



Short Review Paper

An overview of the nonlinear chaos theory in the atmospheric systems

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Abstract

The nonlinear behaviour of the dynamical atmospheric systems may be properly studied by the chaos theory. The atmospheric flows exhibit fractal fluctuations in space and time. Due to nonlinear complexity, the actual physical mechanism of the atmospheric system is yet to be clearly understood. Thus a comprehensive study of the atmospheric instability in the light of nonlinear chaos theory is highly needed. An overview of the developments of the chaos theory in understanding the atmospheric systems is done in this paper. Proper estimation of the nonlinear chaos theory in meteorology may be significant and helpful for accurate prediction of atmospheric instability.

Keywords: Atmosphere, chaos, fractals, instability, non-linearity.

Introduction

The non linear dynamics and chaos theory begins with the advent of Henry Poincare in 1880, when he was studying three body problems of stars and found non periodicity in orbits¹. As atmospheric flows are turbulent in nature, it exhibits fractal fluctuations in space and time ranging from millimetre-sec to the scale of kilometre-year². Atmospheric systems highly sensitive to initial conditions, so its dynamics is been studied by chaos theory in different time. So in the last few years' application of chaos theory in atmosphere was about the evolution of fractal dimensions from the observed data and the existence of low dimensional attractors^{3,4}.

The word chaos first introduced in mathematics in 1975 by Li and Yorke⁵. But in physics Boltzman first used chaos in nineteenth century⁶. Still there is no generally acceptable definition of chaos. Actually chaos refers to unpredictable, irregular, dissipative and nonlinear dynamic systems. Chaos is not a disorder rather it is order without periodicity⁷. Atmosphere is a high dimensional complex dynamical system which is random and uncertain in nature. So chaos theory is highly applicable to a dynamical system like atmosphere^{8,9}.

Prediction is done by using observational data and also noise reduction is very important before data assimilation. Chaos theory introduced a new way to deal with the observational data, especially with the data which was earlier ignored due to its erratic behaviour.

Developments

The development of non linear chaos theory in meteorology has been summarized in Table-1.

Table-1: Developments of chaos theory in atmospheric systems.

Year	Developments
1880	Proposal of chaos theory by Henry Poincare ¹ .
1893	Nikola Tesla predicted about the complexity in nature ¹⁰ .
1898	Jacques Hadamard studied the chaotic motion of free particle sliding in a constant negative curvature surface frictionlessly. He established the sensitivity of solutions to the initial conditions ¹¹ .
1950	Actual physical models of atmosphere came into force and the choice of pattern depends on some arbitrary and unpredictable factors of the past ² .
1961	Nash and Sutcliffe estimated complex non linearity using cybernetics. The general circulations of the atmosphere are created in an arbitrary way. He concluded that the system controls its own faith ¹² .
1963	Edward Lorentz put forward the non linearity in atmospheric phenomenon and foundation of chaos theory ¹³ .
1965	The Butterfly effect explains that the behaviour of dynamic system highly sensitive to initial conditions ¹⁴ .
1970	Mendelbrot identified the fractal geometry of unpredictable irregular fluctuations in space and time ^{15,16} .
1971	Rulle and Takens introduced the concept of strange attractor ¹⁷ .
1990	Rulle described strange attractors as finite number of degrees of freedom and infinite number of frequencies ¹⁸ .
Presently	The progress of chaos theory with the advancement of computers which performs the repeated iteration process and introduced non-linearity in place of linear theory ¹⁹ .

Conclusion

Non-linear chaos theory begins with Henry Poincare and digital computers and later complemented by mathematics of global bifurcation theory and analysis of observed global chaotic data²⁰. Chaos is analysed by using the tools like Poincare maps, Lyapunov exponents and different definitions of dimension. Among the dimensions the fractal dimension is mostly used to measure the strangeness of attractors. It is related to the number of degrees of freedom^{21,22}. It determines the physics of space time evolution which enables the prediction of future evolution of weather pattern. The fractal fluctuation power spectra exhibits power law of form f^α , where f is the frequency and α is the exponent. But the intensity of fluctuation is a function of α only. Natural dynamical systems exhibits space time fractal fluctuations having non Euclidean geometrical structure and fractional dimensions^{23,24}.

The direction of thoughts has changed throughout the years, but the present investigations of non linear dynamics and chaos theory in atmospheric physics is been estimated as follows: i. The identification of fractals in meteorology and atmospheric physics. ii. The application of nonlinear dynamics and chaos in weather prediction. iii. The fractals are used for modelling of some aspects of atmospheric dynamics.

This particular field of research is now been enriched through estimation of accuracy in weather prediction. Such innovative pathways may perform as powerful contributor to determine the influences of the meteorological parameters like temperature, pressure, wind speed etc. of the atmospheric flows having fluctuations of zigzag pattern¹⁶. The actual physics behind the formation of vortices and large eddy circulation of turbulent fluid flows is yet to be discovered.

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