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Method of composing and contents of the Tasks for students' calculation and graphic work in the electrodynamics course

Zharilkassin Iskakov
Almaty University of Power Engineering and Telecommunications, Dzholdasbekov Institute of Mechanics and Machine Science, Almaty, Kazakhstan
iskakov53@mail.ru

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Abstract

The technique of compiling tasks for calculating and graphic works for independent work of students on the elective discipline of electrodynamics for the specialty of electric power engineering is considered in this work. To do this, the study of the electromagnetic field of the plane capacitor and a solenoid is generalized and supplemented. From the relation between the magnetic induction and vector potential of the magnetic field of the capacitor the value of the vector potential is determined as a function of the radial coordinate, which decreases parabolically with distance from the axis of the capacitor. Expressions for energy of electric energy and magnetic field were found assuming that the electric field is concentrated and homogeneous in the condenser volume. The graphs of dependence of displacement current density on the radial coordinate were plotted that indicate a linear increase in the current density displacement inside the coil and hyperbolic decay outside the solenoid. As in the case of the capacitor the energies of the solenoid's electric and magnetic fields are determined, but for this once it is assumed that the magnetic field is concentrated and homogeneous in the solenoid's volume. The proposed method can be used by physics teachers to compose the assignments of calculation and graphic works for the independent work of students majoring in electrical power engineering when choosing a course of electrodynamics as elective discipline.

Keywords: Calculation and graphic work, electrodynamics course, electromagnetic field, a capacitor, a solenoid.

Introduction

In accordance with the State Compulsory Educational Standard and the standard curriculum for the the specialty 5B071800 – Electrical Power Engineering, the physics discipline (Fiz 1205), which is a core component of the basic disciplines, is active in program in the second semester with four maintenance loans[1]. Electrodynamics course with three maintenance loans is presented as one of the basic courses elective components and as a course for the early students' specialization in the third semester at the Almaty University of Power Engineering and Telecommunications[2]. This course does not repeat the content of the electricity and magnetism section, which is an integral part of the physics course, but up-grades at a higher theoretical level, specializing students in reliance on the students knowledge gained in studying of this section, and learns the properties of electric, magnetic and electromagnetic fields based on the Maxwell equations.

Student's individual work in the electrodynamics course in accordance with the number of credits covers three calculation and graphic tasks: the calculation of the electrostatic field and of direct current field, the calculation of the magnetostatic field, studying of the electromagnetic field and the additional questions to each of them[3].

In the framework of the calculation of the electrostatic field, the potential and field strength, the associated charges at the boundary of dielectrics and the force of attraction of the charge to the boundary plane is determined. In the studying of the direct current electric field, the electric field strength and current density in a medium with low conductivity are located at a point at an equal distance from two parallel conductors having a potential difference. The study of the electromagnetic field is made through the calculations of the fields of a flat capacitor and a solenoid.

Content and method of compiling of the tasks for calculation and graphic work in the electrodynamics course

The problem of finding the strength E, the electric field displacement D, the conduction current density j_c and displacement current density j_b, the strength H and induction B of the flat capacitor magnetic field from known properties: permittivity ε and electrical conductivity σ of homogeneous, isotropic and low-conductivity medium filling the gap between the plates , the distance d ($d^2 \ll S$, where S is the area of the plate) and the applied alternating voltage $u = U_m \sin\omega t$ was known[4] (Figure-1). In this paper, this problem is generalized by the representation of the charge magnitude on the plates in the general functionality in the form of $q = q(t)$ and is supplemented by the determination of the vector potential $\mathbf{A}$ of the magnetic field, the energies of the electric W_e and magnetic W_m fields, and their maximum values. We give the procedure for performing the task.

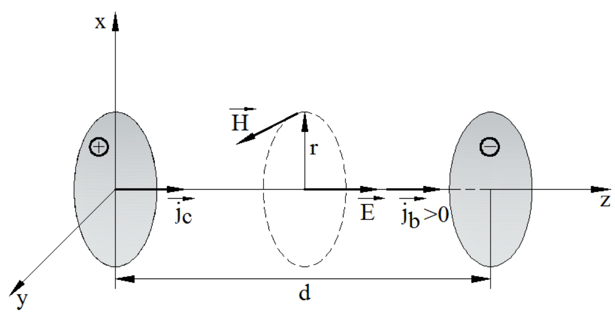

Figure-1: Scheme for selecting the capacitor field calculation contour.

