in their pathways.

Theoretical comparisons: The three models are compared with respect to the variation of D_2/D_1 with dimensionless settling velocity in figure 3. Sumer's equation gave results remarkably larger than the others. This is because it evaluated D_2/D_1 asymptotically and assumed Aris moment[23]. Dispersion evaluated in the diffusive period is larger than that at the convective period[4]. Aris[23] method depends on the second moment and this magnifies the long tail. Figure 3 also shows that for all values of μ_s and μ Sumer's equation gave the same values of D_2/D_1. This is unrealistic since the ratio μ_s/μ should affect D_2/D_1.

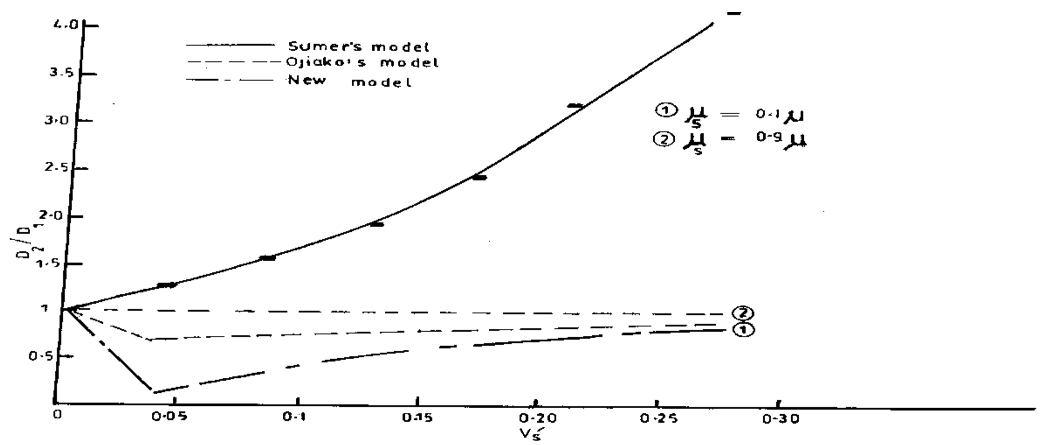

Figure-2

Comparison among three different models for the variation of the ratio of solid to tracer dispersion coefficient with dimensionless setting velocity for various mean particle and flow velocities (μ_s and μ respectively)

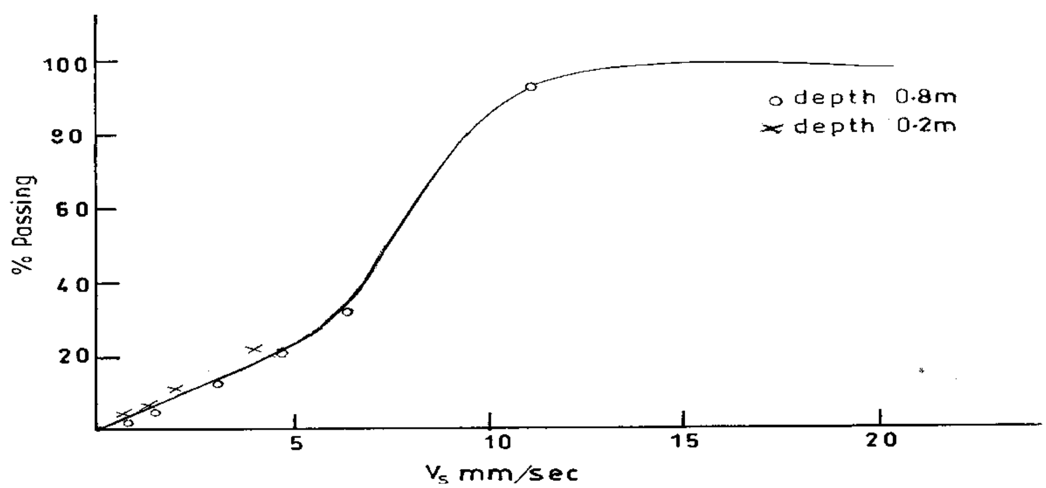

Figure-3

Cumulative frequency distribution of setting velocities

Whereas there is much difference between the graphs of these models at $\mu_s = 0.1\mu$, the difference is insignificant for $\mu_s = 0.9\mu$. This result is expected since in the empirical model, μ_s and μ are assumed equal but are unequal in the new model. Figure-3, therefore, rightly predicts that as the difference between the flow and particle velocities reduces, the two models yield the same results. Practically, however, this is possible when only spherical objects are considered. Otherwise there would be more complicated relationships between the two models.

Equality of Tracer and Pollutant dispersion coefficients (D_1 and D_2 respectively):

From Eq. 25 if $V_s = 0$, $D_2 = D_1$ as should be expected. So long as Vs positive, $D_1 > D_2$ which implies that settling decreases particle dispersion. Resuspension occurs if $V_s < 0$. In this case, $D_2 > D_1$, implying that resuspension may increase dispersion. Resuspension may be caused by wind action, vertical currents generated by density differences, and so on. Resuspension is expected in ponds because of density currents which is prevalent in deeper ponds and may influence detention time. Because pollutants in channels undergo settling and then resuspension unlike non-settleable dyes which stay only in suspension, it may not be very accurate to model settleable pollutants with non-settleable dyes.

