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The Dynamics of Vertical Gyroscope Rotor with Mass Imbalance and Disk Skewing with an Allowance for Hard Nonlinear Resilient Characteristics

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Abstract

The dynamics of vertical unbalanced gyroscope rotor with hard nonlinear resilient characteristics is considered. For the purpose of investigation of rotor vibrations, its dynamic and mathematical model is constructed. The motion equations are recorded as equations of Lagrange of the 2nd kind. The expressions of amplitude and phase of the main harmonic component are expressed using expansion of solutions to the equations of expressed vibrations as well as harmonic balance to the series of Fourier. The equations as variations, and then the equations of Hill type, are recorded to study hardiness on the basis of motion equations. According to the theory of Floquet, these equations are solved, and the stability criterion is found using the harmonic balance method. We concentrate on study of skewing influence, mass imbalance, disk thickness, size of nonlinear characteristics of the resilient mounting as well as external buffering to amplitude and phase and frequency characteristics as well as area boundaries of variability of the main resonance vibration. On the basis of analysis of research results the particularities of dependencies of the amplitude and phase of the main resonance vibration and instability region boundaries are accentuated and described from the angle between the imbalance lines, disk thickness, external resistance and shaft speed without the hard nonlinear resilient characteristics of the rotor. They can be important for development of optimal control of resonance vibrations, determination of optimal construction parameters, working speed range and rotor balancing methods in the pre-design works. The main working expressions are presented in a compact and non-dimensional form.

Keywords: Gyroscope Rotor, Nonlinear Characteristics, Resonance Vibration, Instability Region, Mass Imbalance, Disk Skewing.

Introduction

As is well known, high-speed rotary machines are widely used in many industries (electricity, electronics and radio, food, light, chemical, petroleum, medical, metallurgy, aerospace, nuclear, etc.). High efficiency, low weight and high power density, relatively low cost of production and operation, as well as a small environmental pollution lead to the expansion of applications of rotary machines. Consequently, it is not surprising that the rotary machines are studied for a long time. Despite this, many problems are unsolved, in particular related to the joint action of imbalanced mass and skewed disk to vibrations and stability, taking into account the nonlinear characteristics and nonlinear resistance and subsequently stabilization of nonlinear resonance vibrations of rotary machines.

In 1983, Benson showed that the combined influence of skew and imbalance of disk mass, i.e. the action of centrifugal forces and gyroscopic active torques causes that processing rotor has the unusual phase characteristic, and besides the oscillations phase does not need to match the orientation of imbalance at low speeds[1]. This may significantly change the methods of rotor balancing. Because frictional forces are only included in the equation of rotor translational displacement, the resonance amplitude of translational and angular displacements is indefinitely growing at the second critical speed, which gives a distorted picture of the amplitude- and phase- frequency characteristics of rotor. This omission was considered in studies of the dynamics of two-bearing cantilever rotor with two-generalized imbalance, external damping was taken into account in all four equations of motion[2]. Due to this, we have the ability to properly construct amplitude-and phase- frequency characteristics of rotor, to explore the effects of imbalanced mass and skewed disk, cantilever shaft and external damping, to compare oscillations' amplitude at the critical speeds.

Considerable number of works is devoted to determination of position and orientation of imbalanced mass and skewed disk and, respectively, balancing methods of the rotor vibration control. Here, first of all the methods for determining the rotor mass imbalance and comparison of two methods of balancing are proposed[3]; the first of them is based on the measurement of force, acting on the bearings of balancing machine and the other - the measurement of the corresponding deformation[4]. Additional scales with a few pendulums are used for automatic balancing; motion of rotor is modeled on a computer[5, 6]. The problem, associated with the orientation sensor in the holospectrum technique, is still not resolved[7]. Herewith the holospectrum technique works on the basis of the rotor balancing. Currently, to determine the imbalance, initial phase vector (IPV) is used, but sometimes this method makes the equilibrium state is not defined. It contains the necessary modifications of this method. The principal compound of effects of a value of the major axis and the initial phase angle (IPA) of precession orbit is to be replaced for IPV value. At that the magnitude and angular position of unbalanced mass can be determined exactly for the amount of the major axis and IPA value respectively. Estimation of value will not now depend on the sensor orientation, an optimal result will be provided. It was suggested an effective control scheme for transverse vibrations due to the imbalance of the rotor shaft, and theoretical study[8]. To do this, it's necessary to use an electromagnetic exciter, mounted on the stator in a place, convenient to operate transverse vibrations of the rotor through the air gap around the rotor. Suitable electromagnetic force of response is achieved by changing the control current, proportional to the displacement of the rotor section. This method provides control over the driving force in the air gap, freedom from hardship and loss of service and wear. The centrifuge with a system of automatic removal of centrifuge vibrations, generated by its imbalance is offered[9]. Centrifuge rotor rotates about a fixed axis point. Two or more balloons in the ring, which are attached to the rotor, can automatically eliminate its vibration. The balls, which are also called free elements, can change their position within the ring so as to compensate for dynamic forces. The equations that determine the system behavior, as well as graphics, describe the rotor vibration and behavior of a ball in the presence of imbalance. The article explains that the balls occupy the final position, when the rotor and balls are dynamically stable. Researches of hydroelastic vibrations of vertical gyro rotor, considering the energy source; it also offers practical methods for rotor control[10].

Articles, dedicated to nonlinear oscillations in physical systems (including rotary), attract the attention. The monograph of Hayashi studied in detail nonlinear oscillations in physical systems with one degree of freedom[11]. W. Szemplinska-Stupnicka added these researches by the analysis of resonant curve harmonics of higher order for the solution of oscillations equation, considering the dependence of their amplitudes and frequency phases and the assumption of a constant value of the amplitude (for the driving force) and the damping factor[12]. The study of non-linear resonant oscillations of main harmonics and other forces, taking into account nonlinear resistance[13]. Free vibrations of the rotor Jeffcott, shaft of which has strong nonlinear elastic properties were considered[14]. Here equation of free vibrations of the rotor Jeffcott is non-linear differential equation of second order with the complex deviation. This differential equation was analytically settled on the basis of Krylov –Bogolyubov method at two different initial conditions. The resulting solution describes the oscillatory motions of the rotor. Here investigates the influence of hydrodynamic, gyroscopic forces and damping, the change rotor mass on the rotor vibration, and then the results are analyzed. In the studies of the dynamic characteristics of a flexible rotor on ball bearings, excitation source is a mass imbalance[15]. Rotor shaft with one disc is installed on two elastic supports with bearings and ball bearing. The nonlinearity is due friction to the radial clearance and Herztian contact between races and rolling elements. To solve nonlinear differential equations by the known harmonic balance method, the procedure for the evaluation of harmonic and superharmonic components of the rotor oscillations was developed. The monograph of Grobov V. studies the fluctuations of flexible rotor on elastic supports with non-linear characteristic, but does not concentrate interaction with generalized disk imbalance[16].