The time-varying electric field leads to the appearance in space of a magnetic field, the circulation of the strength vector of which along the closed contour is equal to the algebraic sum of the currents covered by this contour[5,6]. Since the medium is weakly conducting, the following equation holds

$$\oint_l \mathbf{H}d\mathbf{l} = \int_S \left(\mathbf{j}_c + \frac{\partial \mathbf{D}}{\partial t} \right) d\mathbf{S} , \quad (1)$$

where $\frac{\partial \mathbf{D}}{\partial t} = \mathbf{j}_b$ - displacement current density; $\mathbf{D}$ - the electric field displacement; $\mathbf{j}_c$ - the conduction current density.

To solve the problem, we select the auxiliary contour as shown in Figure 1, i.e. in the form of a circle of radius r ($l = 2\pi r$), whose center coincides with the axis z of the cylindrical coordinate system[7]. At all points of such a contour the value of the field strength H is constant, while at each point of the contour the direction of $\mathbf{H}$ coincides with the direction of the tangent to the circle. In addition, the area $S = \pi r^2$ bounded by the contour is perpendicular to the axis of the system and at all points $\mathbf{j}_b = \text{const}$, $\mathbf{j}_c = \text{const}$, i.e. the current densities do not depend on the coordinates. Therefore, expression (1) when selecting a contour inside the condenser, i.e. for r p R can be represented in the form

$$H(r)2\pi r = \left| \mathbf{j}_c + \frac{\partial \mathbf{D}}{\partial t} \right| \pi r^2 , \quad (2)$$

Whence

$$H(r) = \left| \mathbf{j}_c + \frac{\partial \mathbf{D}}{\partial t} \right| \frac{r}{2} . \quad (3)$$

Expression (1) when choosing a contour outside the capacitor, i.e. for r f R can be represented in the form

$$H(r)2\pi r = \left| \mathbf{j}_c + \frac{\partial \mathbf{D}}{\partial t} \right| \pi R^2 , \quad (4)$$

whence the strength of the magnetic field is equal to

$$H(r) = \frac{1}{2} \left| \mathbf{j}_c + \frac{\partial \mathbf{D}}{\partial t} \right| \frac{R^2}{r} . \quad (5)$$

At the boundary of the capacitor, i.e. at $r = R$ the strength of the magnetic field is equal to

$$H(r) = \frac{1}{2} \left| \mathbf{j}_c + \frac{\partial \mathbf{D}}{\partial t} \right| R . \quad (6)$$

According to Ohm's law in differential form, the conduction current density can be represented in the form

$$j_c = \sigma E = \sigma \frac{U}{d} = \sigma \frac{q(t)}{Cd} = \sigma \frac{q(t)}{\varepsilon \varepsilon_0 S} , \quad (7)$$

where ε_0 is the electric constant[8].

The displacement current density by definition is equal to

$$j_b = \frac{\partial D}{\partial t} = \varepsilon \varepsilon_0 \frac{\partial E}{\partial t} = \frac{\varepsilon \varepsilon_0}{d} \frac{\partial U}{\partial t} = \frac{\varepsilon \varepsilon_0}{Cd} \frac{\partial q}{\partial t} = \frac{1}{S} \frac{\partial q}{\partial t} . \quad (8)$$

Substituting formulas (7) and (8) into expression (3), for the H field inside the capacitor, we finally obtain

$$H = \frac{1}{2S} \left| \frac{\sigma q}{\varepsilon \varepsilon_0} + \frac{\partial q}{\partial t} \right| r . \quad (9)$$

Magnetic field induction is determined by the formula

$$B = \mu \mu_0 H = \frac{\mu \mu_0}{2S} \left| \frac{\sigma q}{\varepsilon \varepsilon_0} + \frac{\partial q}{\partial t} \right| r , \quad (10)$$

where μ is the magnetic permeability of the medium, μ_0 is the magnetic constant.

