Research Journal of Recent Sciences ____E-ISSN 2277-2502
Vol. **5(6),** 68-72, June **(2016)** *Res. J. Recent Sci.*

Response of Double-Clamped Micro-Beams to the Casimir Force and SQFD Resting on Strain Gradient Elasticity Theory

Emran Khoshrouye Ghiasi*
Department of Mechanical Engineering, Ferdowsi University of Mashhad, Mashhad, Iran
em.khoshrou@alumni.um.ac.ir

Available online at: www.isca.in, www.isca.me
Received 16th February 2016, revised 14th March 2016, accepted 11th April 2016

Abstract

Present paper shows a numerical assessment to investigate double-clamped micro-beams by considering the Casimir force and SQFD resting on strain gradient elasticity theory using GDQ method. The mathematical formulations are considered to approximately model the non-linear effects of geometry, electrostatic actuation, and Casimir force on the oscillatory system. These equations, in conjunction with boundary conditions, are transformed into dimensionless governing equations and boundary conditions with the aim of simplifying numerical simulations. It is concluded that geometric and material properties both affect the pull-in characteristics of the polysilicon micro-beam. In particular, it is shown that present investigation reveals unexpected influence of the Casimir force on micro-structures.

Keywords: Micro-beam, Strain gradient elasticity theory, GDQ method, Casimir force, Reynolds equation.

Introduction

In just a few years from now micro-electro-mechanical systems (MEMS) have been continually developed into an endless variety of advantages as miniaturization, high accuracy, light weight, low in energy consumption, and high reliability[1-4]. Although there has been a steady growth in the MEMS fabrication technology, MEMS design is known as a difficult task. One of the most serious considerations on the use of MEMS is pull-in instability[5-6] which will result from the electrostatic-force interaction. In this respect, Hasanyan et al.[7] studied pull-in instability in functionally graded (FG) MEMS considering heat generated with respect to the electric current. They coupled non-linear governing equations, whereas the electric conductivity depends on the temperature. In the main, their model accounted for pull-in voltage, graduation of material properties, and mechanical effects. Moreover, a recent study carried out by Jia et al.[8] claims to show pull-in instability and free vibration of poly-SiGe graded micro-beams with a curved ground electrode. They considered geometrically non-linear deformation, electrostatic and intermolecular force, and axial residual stress and then solved the governing equations using differential quadrature (DQ) method.

Practically the electrostatic forces are utilized in MEMS on the grounds of there is distinctive features such as accuracy and stability[9]. To date, there exist many publications in Refs[10-13] regarding MEMS which are considered the electrostatic forces.

When a plate moves vicinity to another surface, for instance, in effect alternately stretching and squeezing any fluid that may be present in free space between the moving plates[14], Squeeze-film-damping happens. It is noted that on the basis of the cause and effect, sideways motion of the air ends in damping and the trapped air also acts as a spring because of its compressibility. As a result of this trend, Homentcovschi and Miles[15] proposed viscous damping and spring force in periodic perforated planar micro. They also developed a finite element model (FEM) to figure out the damping on the cell in the axisymmetric domain.

As for the quantum fluctuations, the Casimir effect is categorized as an attractive force between a pair of parallel plates[16-17]. In this case, Wang et al.[18] studied bending and vibration of an electrostatically actuated circular micro-plate due to the Casimir force to mark out differences of various parameters such as initial gap-thickness ratio, pull-in instability, and natural frequency. Their study indicated that although the Casimir force is on the increase, pull-in parameters decrease continuously from their maximum values at critical Casimir forces where the device collapses with zero applied voltage.

The study, which shows that strain gradient elasticity effect is significant in elastic deformation of small-scale structures, says that the Casimir force combined with SQFD shed some light on pull-in behaviour of present silicon clamped-clamped micro-beam. To this end, the governing equations and associated boundary conditions are derived and then after being discretized by the generalized differential quadrature (GDQ) method, are solved numerically. Final results show that pull-in behaviour may be significantly affected by the parameters when accounting for the full non-linearity.