The above review of the researches shows that combined influence of unbalanced mass and skewed disk to resonant vibrations and stability of rotary machines, taking into account nonlinear factors, presented in real structures was explored a little, and limitation of the study of dimensional nonlinear systems. Thus, the study of resonant oscillations and stability of rotary machines with unbalanced mass and skewed disk (considering nonlinear elastic characteristics of supports), technology of optimal control for resonant vibrations leading to the creation of new working rotary machines with optimal design parameters, are certainly relevant.

A mathematical model of the rotor and the research methodology

Figure-1 shows the geometrical scheme of the rotor. A shaft with a length L is mounted vertically, via lower hinged bearing and an upper elastic support (at a distance L_0). At the free end of the shaft is fixed disk (having a mass m, polar moment of inertia J_p and cross-section moment of inertia J_T that are the same for all directions).

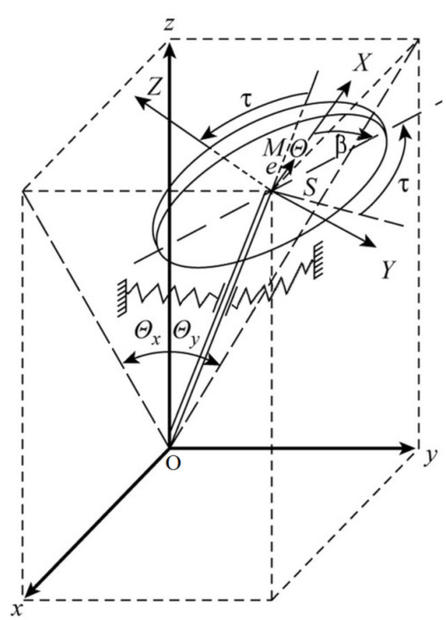

Figure-1
Rotor geometry

The shaft speed ω is so large that the rotor can be regarded as a gyroscope, a fixed point which is the lower shaft bearing. Two coordinate systems are used. The coordinate system Oxy is fixed in space. The geometric center of disk position S is determined by the coordinates x, y, positions of shaft and rotor in the space by angles θ_x, θ_y and the angle of rotation $\varphi = \omega t$. We also assume that the linear eccentricity e lies on the axis SX and lags behind the plane of angular eccentricity τ by an angle β. We confine ourselves to small deviations of the rotor axis, and therefore, in our calculations we only consider terms that are linear relative to small values: $e, \tau, \theta_x, \theta_y$.

Is an expression of the kinetic energy of the rotor and the potential energy of the elastic support, torques of the external forces and dissipation function and substituting them into the Lagrange equation of the second kind[17], we obtain the equations of motion as

$$\left(J_T + mL^2\right)\ddot{\theta}_x + J_P\omega\dot{\theta}_y + \mu_e\dot{\theta}_x + \left(k_1L_0^2 - GL\right)\theta + k_2L_0^4\theta_x^3 = \left(me\omega^2L + Ge\right)\cos\omega t + \left(J_P - J_T\right)\tau\omega^2\cos\left(\omega t + \beta\right)$$
$$\left(J_T + mL^2\right)\ddot{\theta}_y - J_P\omega\dot{\theta}_x + \mu_e\dot{\theta}_y + \left(k_1L_0^2 - GL\right)\theta_y + k_2L_0^4\theta_y^3 = \left(me\omega^2L + Ge\right)\sin\omega t + \left(J_P - J_T\right)\tau\omega^2\sin\left(\omega t + \beta\right) \quad (1)$$

where: k_1 is the support stiffness coefficient, k_2 is the coefficient in the nonlinear term of the elastic force, G is the weight of disc, μ_e is coefficient of resistance. Use of these dimensionless parameters

$$\bar{e} = e/L;\ l_0 = L_0/L;\ \bar{t} = t\omega_0;\ \Omega = \frac{\omega}{\omega_0};\ \bar{J}_p = J_p\Big/\left(mL^2\right);$$
$$\bar{J}_T = J_T\Big/\left(mL^2\right);$$
$$K_1 = k_1\Big/\left(m\omega_0^2\right);\ K_2 = k_2L^2/\left(m\omega_0^2\right);\ P = G\Big/\left(mL\omega_0^2\right);$$
$$\mu = \mu_e\Big/\left(mL^2\omega_0\right), \quad (2)$$

where $\omega_0 = \sqrt{\dfrac{k_1L_0^2 - GL}{mL^2 - \left(J_p - J_T\right)}}$ is the critical speed of the linear system without resistance, using the symbols of amplitude

$$M = \sqrt{\left[\left(\Omega^2 + P\right)\bar{e} + H\tau\Omega^2\cos\beta\right]^2 + H^2\tau^2\Omega^4\sin^2\beta} \quad (3)$$

and the initial phase

$$\gamma = \text{arctg}\frac{H\tau\Omega^2\sin\beta}{\left(\Omega^2 + P\right)\bar{e} + H\tau\Omega^2\cos\beta} \quad (4)$$

The forcing moment to the right parts of the system (8) to yield equations of represented in a compact form:

$$\left(1 + \bar{J}_T\right)\theta_x'' + \bar{J}_p\Omega\theta_y' + \mu\theta_x' + \left(K_1l_0^2 - P\right)\theta_x + K_2l_0^4\theta_x^3 =$$
$$M\cos\left(\Omega\bar{t} + \gamma\right) \quad (5)$$
$$\left(1 + \bar{J}_T\right)\theta_y'' - \bar{J}_p\Omega\theta_x' + \mu\theta_y' + \left(K_1l_0^2 - P\right)\theta_y + K_2l_0^4\theta_y^3 =$$
$$M\sin\left(\Omega\bar{t} + \gamma\right) \quad (6)$$

where: $H = \bar{J}_p - \bar{J}_T$ is the conventional disc thickness.