Resuspension may be expected also if the flow velocity is so high as to pick up and carry away settled-out material from the sludge zone. This begins when the hydraulic shear between the wastewater and the sludge deposits equals the mechanical friction between these deposits and the bottom of the pond.

D_2 is equal to D_1 if: (i) $V_s = 0$, (ii) $D_2 \longrightarrow 0$, (iii) $U \longrightarrow \infty$

The first condition can never be met in an anaerobic pond which acts as a settling basin because of its long term retention. The condition may, however, be approximated in maturation ponds. The second condition is approximated in a plug flow which is however, idealistic. As for the third condition, U is generally small in ponds because of the long detention times. If U can tend to infinity then particles will so much be disturbed on their settling paths that no settling will be possible.

Since the above cases cannot be satisfied in anaerobic or primary facultative ponds, using D_1 instead of D_2 will lead to error, and that error may be quantified by equation (24) or (25). The numerical value of this may be illustrated by using some typical values of D_1, L and h from literature[24]. These are $0.827 m^2/day$, 4m and 0.6m respectively. The flow velocity, U = 1.333m/day. For a particle with a settling velocity of 0.2064m/day, $D_2 = 0.497 m^2/day$. With these values the error difference between D_1 and D_2 is $0.33 m^2/day$. The effect of such errors will be to underestimate the efficiency of the pond, which may lead to allocation of more land than is necessary for waste treatment. This is disadvantageous in congested urban areas where land is scarce or expensive.

Comparison of Predicted and Experimental Data: The values of D_2/D_1 predicted by the empirical equation, the new model and experimental results are compared in figure 4. The new model gave results closer to the measured values than the empirical equation. As mentioned before, the empirical equation is based on Stokes Law and discharge whereas the new equation is based on in situ determined settling velocity and the actual velocity of cloud of pollutants.

Conclusion

A method of predicting the dispersion number of a pollutant from that of a tracer subjected to similar flow conditions was developed using the maximum likelihood method. Compared with the existing empirical formula, the present estimating dispersion model seemed to yield results closer to the experimental data. It was also shown that using D_1 to represent D_2 could lead to error, and subsequently inaccurate pond design. Unlike other methods where the particle settling velocity is computed from stokes equation or settling analysis, the method presented provides a direct method of taking measurements of the settling velocity with the hydrodynamic conditions properly accounted for.

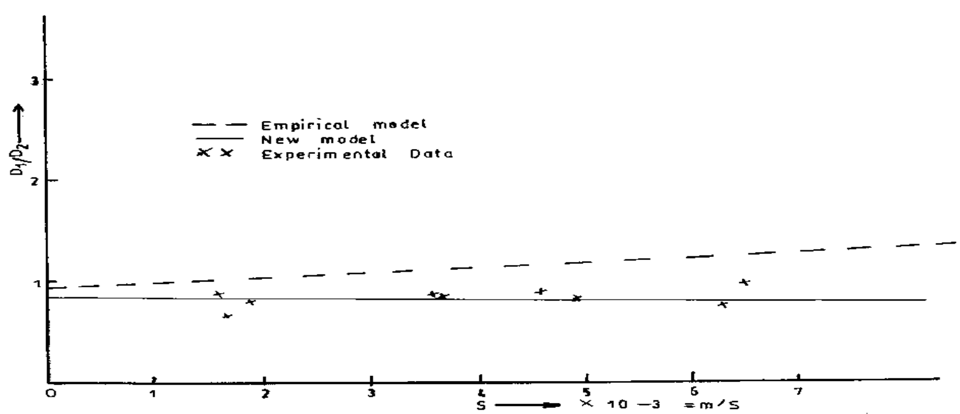

Figure-4
Comparison between experimental D_2/D_1 and computed D_2/D_1 based on different models

Nomenclature: a = function of dispersion number, settling velocity and flow velocity (m/sec), A = pond cross-sectional area (m^2), C = cross-sectional mean concentration (mg/1), d = dispersion number, d_1 = dispersion number of tracer, d_2 = dispersion number of settleable particle, D = dispersion coefficient (m^2/sec), D_1 = dispersion coefficient of settleable particle (m^2/sec), D_2 = dispersion coefficient of settleable particle (m^2/sec), h= pond depth (m), L = pond length (m), m = mass of pond water (g), M = total mass of tracer (g), t = time (secs), U = mean flow velocity (m/sec), U = estimated mean flow velocity (m/sec), U_{*1} = shear velocity (m/sec), U_* = dimensionless shear velocity (U_*/U), V_s = particle settling velocity from New Equation (m/sec), V_{sa} = particle settling velocity from Stokes Equation (m/sec), V^1_s = dimensionless settling velocity (V_s/U_*), w = pond width (m), x = longitudinal axis.