For a field outside the capacitor, the strength H is equal to

$$H = \frac{1}{2S} \left| \frac{\sigma q}{\varepsilon \varepsilon_0} + \frac{\partial q}{\partial t} \right| \frac{R^2}{r} , \quad (11)$$

Magnetic induction B is equal to

$$B = \frac{\mu\mu_0}{2S}\left|\frac{\sigma q}{\varepsilon\varepsilon_0} + \frac{\partial q}{\partial t}\right|\frac{R^2}{r}. \qquad (12)$$

At the boundary of the capacitor, i.e. at $r = R$, the field strength expression H takes the form

$$H = \frac{1}{2S}\left|\frac{\sigma q}{\varepsilon\varepsilon_0} + \frac{\partial q}{\partial t}\right|R, \qquad (13)$$

For magnetic induction we get the following formula

$$B = \frac{\mu\mu_0}{2S}\left|\frac{\sigma q}{\varepsilon\varepsilon_0} + \frac{\partial q}{\partial t}\right|R. \qquad (14)$$

It can be seen from formulas (9) - (14) that the strength and induction of the magnetic field goes up linearly with the distance from the axis in the radial direction, the maximum values are reached at the boundary of the capacitor, then, with distance from the capacitor decays according to the hyperbolic law (Figure-2).

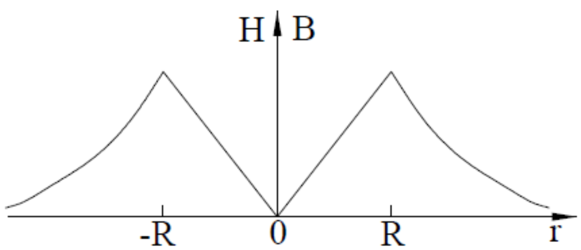

Figure-2: Dependences of the strength and induction of the magnetic field on the distance from the axis of the capacitor.

The vector potential **A** of the magnetic field is defined from the formula[9, 10]

$$\mathrm{rot}\mathbf{A} = \mathbf{B}. \qquad (15)$$

The vector **A** has the same direction as the vector $\mathbf{j}_b$, i.e. it is directed along the capacitor axis. In the cylindrical coordinate system, the vector **A** has only one projection $A_z = A$. Taking this into account we obtain from formula (15),

$$-\frac{dA_z}{dr} = B(r) = \frac{\mu\mu_0}{2S}\left|\frac{\sigma q}{\varepsilon\varepsilon_0} + \frac{\partial q}{\partial t}\right|r. \qquad (16)$$

From the formula (16) we find

$$A = -\frac{\mu\mu_0}{4S}\left|\frac{\sigma q}{\varepsilon\varepsilon_0} + \frac{\partial q}{\partial t}\right|r^2 + \mathrm{const}. \qquad (17)$$

We assume that, if $r = R$: A = 0, then

$$\mathrm{const} = \frac{\mu\mu_0}{4S}\left|\frac{\sigma q}{\varepsilon\varepsilon_0} + \frac{\partial q}{\partial t}\right|R^2 = A_{max}. \qquad (18)$$

After substituting the value of the constant of integration (18) into expression (17), we obtain

$$A(r,t) = \frac{\mu\mu_0}{4S}\left|\frac{\sigma q}{\varepsilon\varepsilon_0} + \frac{\partial q}{\partial t}\right|\left(R^2 - r^2\right). \qquad (19)$$

From formula (19) we see that the vector potential of the magnetic field is reduced in the radial direction according to the parabolic law (Figure-3), and its maximum value is on the capacitor axis.

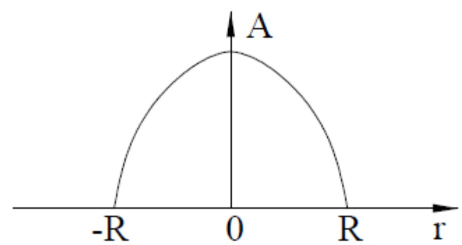

Figure-3: Dependence of the modulus of the magnetic field vector potential on the distance from the axis of the capacitor.