It turns out that the state of stationary motion of the rotor is described by differential equations (5) and (6) of Duffing type.

Typically when considering the periodic solution with a period equal to the period of external influence, the method of solutions expansion ((5) and (6)) to a Fourier range with undetermined coefficients is used. Coefficients can be found by using the well-known method of harmonic balance, taking into account the limited number of terms in the expansion.

Gyroscopic rotor with geometric nonlinear characteristic is examined for resonance to the fundamental frequency. As the elastic support, the materials with pronounced dissipative properties, rubber, rubber resin and other materials in the form of polymers, used as dampers of resultant oscillations, are used.

Approximation of the solutions to equations (5) and (6) in the case of simple resonance of the fundamental harmonic with the oscillation frequency, which is equal to the frequency of disturbance torque, satisfies

$$\theta_x = A\cos(\Omega\bar{t} - \alpha) \quad (7)$$
$$\theta_y = A\sin(\Omega\bar{t} - \alpha) \quad (8)$$

After application of the well-known method of harmonic balance, we obtain the amplitude-and phase-frequency dependence for the rotor system

$$\left\{\left[(1-H)\Omega^2 - \left(K_1l_0^2 - P\right) - \frac{3}{4}K_2l_0^4A^2\right]^2 + \mu^2\Omega^2\right\}A^2 = M^2 \quad (9)$$

$$\text{tg}\alpha = \frac{\left[(1-H)\Omega^2 - \left(K_1l_0^2 - P\right) - \frac{3}{4}K_2l_0^4A^2\right]\text{tg}\gamma + \mu\Omega}{\left[(1-H)\Omega^2 - \left(K_1l_0^2 - P\right) - \frac{3}{4}K_2l_0^4A^2\right] - \mu\Omega\text{tg}\gamma} \quad (10)$$

Equating to zero the forcing torque M and the coefficient of resistance μ, from the formula (9), we obtain the equation for the skeleton curve[18].

$$A = \frac{2}{\sqrt{3K_2l_0^2}}\sqrt{(1-H)\Omega^2 - \left(K_1l_0^2 - P\right)} \quad (11)$$

where

$$\Omega \geq \sqrt{\frac{K_1l_0^2 - P}{1 - H}}$$

From formula (11), it follows the greater coefficient K_2 of nonlinear term of the elastic force, the greater the slope of the supporting curve to the right.

To investigate the stability of the rotor motion we use the Floquet theory. If in the works of Hayashi, W. Szemplinska-Stupnicka and Kydyrbekuly it was used to study the system stability with one degree of freedom, here the Floquet theory we going to use for the rotor system with two degrees of freedom. Let us consider small increments in time $\delta\theta_x$, $\delta\theta_y$ for harmonic solutions θ_{x0} and θ_{y0} of equitation (5) and (6). Stability of resonance vibration modes of the system depends on the behavior of small deviations in time $\delta\theta_x$ and, $\delta\theta_y$ i.e. on solutions of equations of the perturbed system state:

$$\left(1+\bar{J}_T\right)\frac{d^2\delta\theta_x}{d\bar{t}^2}+\bar{J}_p\Omega\frac{d\delta\theta_y}{d\bar{t}}+\mu\frac{d\delta\theta_x}{d\bar{t}}+\left[\left(K_1 l_0^2-P\right)+3K_2 l_0^4\theta_{x0}{}^2\right]\delta\theta_x=0, \quad (12)$$

$$\left(1+\bar{J}_T\right)\frac{d^2\delta\theta_y}{d\bar{t}^2}-\bar{J}_p\Omega\frac{d\delta\theta_x}{d\bar{t}}+\mu\frac{d\delta\theta_y}{d\bar{t}}+\left[\left(K_1 l_0^2-P\right)+3K_2 l_0^4\theta_{y0}{}^2\right]\delta\theta_y=0. \quad (13)$$

If solutions $\delta\theta_x$ and $\delta\theta_y$ of equitations (12) and (13) at $t\to\infty$ are limited, the motion of the system is considered to be stable; if values $\delta\theta_x$ and $\delta\theta_y$ grow indefinitely at $t\to\infty$, the motion (according to Lyapunov study) is unstable.
Using the transformations

$$\delta\theta_x=e^{-0,5\mu\bar{t}}\xi \text{ и } \delta\theta_y=e^{-0,5\mu\bar{t}}\eta \quad (14)$$

and changing values θ_{x0} and θ_{y0} by their decompositions (7) and (8), bring (12) and (13) to equations of the Hill:

$$\left(1+\bar{J}_T\right)\frac{d^2\xi}{d\bar{t}^2}-\mu\bar{J}_T\frac{d\xi}{d\bar{t}}+\left(\theta_{01}+\theta_{2C}\cos2\Omega\bar{t}+\theta_{2S}\sin2\Omega\bar{t}\right)\xi+\bar{J}_p\Omega\frac{d\eta}{d\bar{t}}-\frac{1}{2}\mu\bar{J}_p\Omega\eta=0 \quad (15)$$

$$\left(1+\bar{J}_T\right)\frac{d^2\eta}{d\bar{t}^2}-\mu\bar{J}_T\frac{d\eta}{d\bar{t}}+\left(\theta_{01}+\theta_{2C}\sin2\Omega\bar{t}-\theta_{2S}\cos2\Omega\bar{t}\right)\eta-\bar{J}_p\Omega\frac{d\xi}{d\bar{t}}+\frac{1}{2}\mu\bar{J}_p\Omega\xi=0 \quad (16)$$

where: $\theta_{01}=\frac{1}{4}\left(1+\bar{J}_T\right)\mu^2-\frac{1}{2}\mu+\left(K_1 l_0^2-P\right)+\frac{2}{3}K_2 l_0^4 A^2$;

$$\theta_{2C}=\frac{2}{3}K_2 l_0^4 A^2\cos2\alpha;\ \theta_{2s}=\frac{2}{3}K_2 l_0^4 A^2\sin2\alpha \quad (17)$$

