

Grouping of Significant Geomorphic Parameters using Multivariate Technique

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Available online at: www.isca.in, www.isca.me

Received 7th September 2016, revised 1st October 2016, accepted 19th October 2016

Abstract

The hydrologic modelling play vital role in study of the hydrological behaviour of any watershed. The dimension reduction technique like Principal Component Analysis (PCA) which uses an orthogonal transformation is used in this study. The PCA technique has been applied in upper and middle sub basins of Godavari river basins for 11 selected watersheds, Maharashtra (India). For grouping geomorphic parameters on the basis of their significant correlations 13 dimensionless geomorphic parameters are considered. PCA clearly shows that first two PC are strongly correlated among some geomorphic parameters. The results show that the 3rd PC is not showing strong correlation with any parameter but shows moderate correlation with L_p/L_w . The result clearly reveals that, due to poor correlation of the hypsometric integral and main stream channel slope with others could not be grouped with any of the component. The PC loading matrix which is obtained from finally selected 11 parameters correlation matrix, clearly showed first three component gives 94.283% explained variance. Hence it is concluded that PCA is very effective and useful tool to screen out the insignificant parameters for watersheds hydrologic behavioural study such as runoff and sediment yield modelling.

Keywords: PCA, Multivariate, Data reduction, Geomorphic parameter, Watershed, Godavari.

Introduction

The natural resources are very important for the survival of all living beings on the blue earth. The conservation of natural resources such as soil and water conservation are planned considering watershed an unit. The watershed is a natural unit bounded by ridge line having single outlet. Watershed is ideal unit for land and water resources planning and management purposes¹. India is very divergent country in its physio-topographical nature and in terms of availability natural resources. In such divergent country monitoring and gauging of the watersheds very difficult. Such monitored data is essential for the planning and management of the watershed. To overcome such problem hydrological modelling is the one of the best solution. Hydrologic modelling is good tool for prediction of hydrologic behaviour of a basin². The hydrological behaviour can be interrelated from the morphological characteristics of the watershed. The geomorphological characteristic are also useful for the development of the regional hydrological models which may be used to solve the complex problems of ungauged watersheds or in the case of unavailability of the data. In prioritization of the watershed is done on the basis of runoff and sediment data, but the monitoring of such data in any watershed is laborious and time consuming job. Collection of such data requires the continuous monitoring and analysis. In the absence of such data the geo-morphological characteristics are helpful to assess most critical sub-watershed¹. The morphometric analysis of drainage basins was began in middle of the 20th century³. The advent of spatial technology like remote sensing and GIS allows

the digitally extraction of geo-morphometric parameters for the hydrological analysis of the watershed.

The collection of the morphological characteristics is laborious and time-consuming, hence it's better to reduce the number of parameters which can simulate the behavior of the drainage basin on the basis of their morphological characteristics. When there are so many variables in analysis, it's often helpful to reduce the variables to a smaller set of factors⁴. The screening of such interrelated variables in under-lying dimensions can be achieved by dimension reduction technique called as Principal Component Analysis (PCA)². The PCA is the techniques which reduces the number of inter-correlations variables in small number of sets of the uncorrelated factors, which together summarizes data into the original matrix as well as explains the relationships and influences among them. The present research study was planned to carry out the inter-relationship among the variables for screening less significant variables out of the analysis and in order to arrange the remaining variable into physically most significant groups after the applying PCA technique.

Study Area: The present study was carried out for 11 watershed located in upper and middle sub basins of Godavari river basin in Maharashtra (India). The Godavari river originates at Brahmagiri hills, Trimbakeshwar in Nashik district (Maharashtra) at an elevation of 1,067 m⁵. It is also known as "Vridha Ganga" or "Dakshin Ganga"⁶. The Godavari river basin extends over the state like Maharashtra, A.P.,

Chhattisgarh, Telangana and Odisha in addition to smaller parts in M.P., Karnataka and Union territory of Puducherry. The basin lays in latitudes 16°16'0" North and 23°43' longitudes 73°26' to 83°07' East. The basin is roughly triangular in shape having an area of 3,12,813 km² (approx.10% of India's geographical area).⁶.