Strain gradient elasticity theory

Based on the strain gradient theory[19], the strain energy U in a deformed isotropic linear elastic material occupying region Ω is given by

$$U = \frac{1}{2}\int_{\Omega}\left(\sigma_{ij}\varepsilon_{ij} + p_i\gamma_i + \tau^{(1)}_{ijk}\eta^{(1)}_{ijk} + m^s_{ij}\chi^s_{ij}\right)dv, \quad (1)$$

in which

$$\varepsilon_{ij} = \frac{1}{2}\left(\partial_i u_j + \partial_j u_i\right), \quad (2)$$

$$\gamma_i = \varepsilon_{mm,i}, \quad (3)$$

$$\eta^{(1)}_{ijk} = \frac{1}{3}\left(\varepsilon_{jk,i} + \varepsilon_{ki,j} + \varepsilon_{ij,k}\right) - \frac{1}{15}\delta_{ij}\left(\varepsilon_{mm,k} + 2\varepsilon_{mk,m}\right)$$
$$-\frac{1}{15}\left[\delta_{ij}\left(\varepsilon_{mm,i} + 2\varepsilon_{mi,m}\right) + \delta_{ki}\left(\varepsilon_{mm,j} + 2\varepsilon_{mj,m}\right)\right], \quad (4)$$

$$\chi^s_{ij} = \frac{1}{2}\left(e_{ipq}\varepsilon_{qj,p} + e_{jpq}\varepsilon_{qi,p}\right), \quad (5)$$

where: u_i is the displacement vector, ε_{mm} is the dilatation gradient vector, $\eta^{(1)}_{ijkl}$ is the deviatoric stretch gradient tensor, and χ^s_{ij} is the symmetric rotation gradient tensor. Likewise, the corresponding stress measures defined as[19]

$$\sigma_{ij} = k\,\delta_{ij}\varepsilon_{mm} + 2\mu\varepsilon'_{ij}, \quad (6)$$

$$p_i = 2\mu l_0^2\gamma_i, \quad (7)$$

$$\tau^{(1)}_{ijk} = 2\mu l_1^2\eta^{(1)}_{ijk}, \quad (8)$$

$$m^s_{ij} = 2\mu l_2^2\chi^s_{ij}, \quad (9)$$

where ε'_{ij} is deviatoric strain, p_i, $\tau^{(1)}_{ij}$, and m^s_{ij} are the higher order stresses, k and μ are bulk and shear modulus, respectively, and l_0, l_1, and l_2 are additional independent material parameters associated with gradients, deviatoric stretch gradients, and rotation gradients, respectively.

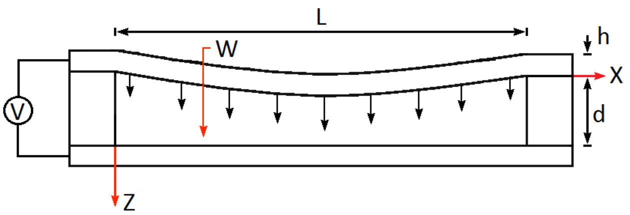

Figure-1
Schematic configuration of a double-clamped micro-beam

Theoretical formulation

A double-clamped micro-beam is modelled as an electrode beam with a stationary ground electrode underneath as shown in Figure-1. The length, width, and thickness of micro-beam are L, b, and h, respectively. Also, d denotes the air initial gap.

The Casimir force takes the form of[20]

$$F_{\text{Casimir}} = \frac{\pi^2\bar{h}cb}{240\times(d - W)^4}, \quad (10)$$

where $\bar{h}$ = 1.055 × 10^{-34} J is Plank's constant divided by 2π, c = 3 × 10^8 ms^{-1} is the speed of light in vacuum, and W is the deflection of micro-beam.

The equation of motion of the double-clamped micro-beam subjected to the combined effects of electrostatic actuation and the Casimir force can be expressed as

$$EIW''''_X + \rho A\ddot{W} + Pb$$
$$= EA\left[\int_0^L W'^2_X dX + N_0\right]W''_X + F_{\text{Electrostatic}} + F_{\text{Casimir}}, \quad (11)$$

where EI is the flexural rigidity, ρ is density, $A = bh$ is the cross-sectional area, P is the pressure, and N_0 is a constant denoting the residual force acting on the micro-beam, respectively. Starting with the assumption of isothermal and small pressure variation, the SQFD is then given by

$$P''_X + P''_Y = \frac{12\mu\dot{Z}}{d^3}, \quad (12)$$

where μ is viscosity of the medium in the gap. The micro-beam is subjected to the following boundary conditions