Some functions of the arguments A,α . According to the Floquet theory, particular solutions of equations (15) and (16) are in the form

$$\xi=e^{\lambda\bar{t}}a\cos\left(\Omega\bar{t}-\delta\right) \quad (18)$$

$$\eta=e^{\lambda\bar{t}}a\sin\left(\Omega\bar{t}-\delta\right) \quad (19)$$

where: λ - characteristic exponent (real or imaginary). Substituting the solutions (18) and (19) to the equations (15) and (16) and applying the harmonic balance method, i.e. equating the coefficients of the same frequencies, we obtain a system of linear homogeneous equations that must be satisfied for any nontrivial values of the variables a and δ . Then, using Floquet theory for the oscillating system with two degrees of freedom and taking into account (17) and $\lambda=\frac{\mu}{2}$, we make the characteristic determinant, defining the boundaries of instability region for the harmonic solutions, characterizing the resonance on the fundamental frequency. Expanding the determinant, we obtain an expression that defines the instability region, depending on the rotor parameters:

$$R\left(A,\Omega,\mu,K_2,H\right)=$$
$$=\left[0,25\left(1+\bar{J}_T\right)\mu^2-0,5\mu^2\bar{J}_T-(1-H)\Omega^2+\theta_{01}\right]^2+\left[0,5\left(2+2\bar{J}_T-\bar{J}_p\right)\Omega\mu+\mu\left(\frac{1}{2}\bar{J}_P-\bar{J}_T\right)\Omega\right]^2-$$
$$-\frac{1}{4}\left(\theta_{2C}^2+\theta_{2S}^2\right)=0 \quad (20)$$

In this, region of stability is characterized by the inequality

$$R>0 \quad (21)$$

The geometric locus of points at which the amplitude curves for the main resonance oscillations have vertical tangents, is determined by the equation

$$\frac{d\Omega}{dA}=0. \quad (22)$$

In accordance with the equation (9), we have the following equality

$$f\left(\Omega,A\right)=\left\{\left[(1-H)\Omega^2-\left(K_1 l_0^2-P\right)-\frac{3}{4}K_2 l_0^4 A^2\right]^2+\mu^2\Omega^2\right\}A^2-M^2=0 \quad (23)$$

Let us differentiate the last equation by frequency[19], then we get

$$\frac{\partial f}{\partial\Omega}+\frac{\partial f}{\partial A}\frac{\partial A}{\partial\Omega}=0$$

It follows that

$$\frac{\partial\Omega}{\partial A}=-\frac{\partial f/\partial A}{\partial f/\partial\Omega} \quad (24)$$

Condition (24) is approximately satisfied with the equity

$$\frac{\partial f}{\partial A}=0 \quad (25)$$

The equation (25) leads to the equation

$$\left[(1-H)\Omega^2-\left(K_1 l^2-P\right)-\frac{3}{2}K_2 l^4 A^2\right]^2+\left(\mu\Omega\right)^2-\left(\frac{3}{4}K_2 l^4 A^2\right)^2=0 \quad (26)$$

Thus, the tangent to the resonance curves is vertical at the edge of the main region of instability. Equation-26 describes the boundary curves of the stability range for oscillations at the fundamental resonance frequency.

Results and Discussion

Calculations using formulas (3), (4), (9), (10) and the solving of set of Equations (20) and (26) were obtained using numerical methods in the symbolic computation system «Maple 11» for the following rotor parameters: $\bar{e}=0,01$; $\tau=0,02$; $l_0=0,88$; $\mu=0,01$;
$H=+0,1$: $\bar{J}_p=0,2$; $\bar{J}_T=0,1$; $K_1=1,19$;
$K_2=1,19;2,19$; $P=0,012$;

$H = -0{,}1: \ \bar{J}_p = 0{,}1; \ \bar{J}_T = 0{,}2; \ K_1 = 1{,}45;$

$K_2 = 1{,}45; 2{,}45; \ P = 0{,}014.$

Analysis of formula (10) shows, the amplitude of the driving torque reaches a maximum value at $\beta=0^0$ in the case of a thin disc (at $\beta=180^0$ in the case of a thick disc), a minimum value – at $\beta=180^0$ in the case of a thin disc (at $\beta=0^0$ in the case of a thick disc) and an intermediate value – at $\beta=\pm 90^0$. It is illustrated by graphics M = M (Ω) for different values of β and H, shown in Figures-2 and 3.

On Figures 4 and 5 and on graphs γ=γ (Ω) at Ω→∞ the phase of the driving torque strives to the asymptotic value: maximum at $\beta=+90^0$ in the case of a thin disc (at $\beta=-90^0$ in the case of a thick disc), minimum at $\beta=+90^0$ in the case of a thick disc (at $\beta=-90^0$ in the case of a thin disc) and assumes a zero value at $\beta=0^0, 180^0$.

Figures 6-9 show amplitude- and phase-frequency characteristics of the rotor in cases of thin and thick disks. From the figures above it is clear that the resonance curves are extended from the end to the right due to the influence of nonlinearity in the system. The greater the magnitude of nonlinearity K_2, the stronger this effect.