Conduction current density $j_c = \frac{\sigma q}{\varepsilon\varepsilon_0 S}$ and electric displacement current density $j_b = \frac{\partial q}{S\partial t}$ are the same over the plate area and are only time-dependent.

Then the vector potential of the magnetic field can be expressed in terms of the conductivity current density and the bias current

$$A(r,t) = \frac{1}{4}\mu\mu_0\left|j_c + j_b\right|\left(R^2 - r^2\right). \qquad (20)$$

Electric field energy is determined by the formula

$$W_e = \frac{q^2}{2C} = \frac{q^2 d}{2\varepsilon\varepsilon_0 S}. \qquad (21)$$

The electric field is considered to be concentrated inside the capacitor and homogeneous in volume here.
Magnetic field energy is defined from the formula

$$W_M = \int_V \frac{B^2}{2\mu\mu_0}dV, \qquad (22)$$

where

$$dV = 2\pi r dr \times d \qquad (23)$$

is the volume of the elementary ring (Figure-4). After substituting the value of B inside the capacitor (19) and expression (23) into the formula (22) and after integration, we obtain the expression for the energy of the capacitor magnetic field

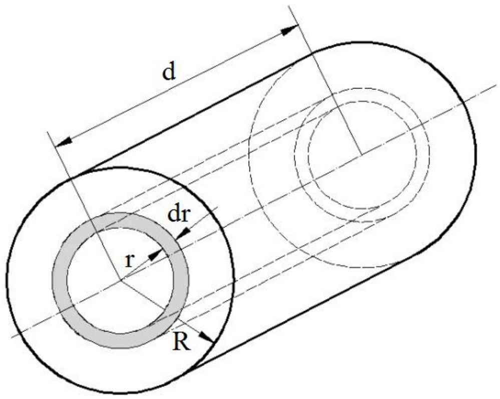

Figure-4: Scheme for calculating the field energy.

$$W_M = \frac{\pi\mu\mu_0 R^4 d}{16S^2}\left(\frac{\sigma q}{\varepsilon\varepsilon_0} + \frac{\partial q}{\partial t}\right)^2 . \qquad (24)$$

The maximum values of W_e and W_M can be found after srendering concrete of the functional dependence q=q(t).

For specific assignments for students, the functional dependence q=q(t) can be provided in the more common forms in engineering: $q=CU_m \sin\omega t$, $q=CU_m |\sin\omega t|$, q=ECexp[-t/(rC)], q=EC{1-exp[-t/(rC)]} (E – constant-current source electromotive force) etc.

The problem of finding the magnetic induction (magnetic field strength), the electric field strength (displacement), as well of the electric displacement current density j_b dependence on the distance from the solenoid axis to the transverse section radius R, the number of turns per unit length n and the alternating current passing through it $i = I_m \sin\omega t$ is known[11]. This problem is generalized with current strength presentation through the solenoid in the general functional dependence in the form of i=i(t) and is supplemented by calculating of electric field energy and magnetic field energy. We give the procedure for the task.

The time-varying magnetic field leads to the appearance in space of an electric field, the circulation of the strength vector of which is equal to the time derivative of the magnetic flux through the surface bounded by this contour, with the minus sign[12, 13]

$$\oint_l \mathbf{E}d\mathbf{l} = -\frac{\partial\Phi}{\partial t}. \qquad (25)$$

Here the symbol of the partial time derivative $(\partial/\partial t)$ emphasizes the fact that the contour and the surface stretched on it are immovable. Since the magnetic flux through the stretchable surface to the contour is equal to

$$\Phi = \int_S \mathbf{B}d\mathbf{S}, \qquad (26)$$

then

$$\frac{\partial}{\partial t}\int_S \mathbf{B}d\mathbf{S} = \int_S \frac{\partial \mathbf{B}}{\partial t}d\mathbf{S}.$$