$$\underbrace{W = W'_X}_{X=0} = \underbrace{W = W'_X}_{X=L} = 0. \quad (13)$$

Then, the pressure boundary conditions for the case shown in Figure-1 is

$$\underbrace{P'_X}_{X=0,L} = 0. \quad (14)$$

By introducing the following dimensionless variables

$$\hat{W} = \frac{W}{d},\ \hat{X} = \frac{X}{L},\ \hat{T} = \frac{T}{\tau},\ \tau = \sqrt{\frac{\rho bhL^4}{EI}},\ \hat{P} = \frac{P}{P_a},\ \hat{h} = \frac{d}{d+W}, \quad (15)$$

where P_a is the ambient pressure. Afterwards, upon substitution of the dimensionless quantities given in Equation-15 into Equations-11 and 12 leads to

$$\hat{W}''''_{\hat{X}} = \left[\alpha\int_0^L \hat{W}'^2_{\hat{X}}dX + N\right]\hat{W}''_{\hat{X}} - \ddot{\hat{W}} - \beta P + \frac{\gamma}{\left(1-\hat{W}_{\hat{X}}\right)^2} + \frac{\Delta}{\left(1-\hat{W}_{\hat{X}}\right)^4}, \quad (16)$$

$$\hat{P}''_X = \mathrm{I}\left(\dot{\hat{P}}_{\hat{X}} + \dot{\hat{h}}\right), \quad (17)$$

in which

$$\alpha = 6\left(\frac{h}{d+W}\right)^2,\ N = \frac{N_0L^2}{EI},\ \beta = \frac{P_abL^4}{EI(d+W)},\ \gamma = \frac{6\varepsilon V_{DC}^2L^4}{E(d+W)^2h^3},$$
$$\Delta = \frac{12L^4\pi^2\bar{h}c}{240Eh^3d^5},\ \mathrm{I} = \frac{12\mu}{P_a(d+W)^2}\sqrt{\frac{\rho bh}{EI}}, \quad (18)$$

where $\varepsilon = 8.854187817620 \times 10^{-12}$ F/m is the permittivity of the free space.

Applying the GDQ Method

Here, the GDQ method[21-22] is employed to solve the non-linear partial differential equations. To this end, by considering Equations-16 and 17 one can obtain as

$$\sum_{j=1}^{N} C_{ij}^{(4)} \hat{W}_j = \left[\alpha \sum_{m=1}^{N} \sum_{n=1}^{N} C_m \hat{W}_m C_{ij}^{(2)} \hat{W}_n + N \right] \sum_{j=1}^{N} C_{ij}^{(2)} \hat{W}_j - \ddot{\hat{W}}_i - \beta P_i + \frac{\gamma}{\left(1-\hat{W}_i\right)^2} + \frac{\Delta}{\left(1-\hat{W}_i\right)^4}, \tag{19}$$

$$\sum_{j=1}^{N} C_{ij}^{(2)} \hat{P}_j + \sum_{j=1}^{N} C_{ij} \hat{W}_j = \mathrm{I}\left(P_a \dot{\hat{W}}_i + \hat{h} \dot{\hat{P}}_i \right), \tag{20}$$

where $C_{ij}^{(k)}$ is the weighting function. In addition, boundary conditions can be rewritten as

$$\sum_{j=1}^{N} C_{1j} \hat{W}_j = \sum_{j=1}^{N} C_{Nj} \hat{W}_j = 0, \tag{21}$$

$$\sum_{j=1}^{N} C_{1j} \hat{P}_j = \sum_{j=1}^{N} C_{Nj} \hat{P}_j = 0. \tag{22}$$

It is important to note that the procedure explained above leads to a system of 15 ($n \times m$) algebraic equations with the same number of unknowns.

Simulation and Discussion

Consider a clamped-clamped micro-beam made of polysilicon subjected to an electrostatic actuation and the Casimir force so that the geometric and material properties are listed in Table-1. Note that the numerical assessment of Equation-19 is carried out until the error norm becomes less than 10^{-4}.