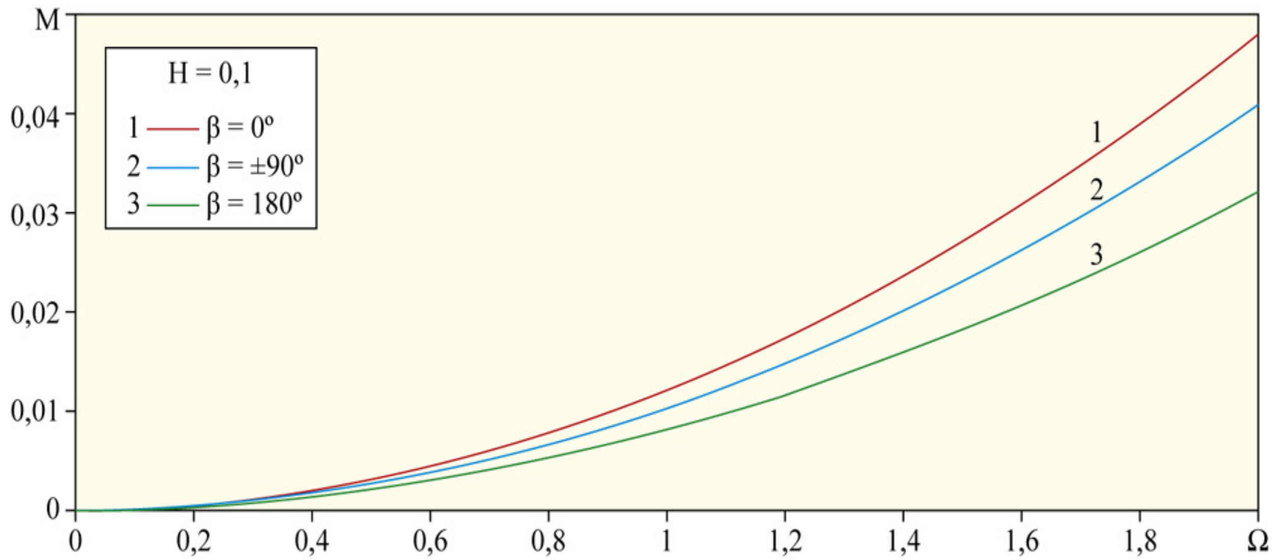

Figure- 2
Effect of the angle between the orientations of the imbalances on forcing torque Case of a thin disk

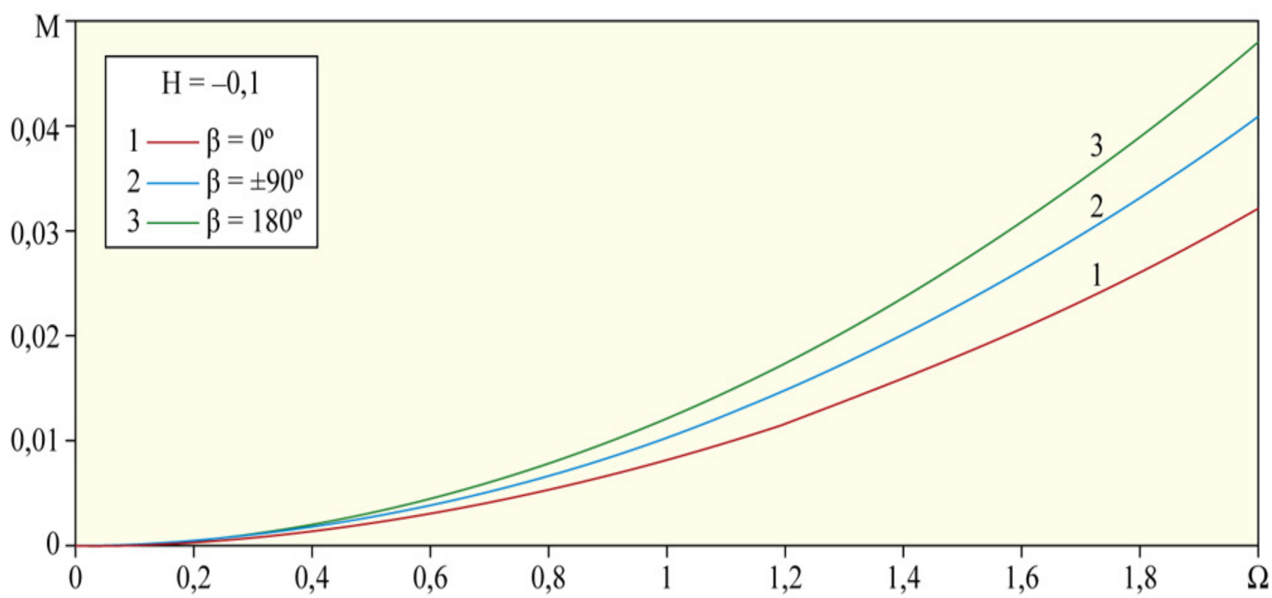

Figure- 3
Effect of the angle between the orientations of the imbalances on forcing torque Case of a thick disk

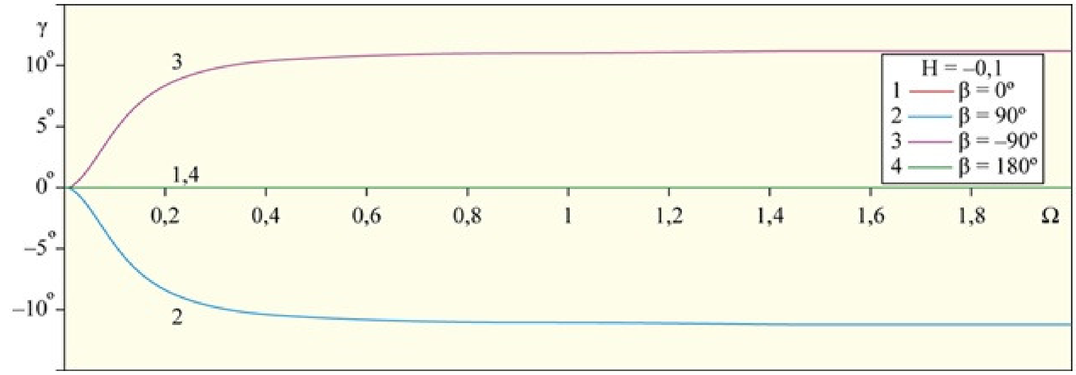

Figure- 4
Effect of the angle between the orientations of the imbalances on the phase of forcing torque. Case of thin disk

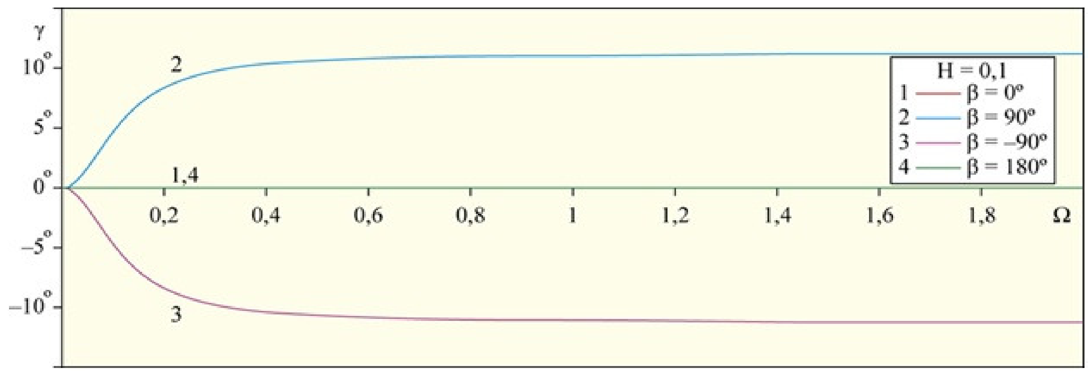

Figure- 5
Effect of the angle between the orientations of the imbalances on the phase of forcing torque
Case of a thick disk

