The fractal dimension study of butterfly wing pattern

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Abstract

The geometrical complexity in the wing pattern of the butterfly species of different taxa was analyzed for their fractal dimension, using mathematical models in Matlab. Fractals are geometrical figures which repeat themselves at progressively smaller scale and exhibit more complex structure at larger scale. The analysis was restricted to a small number of butterfly species of Saurashtra region, Gujrat, India. The analysis of FD value of wing pattern was conducted between species of each genus, within sub-families and among families of Lepidoptera. Preliminary results provide some evidence for the similarity of the FD value of the wing pattern. Among five families, Nymphalidae and Pieridae were found to have the same mean value for the FD of their wing pattern, in spite of their taxonomical difference. It was also found that within each family, the sub-families showed a significant variation in the mean FD value (one way ANOVA, p < 0.05). It was observed that different species in the same genus exhibit significant variation for the mean FD value (unpaired t-test, p < 0.05, one way ANOVA). The study thus exhibits that taxonomically diverse species may hold similar FD values and the determining factors for these results need to be investigated.

Keywords: MatLAB, Fractals, Lepidoptera.

Introduction

Referring to the literature, published on the butterfly fauna of Saurashtra region, and by understanding the subjective characters of wings pattern of butterfly species, the geometrical features of wing pattern were analyzed based on applied mathematics. The pilot study was conducted to measure the fractal dimension of complex wing structure by using well known Box-counting method[1].

The diversity in butterfly species was recorded in the several parts of the Saurashtra region[2]. Five different families manifested these butterfly species, are family Nymphalidae (Brush footed) followed by Papilionidae (Swallowtails), Lycaenidae (Blues), Pieridae (Sulpher) and Hesperiidae (Skippers)[3].

As on date, the taxonomic distribution of Lepidopteron is cited by number of publications based on their morphology and molecular study. There is no evidence of study exhibits topological view, technically could define taxonomic issues of lepidopteron. For most there is no field study that supports the identification of mimic species of butterfly. Withstanding this review, the present study was attempted to initiate the practice of implantation of mathematical tool on biological structure. To carry out the analysis, the concept of fractal property of butterfly wing pattern was studied and related fractal dimension was measured. As described later measure of fractal dimension gives an idea of complexity at different scales of an object[4], obtained data may provide an adequate tool to defining evolutionary pattern in butterfly taxa as well as insulation of mimic species. Moreover the similarity of FD values may coin reliability in indication of phylogenetic relationship. This study provides possible emergence for using of measurement of fractal dimension of wing pattern for classifying the butterfly species and searching the members to forming groups, were the hypothesis have been tested to study the diversity in butterfly wing pattern based on their geometrical complexity, viz. fractal dimension.

Fractal dimension: The fractal is coined by Mandelbrot (1977) means self similar pattern[5]. The fractal pattern mainly defined as partially self-similar pattern in which, small part or unit of the object is similar to another or remaining whole[6].

One of the applications of fractal theory is directly related to the measurement of complexity of biological structure in form of very small unit and at higher magnification. Many objects in nature show self-similarity. This fractality of the object can be measured in terms of their fractal dimension by using, well known Box-counting method, which can be demonstrated from scaling law.

Materials and methods

The present study was conducted during the time period of September to March 2012-2013. The sampling was carried out by collecting high-resolution photographs of total 41 species (Table-1) of butterfly of several areas of Saurashtra from reliable published literature.

Table-1: Butterfly species of several areas of Saurashtra region.

Common name	Scientific name	Common name	Scientific name
Family 1	*Nymphalidaea*	**Family 3**	*Lycaenidae*
Sub Family 1	*Acradae*	Sub Family 1	*Polymmatinae*
Tawny castor	*Acraea violae*	Pale grass blue	*Pseudozizeeria maha*
		Bright babul blue	*Azanus ubaldus*
Sub Family 2	*Danainae*	Zebra blue	*Syntarucus plinius*
Blue tiger	*Tirumala limniace*	Lime blue	*Chialdes laius*
Plain tiger	*Danaus chrysippus*	Gram blue	*Euchrysop cnejus*
Striped tiger	*Danaus genutia*		
Common crow	*Euploea core*	Sub Family 2	*Theclinae*
		Common silverline	*Spindasis vulcanus*
Sub Family 3	*Nymphalinae*		
Danaid egg fly	*Hypolimnas misippus*		
Blue Pansy	*Junonia orithya*	**Family 4**	*Pieridae*
Yellow pansy	*Junonia hierta*	Sub Family 1	*Coliadnae*
Lime pansy	*Junonia lemonias*	Common Emigrant	*Catopsilia Pomona*
Baronet	*Ethualia nais*	Mottled Emi	*Catopsilia pyranthe*
Common leopard	Phalanta phalantha	Small yellow	Eurema brigitta
Painted lady	Cynthia cardui	Common yellow	Eurema hecabe
Angled castor	*Ariadne ariadne*	Spotless Yellow	*Eurema laetaBoisduval*
Joker	*Byblia ilithyia*		
		Sub Family 2	*Pierinae*
Sub Family 4	*Satyridea*	Common gull	*Cepra nerissa*
Common threering	*Ypthima asteropemahrattta*	Plain orange tip	*Colotis eucharis*
Common evening brown	*Melanitis leda*	White orange tip	*Ixias marianne*
		Yellow orange tip	*Ixias pyrene*
		Small Salmon Arab	*Colotis amata*
Family 2	*Papilionedae*	Large Salmon Arab	*Colotis fausta*
		Crimson Tip	*Colotis danae*
Sub Family 1	*Papilioninae*	Pioneer	*Anaphaeis aurota*
Common mormon	*Papilio polytis*		
Common rose	*Pchlio ptaaristolochiae*		
Lime	*Papilio demoleus*	**Family 5**	*Hesperiidae*
		Subfamily	*Pyriginae*
		Conjoid swift	*Baoris conjuncta*
		Dart	*Patanthus sp.*
		Straight swift	*Paranara naso boda*