Table-1
Geometric and material properties

Parameters	Values
Young's modulus (E) (N/m^2)	$169 \cdot 10^9$
Density (ρ) (kg/m^3)	2332
Viscosity (μ) (N/sm^2)	$1.82 \cdot 10^{-5}$
Length (L) (µm)	400
Width (b) (µm)	40
Thickness (h) (µm)	15
Initial gap (d) (µm)	60

Figure-2 exhibits the midpoint deflection versus time in the absence of applied voltage with and without the Casimir force for present polysilicon clamped-clamped micro-beam. As could be seen in this figure, effects of the Casimir force leads to very inaccurate results. On the other hand, it is deduced that as the initial gap between micro-beam and stationary ground electrode is precisely equal to or smaller than d = 60 µm, the oscillatory system may collapse independently of components. In addition, it is noted that the results shown in Figure-2 are obtained based on the full non-linear equation of motion of micro-beam. However, by neglecting the non-linear terms, the obtained midpoint deflection may drop in value. Against, Jia et al.[23] showed the maximum deflection under the applied voltage may be increased, if the effects of pull-in voltage are taken into account. Furthermore, pull-in voltage of the micro-structure will also be varied by variation of gap ratio, $6(\frac{h}{d+W})^2$.

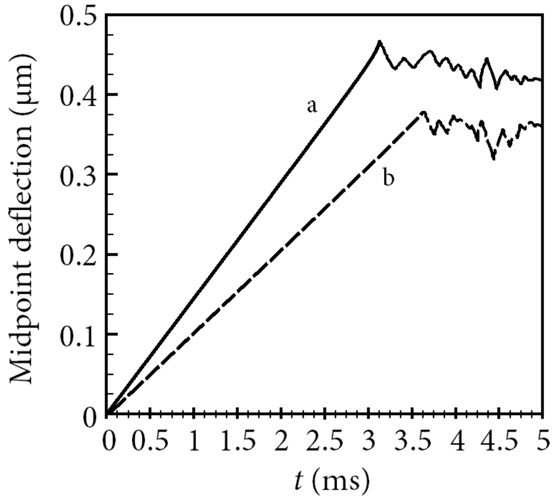

Figure-2
Variation of the midpoint deflection versus time (a) with and (b) without the Casimir force

The results given in Figure-3 clearly show both linear and non-linear result of the strain gradient elasticity theory for the pull-in voltage of micro-beam versus gap ratio with and without the Casimir force effect. Based on this figure, by increasing the gap ratio, the pull-in voltage calculated from both the linear and non-linear analysis increases. It is noted that neglecting the stress components in Equation-6 may lead to under-estimation of pull-in voltage. Nevertheless, for a constant gap ratio, an increase in initial gap d results a decrease in the pull-in time when the oscillatory system is yet statically stable. Hence, it can be concluded that the Casimir force varies almost linearly according to geometrical parameters.

Figure-4 has been provided to validate the accuracy of pull-in gap versus length from present numerical assessment. It should be mentioned that the critical pull-in gap with the Casimir force is identical to that without the Casimir force as the micro-beam is shorter. This is due to the fact that effect of the Casimir force, both in terms of pull-in voltage and gap, could be considerable.

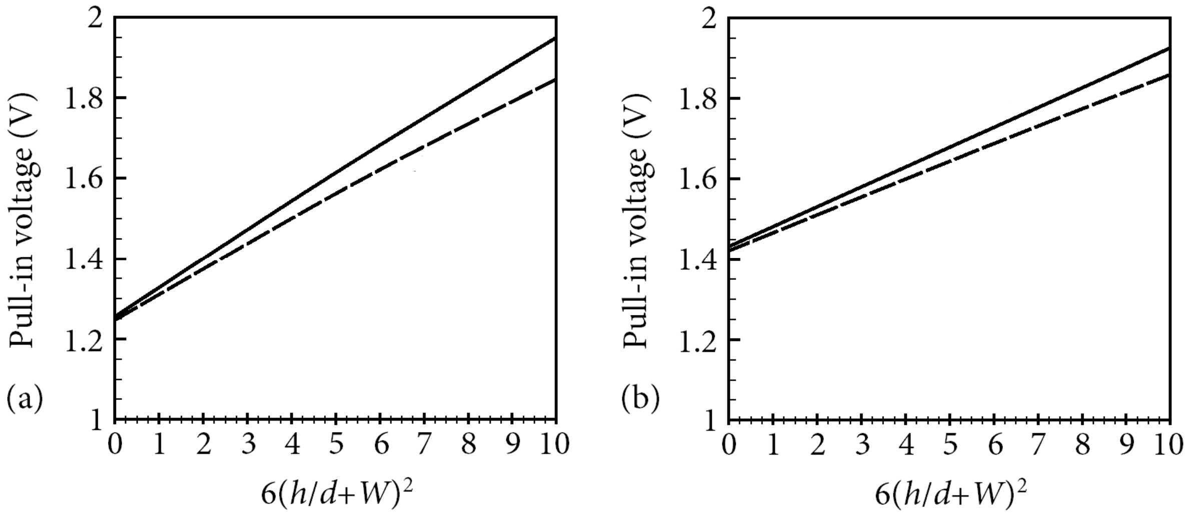

Figure-3
Influence of the gap ratio on the pull-in voltage response considering linear analysis (dotted line) non-linear analysis (direct line) (a) With and (b) without the Casimir force