Dependence of the amplitude A on the angle β between the lines of mass imbalance and maximum disc skew (Figures-6 and 8) is similar to the case of a rotor linear model[20]. The skew effect may lead to an increase or a decrease of the amplitude of the oscillations. In the case of a thin disc (H = 0,1) at the angle $\beta = 0$, the maximum skew line coincides with the line of the eccentricity mass vector; the directions of gravity torques, centrifugal force and a gyroscopic moment coincide, and exciting moment reaches its maximum (Figure-2), and these lead to intensification of shaft deviation from the vertical (Figure-6). At angle $\beta = 180^0$, the gyroscopic torque is directly opposite to the total torque of centrifugal and gravity forces; the forcing torque and amplitude of the oscillations are at their smallest (discussion of the formula (10) and the graph of Figures-2 and 6). At angle $\beta = \pm 90^0$, the disturbances are perpendicular to each other, and we obtain intermediate values of the exciting moment and the oscillation amplitude (Figure-2, Figure-6). In the case of the thick disk ($H = -0,1$), dependence of the oscillation amplitude *A* on the angle β changes due to the nature of the opposite sign of the gyroscopic moment (Figure-8).

Left to the skeleton curve regimes, for which the amplitude increases at growth, will be stable; right to the skeleton curve regimes, for which the amplitude decreases at growth, will be stable. Segments MAB and ND of the resonance curve correspond to stable amplitudes, and section BD – to unstable. At very slow increase of the angular velocity of the rotor, an oscillation amplitude increases along the curve MAP, and as it can be seen from calculations at the point corresponding to coincidence of large amplitude values, the amplitude changes its value by a «leap» and subsequently decreases when increasing. At very slow decrease of the angular velocity, an oscillation amplitude varies at the curve ND; at the point D the amplitude changes its value by a «leap» to a value corresponding to point A on the resonance curve; at further decrease of the angular velocity, the amplitude varies at the curve AM.

In the phase-frequency characteristic of the fundamental harmonic, the phase curves at a critical velocity rise up, and then in accordance with the values of the amplitude A bifurcate: one group of branches looks toward to the horizontal constant level, the other descends from a given level, and the third rises above this level; the values affect the relative positioning of the curves in each group. Calculations show that when amplitude A takes one instead of two values, curves at three values A are grouped into one line, and the phase-frequency curves in the case of linear model[20] will vary on values.

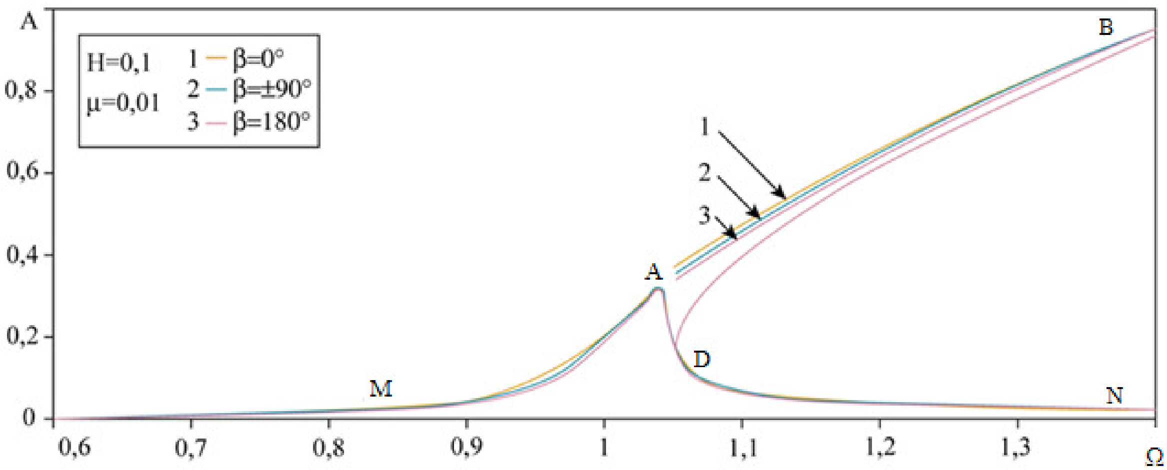

Figure- 6
Effect of the angle between the orientations of the imbalances on the resonance amplitude of the fundamental frequency. Case of a thin disk

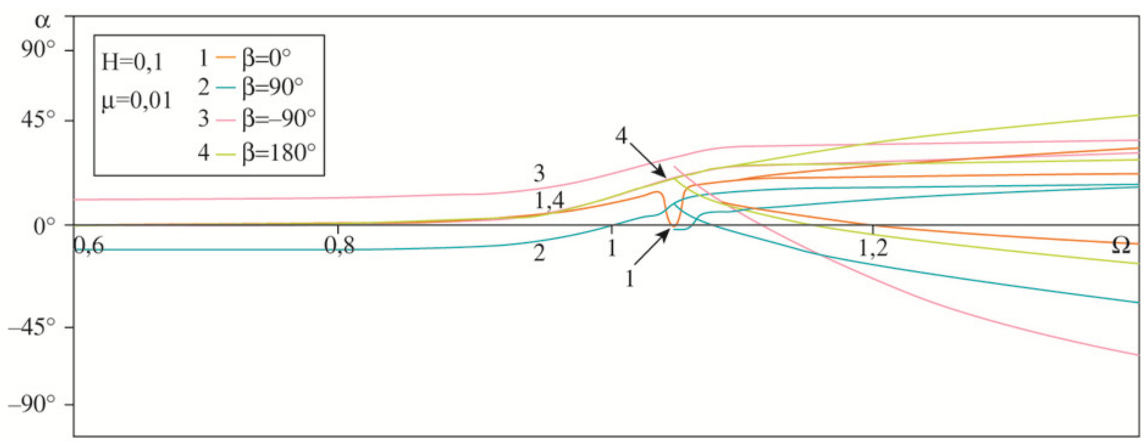

Figure- 7
Effect of the angle between the orientations of the imbalances on phase shift angle at the fundamental frequency. Case of a thin disk