For each photograph of butterfly species, ten high-resolution photographic samples were collected and further converted into black and white image. The fractal dimension was calculated for each sample image, using MatLab (R2012b).

By Considering the Box-counting method in which object imposed on square (Boxes) plane with contrast background. The defined formula for measuring fractal dimension D of an object having two dimensions is given below,

$$D = \lim_{\varepsilon \to 0} \frac{\ln N(\varepsilon)}{\ln 1/\varepsilon} \qquad (1)$$

Where: $N(\varepsilon)$ is the number of boxes in a square grid of side size ε required to cover the object[1]. This definition can be illustrate based on scaling law (Equation-2).

$$N(\varepsilon) = C\left(1/\varepsilon\right)^D \qquad (2)$$

Where: D is the dimension in which one need C/ε boxes with side size ε for measuring dimension at each scale and C is the constant.

As the fractal dimension includes information from all possible scales, only the fractal structure of counter part of the selected portion was considered to study the geometrical arrangement of wing, formed by loop and network of vessels[7]. The study was also proposed to check, how distinctive the fractal dimension may be between related species or families. The aspects were also put forward to found the distinction in FD value of mimic species.

Results and discussion

The obtained average values of fractal dimension for five different families are given in Table-2. As shown in Table-2, there is no possible distinction for average FD value among different families (one way ANOVA, $p < 0.05$). The findings about, each sub-family within their family and each species, sharing common genus also shows the similarity for the fractal dimension of wing (unpaired t-test, $p < 0.05$, one way ANOVA, $p < 0.05$). Some of the data of FD values have been plotted in the graph (Figure-1).

Table-2: Average FD values of Butterfly Families.

| Family	FD
Nymphalidae	1.80483
Papilionedae	1.79258
Lycaenidae	1.78554
Pieriidae	1.79431
Hesperiidae	1.81188
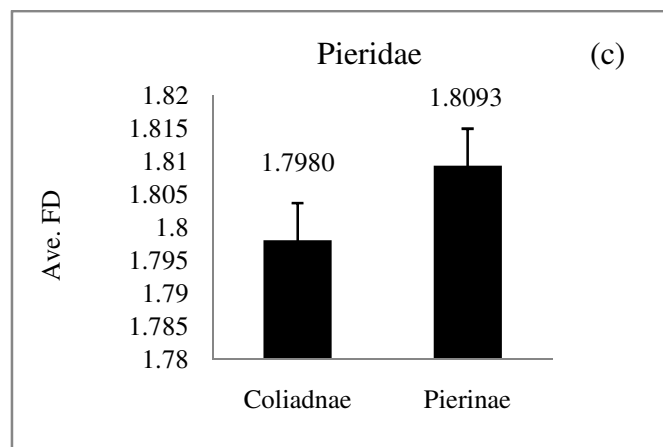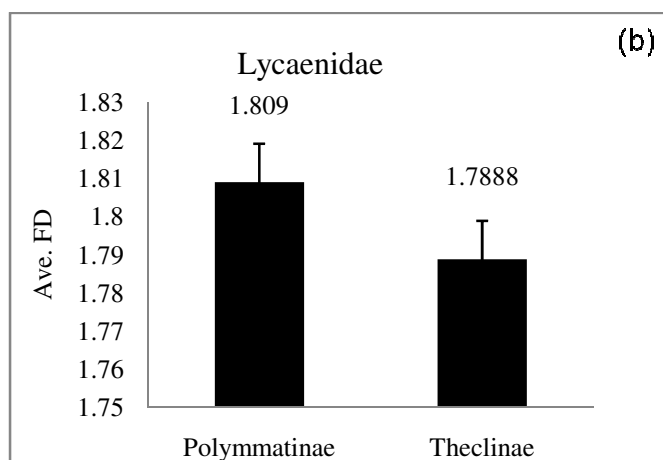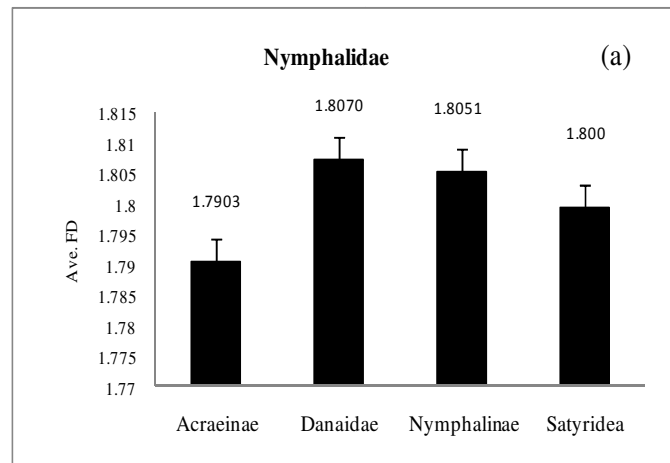
Figure-1a, b and c: Histogram shows an average FD value of different subfamilies for each family.