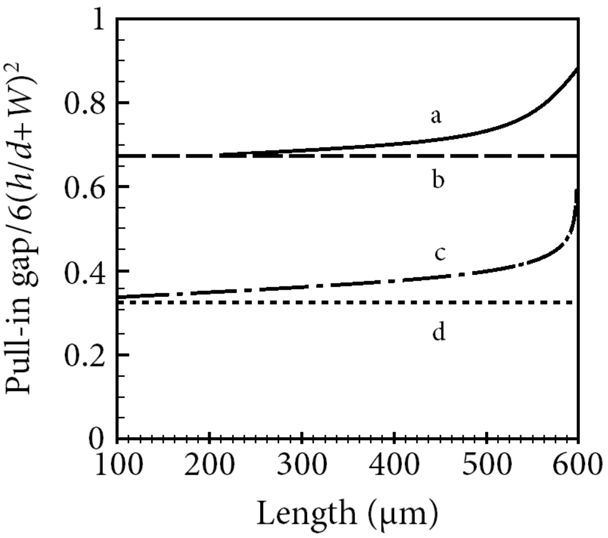

Figure 4
Effect of the length on pull-in gap / 6(*h*/*d*+*W*)^2 with and without the Casimir force (a) with the Casimir force[24], (b) without the Casimir force[24], (c) with the Casimir force by present analysis and (d) without the Casimir force by present analysis

According to Equation-18, Δ is a parameter of geometrical and material properties such as initial gap between micro-beam and stationary ground electrode. As seen in Figure 5, the calculated values of γ increase with an increase in Δ. It means that in the case of $\frac{L^4}{h^3 d^5} >> 9.75 \times 10^{-3}$, both parameters of pull-in time and pull-in voltage can be replaced by each other with any changes in the properties of micro-beam.

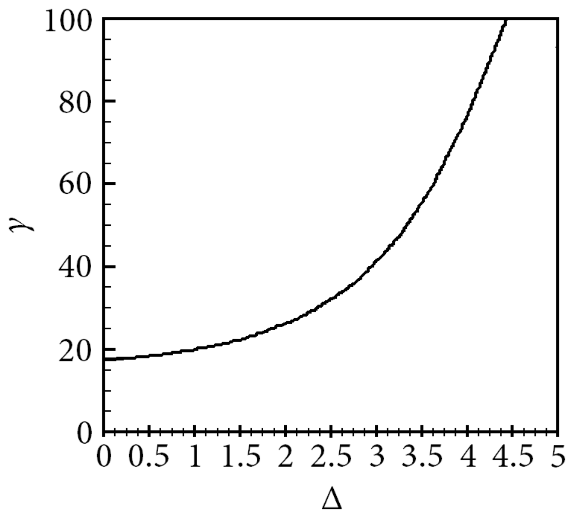

Figure-5
Variation of γ versus Δ

Conclusion

In this paper, the GDQ method was employed to investigate effect of the Casimir force and SQFD on polysilicon double-clamped micro-beam response resting on strain gradient elasticity theory. To do so, a non-linear Euler-Bernoulli beam model was utilized which accounts for the axial residual stress, the geometric non-linearity of mid-plane stretching as well as an electrostatic and the Casimir force. Furthermore, non-linear Reynolds equation was modelled numerically for SQFD in an unsteady condition. Overall, the numerical results showed that decreasing the applied voltage may lead to early instability in the micro-beam. The results also were compared with those obtained by previous experiments in the literature.