In this equation we have exchanging places of the operations of differentiation by time and integration over the surface, since the contour and the surface are fixed. Then equation (25) can be represented in the form

$$\oint_l \mathbf{E}d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t}d\mathbf{S}. \qquad (27)$$

To solve the problem, we select the auxiliary contour as shown in Figures-5 and 6, i.e. in the form of a circle of radius r ($l = 2\pi r$), whose center coincides with the axis of the system. At all points of such a contour the value of the field strength is constant, while at each point of the contour the direction of **E** coincides with the direction of the tangent to the circle. In addition, the area $S = \pi r^2$, bounded by the contour is perpendicular to the axis of the system, and hence the directions of $\mathbf{B}$ and $d\mathbf{S}$ coincide.

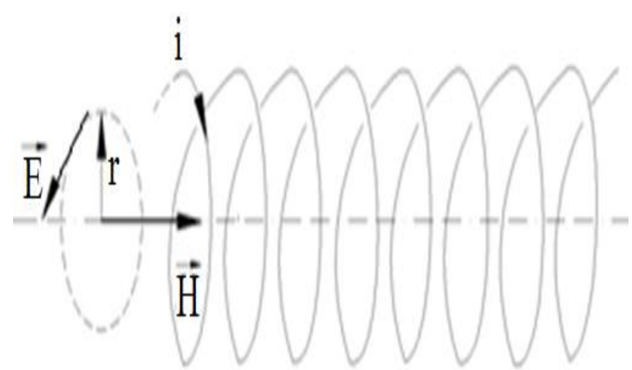

Figure-5: Scheme for selecting the solenoid field calculation contour inside the solenoid.

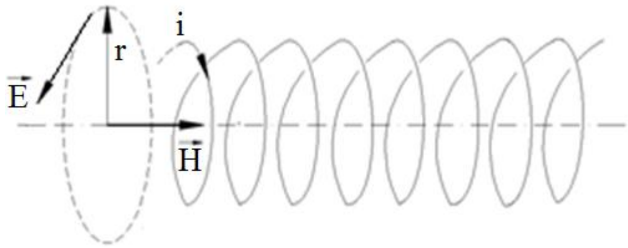

Figure-6: Scheme for selecting the solenoid field calculation contour outside the solenoid.

The magnetic field of the solenoid is concentrated in its inside, its magnetic induction is the same at all its points and is independent of the coordinates and is equal to

$$B = \mu\mu_0 ni. \tag{28}$$

Therefore, expression (27) with a radius of the contour smaller than the radius of the solenoid, i.e. r p R (Figure-5) can be represented in the form

$$E(r)2\pi r = -\frac{\partial B}{\partial t}\pi r^2, \tag{29}$$

whence, taking into account the relation (28), the electric field strength inside the solenoid is

$$E(r) = -\frac{\partial B}{\partial t}\frac{r}{2} = \frac{1}{2}\mu\mu_0 n\frac{di}{dt}r \tag{30}$$

and the displacement of the electric field is

$$D = \varepsilon\varepsilon_0 E = -\frac{1}{2}\varepsilon\varepsilon_0\mu\mu_0 n\frac{di}{dt}r. \tag{31}$$

When a radius of the contour greater than the radius of the solenoid, i.e. r f R (Figure-6), equation (27) can be represented in the form

$$E(r)2\pi r = -\frac{\partial B}{\partial t}\pi R^2. \tag{32}$$

Whence, respectively, taking into account the relation (28), the strength and the displacement of the electric field outside the solenoid are equal to

$$E(r) = -\frac{1}{2}\frac{\partial B}{\partial t}\frac{R^2}{r} = -\frac{1}{2}\mu\mu_0 n\frac{di}{dt}\frac{R^2}{r}, \tag{33}$$

$$D = -\frac{1}{2}\varepsilon\varepsilon_0\frac{\partial B}{\partial t}\frac{R^2}{r} = -\frac{1}{2}\varepsilon\varepsilon_0\mu\mu_0 n\frac{di}{dt}\frac{R^2}{r}. \tag{34}$$