As previously pointed, our findings found no distinction in FD value for each family, sub- family and species.

The results also provide important note about mimic butterfly species. The butterfly species *Danaus chrysippus* is mimicked by the member of other sub-family *Hypolimnas misippus.* These visually looking similar members (Figure-3) don't have similar fractal dimension (Figure-2).
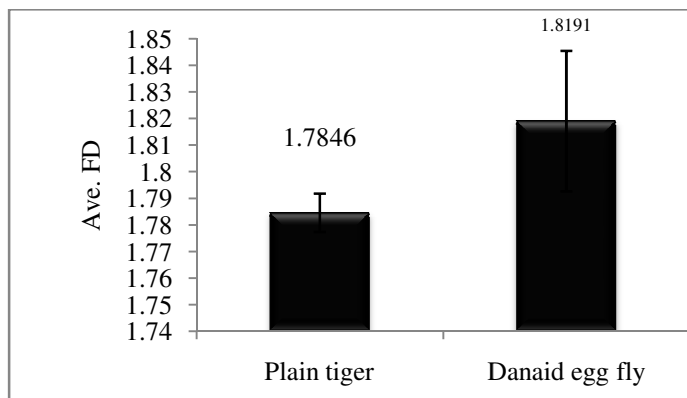
Figure-2: Histogram shows an average FD value of mimic species.
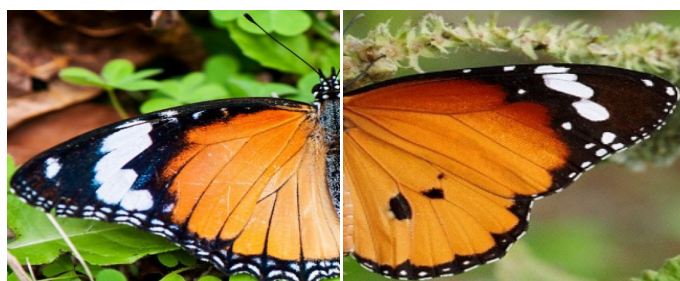
Figure-3: Similarity in wing pattern of mimic species. (a) Left hand: Danaid eggfly (Hypolimnas   misippus) wing pattern (b) Right hand: Plain Tiger (Danaus chrysippus) wing pattern.

Discussion: The butterfly fauna from the several part of Saurashtra region have been put forward to study the fractality of wing pattern of butterfly. The study was made to classify and identify the butterfly species, based on the fractal dimension, which exhibits the geometrical complexity formed in butterfly wing pattern. Thus from the results it can be found that, the different species exhibit almost similar value for their fractal dimension though, phylogenetically they are from different taxa. This is regarded as a fact, especially when the measurement includes important facts from all possible scales like, smallest unit to whole object of geometrical arrangement. Thus fractal dimension may provide an application to form group of species that look different, but might be show identical value of fractal dimension.

The similarity in FD value of different species from different families raises question about the evolutionary pattern of butterfly taxa. It can be assumed that the butterfly species do show possible distinction in FD value because they may have common elements as a raw material through which they developed depending on external factors that they exposed to. Thus measurement of fractal dimension of wing pattern can be used as a supplementary parameter to study butterfly taxa.

With respect to mimic species of butterfly, our analysis provide an adequate tool to identify those (the species which look similar at first site) based on their fractal property of wing pattern. The Plain tiger (*Danaus chrysippu)* species can be appropriately identified, based on their FD though it mimic by the Danaid egg-fly (*Hypolimnas misippus).*

Conclusion

This is the preliminary investigation, about the study of biological structure, in terms of their geometry by introducing the mathematical model. Although these results shows the fractality and fractal dimension of complex biological structure, they say nothing about the mechanism involve in this kind of particular pattern. Withstanding this critic, the study was established to initiate the application of mathematical programmer, for identifying group where, there a developmental raw material is common feature. Our findings are interesting, and also raise some questions about the topology that leads behind the biological structure. Thus the study warrants further work, to broaden the basis, to confirm the results, and in case to extend them.

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