Greek Symbols: β = dimensionless settling velocity parameter (V^1/4), Κ = Von Karman constant – dimensionless, ρ = density of pond water (g/m^3), μ = dimensionless mean flow velocity, μ_s = dimensionless particle flow velocity, ψ = psi function – dimensionless

References

1. Crook J., Quality criteria for reclaimed water, *Water Science and Technology,* **24(9),** 109-121 **(1991)**

2. Marecos do Monte, M.H.F. and Mara D.D., The hydraulic performance of waste stabilization ponds in Portugal, *Water Science and Technology,* **19(12),** 219-227 **(1987)**

3. Agunwamba J.C., Dispersion Modeling of Waste Stabilization Pond, *Ph.D Thesis,* Department of Civil Engineering, University of Nigeria, Nsukka **(1992)**

4. Agunwamba J.C., Dispersion number determination in water stabilization ponds, *Water, Air and Soil Pollution,* **59**, 241-247 **(1991)**

5. Polprasert C. and Bhattarai K.K., Dispersion model for waste stabilizations ponds, *J. Env. Eng. Div. ASCE,* **111(1)**, 45-59 **(1985)**

6. Ojiako G.U., An integrated study of tracer and solid particle dispersions, *Indian Engineering Journal – Environmental section*, **69**, 25-31 **(1988)**

7. James A., An Alternative Approach to the design of Waste Stabilization ponds, *Water Science and Technology,* **19(22),** 213-218 **(1987)**

8. Summer B.M., Mean velocity and longitudinal dispersion of heavy particles in turbulent open channel flow, *J. S Fluid Mechanics,* **65**, 11-28 **(1974)**

9. Agunwamba J.C., Dispersion of settleable pollutants modified Levenspiel's approach, *J. Sci. Engr. Tech*., **9(3)**, 4276-4288 **(2002)**

10. Huisman L., Sedimentation and floatation models for waste stabilization ponds, *Journal of Environmental Engineering Division ASCE,* **110,** 550-561 **(1973)**

11. Harris E.K., A new Statistical Approach to the one dimensional diffusion model, *International Journal of Air and Water Pollution,* **7**, 799 **(1963)**

12. Thackston E.L., Hays J.R. and Krenkel P.A., Least squares estimation of mixing coefficient, *J. San. Eng. Div., ASCE,* **93(3)**, 47–58 **(1967)**

13. Agunwamba J.C., Egbuniwe N. and Ademiluyi J.O., Prediction of the dispersion number in waste stabilization ponds, *Water Research,* **26(1)**, 85-90 **(1992)**

14. Bickel P.J. and Doksun K.A., *Mathematical Statistics,* Holden-Day, Inc., San Francisco **(1977)**

15. Smith R., Vertical drifts and reaction effects upon contaminant dispersion in parallel shear flows, *J. Fluid Mech.,* **165**, 425-444 **(1986)**

16. Abramowitz M. and Stegun I.A., *Handbook of Mathematical Functions* Dovers Publications, INC., New York, **(1965)**

17. Arora K.R., Soil Mechanics and Foundation Engineering, Standard Publishers, Delhi **(1997)**

18. Imhoff K. and Fair G.M., *Sewage Treatment.* 2nd ed., John Wiley and Sons, Inc. New York **(1956)**

19. Fair G.M., Geyer J.C. and Okun D.A., *Elements of Water Supply and Waste Water Disposal,* 2nd ed. John Wiley and Sons, Inc. New York **(1971)**

20. Khurmi R.S., A Textbook of Hydraulics, Fluid Mechanics and Hydraulic Machines, S. Chand and Company Ltd., New Delhi **(2003)**

21. APHA, *Standard Methods for the Examination of Water and Wastewater,* 17th Ed. American Public Health Association, American Water Works Association, Water Pollution Control Federation, Washington, D.C. **(1992)**

22. Levenspiel O. and Smith W.K., Notes on the diffusion type model for the longitudinal mixing of fluids in flow, *Chemical Engineering Science,* **6**, 227-233 **(1957)**

23. Aris R., On the dispersion of a solute in a fluid flowing through a tube. *Proceedings Royal Society,* London Series A, 295:67 **(1956)**

24. Polprasert C., Dassanayake M.G. and Thanh N.C., Bacterial die – off kinetics in waste stabilization ponds, *J. Wat. Pollut. Contr. Fed.,* **55(3)**, 285-296 **(1983)**

25. Agunwamba J.C., Boundary conditions for waste stabilization pond, *Water Air and Soil Pollution,* **52**, 237-245 **(1990)**

26. Lambe T.W., *Soil Testing for Engineers,* John Wiley and Sons, Inc. New York **(1951)**

27. Ryshik L.M. and Gradstein I.S., *Tables of Series, Products and Integral,* Veb Deutscher Verlay Der Deutscherr Verlay Der Wissenschaften, Berlin **(1957)**