References

1. Shi H., Fan S., Xing W. and Sun J. (2015). Study of weak vibrating signal detection based on chaotic oscillator in MEMS resonant beam sensor. *Mechanical Systems and Signal Processing*, 50-51, 535-547.
2. Caruntu D.I. and Martinez I. (2014). Reduced order model of parametric resonance of electrostatically actuated MEMS cantilever resonators. *International Journal of Non-Linear Mechanics*, 66, 28-32.
3. Hu Y., Shen X., Zhang Y., Wang Z. And Chen, X. (2014). Research reviews and prospects of MEMS reliability. *Integrated Ferroelectrics*, 152, 8-21.
4. Sheng H. and Zhang T. (2015). MEMS-based low-cost strap-down AHRS research. *Measurement*, 59, 63-72.
5. Zahng W.M., Yan H., Peng Z.K. and Meng G. (2014). Electrostatic pull-in instability in MEMS/NEMS: A review. *Sensors and Actuators*, 214(A), 187-218.
6. Joglekar M.M. and Pawaskar D.N. (2011). Closed-form empirical relations to predict the dynamic pull-in parameters of electrostatically actuated tapered microcantilevers. *Journal of Micromechanics and Microengineering*, 21, 1-12.
7. Hasanyan D.J., Batra R.C. and Harutyunyan S. (2008). Pull-in instability in functionally graded microthermoelectromechanical systems. *Journal of Thermal Stresses*, 31, 1006-1021.
8. Jia X.L., Yang J., Kitipornchai S. and Lim C.W. (2012). Pull-in instability and free vibration of electrically actuated poly-SiGe graded micro-beams with a curved ground electrode. *Applied Mathematical Modelling*, 36, 1875-1884.
9. Ai S. and Pelesko J.A. (2007). Dynamics of a canonical electrostatic MEMS/NEMS system. *Journal of Dynamics and Differential Equations*, 20(3), 609-641.
10. Chuang W.C., Lee H.L., Chang P.Z. and Hu Y.C. (2010). Review on the modelling of electrostatic MEMS. *Sensors*, 10, 6149-6171.
11. Sedighi H.M. (2014). Size-dependent dynamic pull-in instability of vibrating electrically actuated microbeams based on the strain gradient elasticity theory. *Acta Astronautica*, 95, 111-123.
12. Son C. and Ziaie B. (2008). Pull-in instability of parallel-plate electrostatic microactuators under a combined variable charge and voltage configuration. *Applied Physics Letters*, 92, 1-4.
13. Yazdanpanahi E., Noghrehabadi A. and Ghalambaz M. (2013). Balance dielectric layer for micro electrostatic switches in the presence of capillary effect. *International Journal of Mechanical Sciences*, 74, 83-90.
14. Altuğ Bıçak M.M. and Rao M.D. (2010). Analytical modelling of squeeze film damping for rectangular elastic plates using Green's functions. *Journal of Sound and Vibration*, 329, 4617-4633.
15. Homentcovschi D. and Miles R.N. (2010). Viscous damping and spring force in periodic perforated planar microstructures when the Reynold's equation cannot be applied. *Journal of the Acoustical Society of America*, 127(3), 1288-1299.
16. Buks E. and Roukes M.L. (2001). Stiction, adhesion energy, and the Casimir force effect in micromechanical systems. *Physical Review*, 63(B), 1-4.
17. Bordag M., Mohideen U. and Mostepanenko V.M. (2001). New developments in the Casimir effect. *Physics Reports*, 353, 1-205.
18. Wang Y.G., Lin W.H., Li X.M. and Feng Z.J. (2011). Bending and vibration of an electrostatically actuated circular microplate in presence of Casimir force. *Applied Mathematical Modelling*, 35, 2348-2357.
19. Lam D.C.C., Yang F., Chong A.C.M., Wang J. and Tong P. (2003). Experiments and theory in strain gradient elasticity. *Journal of the Mechanics and Physics of Solids*, 51, 1477-1508.
20. Lamorreaux S.K. (2005). The Casimir force: background, experiments, and applications. *Reports on Progress in Physics*, 68, 201-236.
21. Shu C., Khoo B.C., Chew Y.T. and Yeo K.S. (1996). Numerical studies of unsteady boundary layer flows past an impulsively started circular cylinder by GDQ and GIQ approaches. *Computer Methods in Applied Mechanics and Engineering*, 135, 229-241.
22. Bellman R. and Casti J. (1971). Differential quadrature and long-term integration, *Journal of Mathematical Analysis and Applications*. 34, 235-238.
23. Jia X.L., Yang J. and Kitipornchai S. (2011). Pull-in instability of geometrically nonlinear micro-switches under electrostatic and Casimir forces. *Acta Mechanica*, 218, 161-174.
24. Lin W.H. and Zhao Y.P. (2005). Casimir effect on the pull-in parameters of nanometer switches. *Microsystem Technologies*, 11, 80-85.