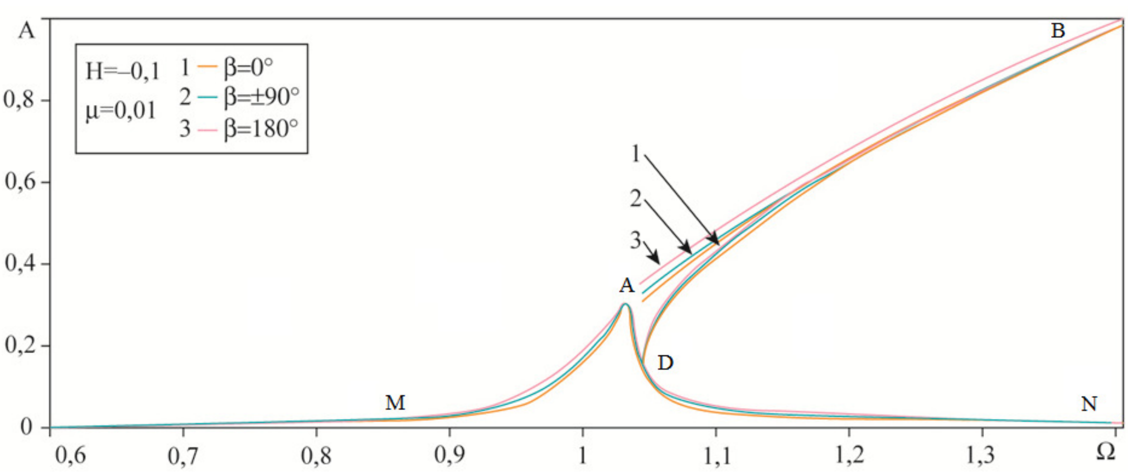

Figure- 8
Effect of the angle between the orientations of the imbalances on the resonance amplitude of the fundamental frequency. Case of a thick disk

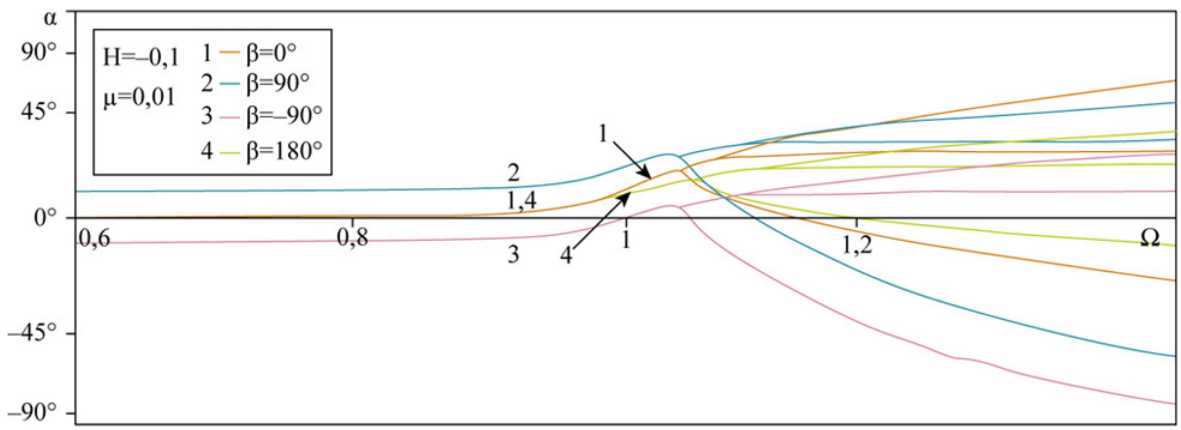

Figure- 9
Effect of the angle between the orientations of the imbalances on phase shift angle at the fundamental frequency. Case of a thick disk

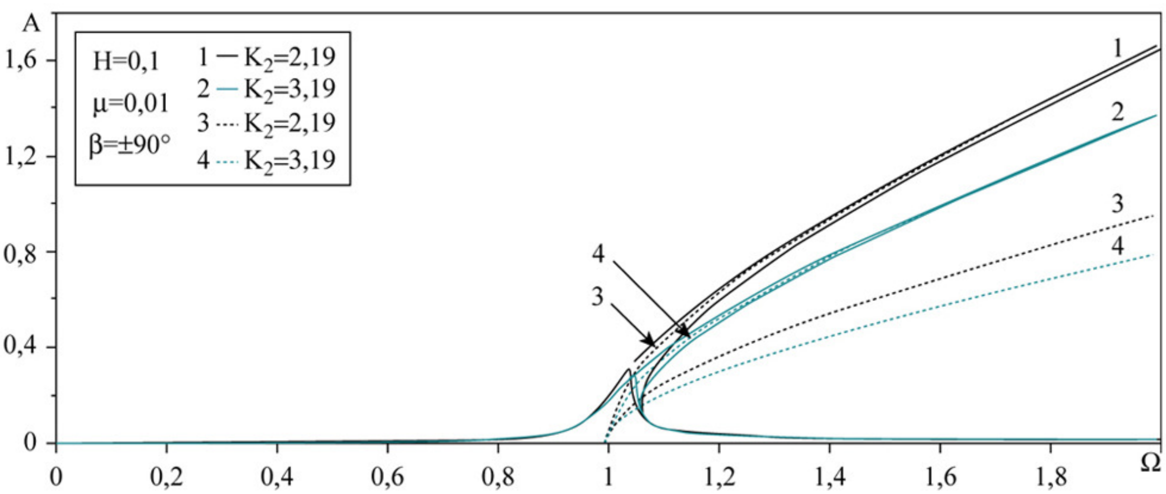

Figure- 10
Effect of the nonlinearity magnitude on the boundary of the instability region of the fundamental resonance. Case of a thin disk