At the boundary of the solenoid, the strength and displacement of the electric field are equal to

$$E(r) = -\frac{1}{2}\frac{\partial B}{\partial t}R = -\frac{1}{2}\mu\mu_0 n\frac{di}{dt}R, \tag{35}$$

$$D = -\frac{1}{2}\varepsilon\varepsilon_0\frac{\partial B}{\partial t}R = -\frac{1}{2}\varepsilon\varepsilon_0\mu\mu_0 n\frac{di}{dt}R. \tag{36}$$

Using the formula for determining the displacement current density and the field displacement formula, we obtain an expression determining the displacement current density inside the solenoid

$$j_b = \frac{\partial D}{\partial t} = -\frac{1}{2}\varepsilon\varepsilon_0\frac{d^2B}{dt^2}r = -\frac{1}{2}\varepsilon\varepsilon_0\mu\mu_0 n\frac{d^2i}{dt^2}r, \tag{37}$$

also the expression for the displacement current density outside the solenoid is in the form

$$j_b = -\frac{1}{2}\varepsilon\varepsilon_0\frac{d^2B}{dt^2}\frac{R^2}{r} = -\frac{1}{2}\varepsilon\varepsilon_0\mu\mu_0 n\frac{d^2i}{dt^2}\frac{R^2}{r} \tag{38}$$

It can be seen from the formulas (30), (31), (33), (34), (35) - (38) that the strength (and displacement) of the electric field and the displacement current density modulo (Figure-6) grow linearly with distance from the axis of the solenoid in the radial direction and at the boundary of the solenoid take the maximum values, are decreases according to the hyperbolic law (Figure-7) with the distance from the solenoid.

Magnetic field energy is determined by the formula

$$W_{\text{м}} = \frac{1}{2}Li^2 = \frac{1}{2}\mu\mu_0 n^2\pi R^2 li^2. \tag{39}$$

Here it is assumed that the magnetic field is concentrated within the solenoid and it is homogeneous in volume.

Electric field energy is defined from the formula

$$W_e = \frac{1}{2}\int_V \varepsilon\varepsilon_0 E^2 dV. \tag{40}$$

Here are

$$dV = 2\pi r dr \times d \tag{41}$$

is the volume of the elementary ring (Figure-2). After substituting (30) and (41) in (39) and after integration, we obtain the expression for the solenoid electric field energy

$$W_e = \frac{1}{16}\pi\varepsilon\varepsilon_0\left(\mu\mu_0\right)^2 n^2 d\left(\frac{di}{dt}\right)^2 R^4. \tag{42}$$

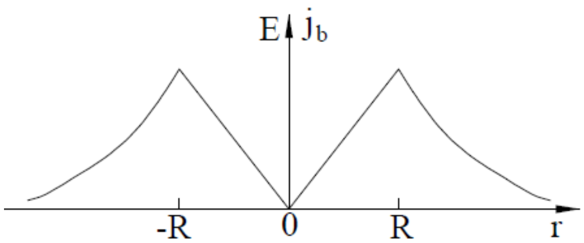

Figure-7: Dependences of the strength of an electric field and the displacement current density on the distance from the axis of the solenoid.

The maximum values of W_e and W_M can be found after dependence type determining i=i(t). The functional dependence i= i(t) can be provided in more common in engineering types for specific tasks for students:
$i=I_m \sin\omega t$, $i=I_m |\sin\omega t|$, $i=I_m \exp(-rt/L)$, $i= I_m [1-\exp(- rt/L)]$ etc.

Conclusion

The Method of composing and contents of the tasks for students' calculating and graphic work by the elective discipline - electrodynamics for the specialty - Electrical Power Engineering has been proposed. A brief summary of the two works is given.

The third calculation and graphic work on the calculation of the electromagnetic field of capacitor and solenoid is performed on the basis of Maxwell's equations and of displacement current density determination is generalized with representation of the charge magnitude on the capacitor plates and current strength through the solenoid in the general functional dependences. It is supplemented by the determination and investigation of the vector potential as a function of the distance from the capacitor axis and by the calculation of the electric field energy and magnetic field energy.

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