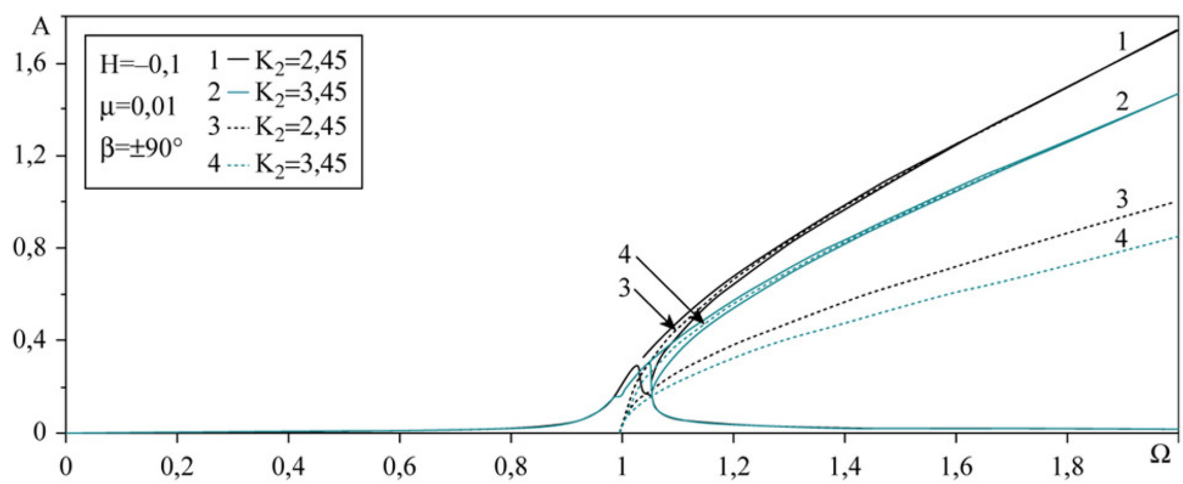

Figure- 11
Effect of the nonlinearity magnitude on the boundary of the instability region of the fundamental resonance. Case of a thick disk

Graphs on Figures 10 and 11 show, that with increasing magnitude of the nonlinearity, the instability regions are moved downwards and their width decreases. From Figures 10 and 11, the influence of the disk thickness on the position and width of the instability region can be seen. At the same speed of rotation, the width of instability region for a thin disk is less than the same parameter for a thick disk.

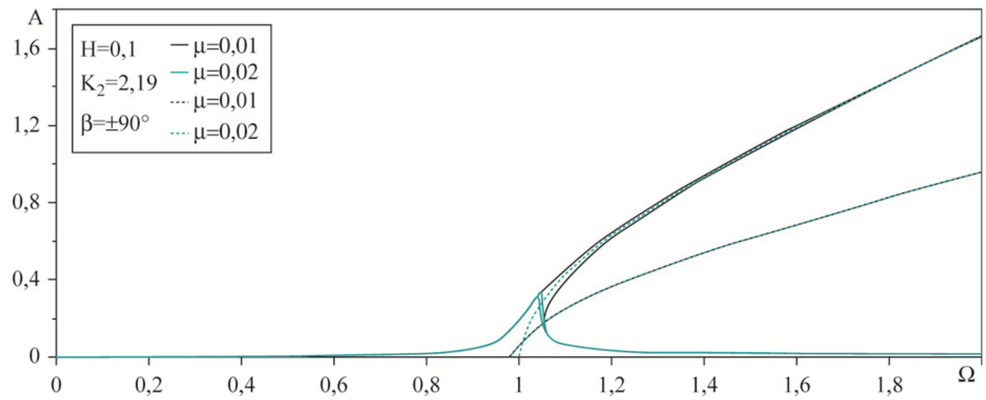

Figure- 12
Effect of external resistance on the oscillation amplitude and the boundary of the instability region of the fundamental resonance. Case of a thin disk

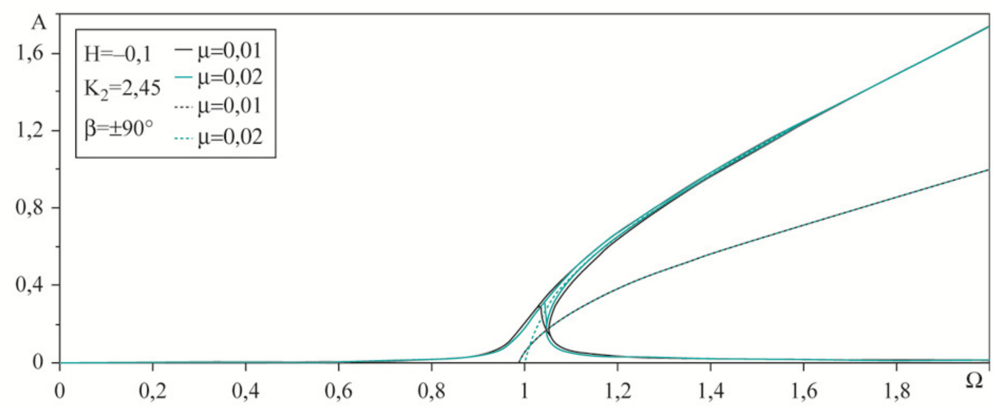

Figure- 13
Effect of external resistance on the oscillation amplitude and the boundary of the instability region of the fundamental resonance. Case of a thick disk

As it can be seen from the formula (20), design parameters are reflected in the terms of the rotor instability region. Therefore, by varying them, we can choose the optimal modes of the machine operation, excluding resonance phenomena.

Conclusion

The basic resonant oscillations and stability of a vertical rigid gyro rotor with rigid nonlinear elastic characteristics and nonlinear resistance, a skewed disk and an imbalanced mass are investigated. It is found that the disk skew affects the amplitude and phase of the driving torque and consequently the magnitude of the amplitude and phase of the fundamental resonant oscillations. Variants such as thin and thick disks are studied. Changing the disk thickness affects the location of the resonance curves and the width of the instability region. Under the influence of nonlinear components of the elastic force, the resonance curves are drawn from the end and bent to the right, to the high speed region, the amplitude decreases, and the instability region shifts downward; there at the shift in the upper bound is greater than that of the lower bound and the width decreases.

External resistance shortens the resonance amplitude and virtually has no effect on the boundary of instability region. The results of studies of the effect of thickness, weight imbalance and skewed disk, their orientations, the nonlinear parameter of the elastic support, as well as the viscous external friction at the main resonant oscillations and their stability can create balancing methods, to determine the safe operating range of velocities and to find effective parameters for the reliable operation of the rotor.

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