Major soils in the Godavari basin as well its adjoining areas are black (Regur), red, laterites and lateritic, alluvium, mixed red and black soils, mixed red and yellow and saline and alkaline soils. The Pravara, the Manjra, the Maner, the Purna, the Penganga, the Pranhita, the Wardha, the Indravati and the Sabari are major tributaries of the Godavari. The Godavari basin is sub divided in to Upper, Middle, Lower, Indravati, Manjara, Wainganga, Wardha and Pranhita and other (Anonymous, 2012). The major Land Use Land Cover (LULC) in Godavari basin are agricultural land 1, 86,347.17 km² (59.57%), built up land 5,187.26 km² (1.66%), Forest 93142.06 km² (29.78%), Grassland 85.84 km² (0.03%), Wasteland 16785.92 km² (5.36%), Water bodies 11,263.75 km² (3.60%)⁵.

The Central Water Commission (CWC) assessed water resources potential of the Godavari basin is about 110.54 km³ out of that the utilizable surface water is approx. 76.3 km³ and replenishable GW is approx. 45 km³. The basin is having very good potential for development of irrigation activities and hydropower generation. The present utilization is only 40 km³ in the case of surface water and 6 km³ in the case of GW. Considering the potential of the basin the present study was undertaken for hydrologic modelling purposes. The selected 11 sub watershed are lies in Nashik, Ahmednagar, Beed, Jalna, Parbhani, Aurangabad and Nanded district as given in Figure-1.

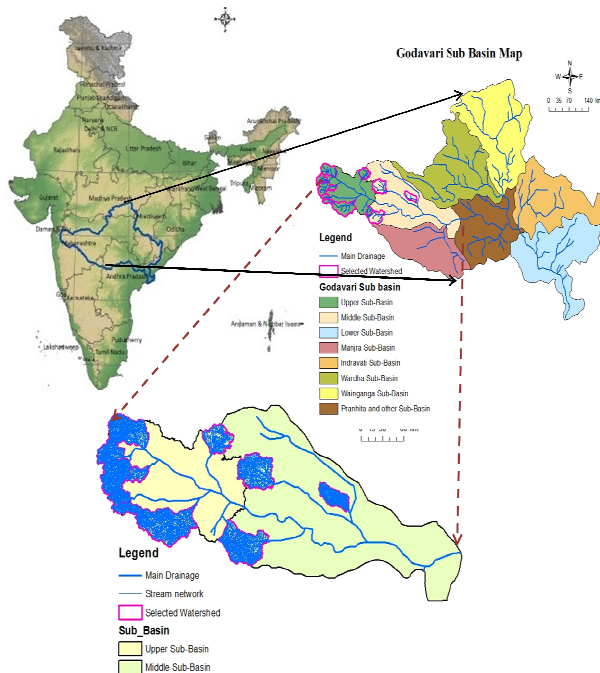


Figure-1
Location map of the study area

Methodology

Geomorphological characteristics are very important in the study of hydrologic behaviour of the watersheds. Hence geomorphological characteristics were extracted from the Survey of India (1:50,000 scale) toposheets. Thirteen dimensionless geomorphic parameters of 11 selected watersheds were considered the upper and middle Godavari basin, Maharashtra (India) were extracted in the Geographic Information System (GIS) environment using Arc GIS 10.2 software. The various formulae used to extract the parameters are tabulated in Table-1.

PCA technique: The PCA is one of the earlier work carried out by Pearson with the proper adaptations of PCA suggested by the Hotellings⁷. The PCA is used to find out a small number of independent linear combinations (known as principal components) of a set of measured variables which gives more variability in the original variables. The PCA used for reducing the complexity of high dimensional data which is also known as dimensional reduction technique. It is also used as exploratory data analysis tool. It is also useful for constructing predictive models, as in PCA regression. The PCA, is used to maximize the variance of a linear combination of the variables.

Many times among geomorphic parameters are correlated with each other. The correlation between these parameters gives that, some of the information contained in one variable is also present in other. The Ist PC is nothing but the linear combination of the original variables which may contributes maximally in total variance; the IInd PC, uncorrelated with the first, contributes a maximally in residual variance, and so on up to the analyses of total variance. The method is most suitable when all the variables are expressed in the same dimension, as its dependent up on the total variance of the original variables. Hence the expression of variables standardised form essential, i.e. the selection of the unit of measurement for each variable so that its sample variance will be: i. The analysis is performed on the basis of matrix of correlation, with the total variance equal to n. It is achieved by adopting the following two steps:
Step I: Calculate the matrix of correlation i.e. *R*
Step 2: Calculate the PC loading matrix by PCA.
Step 3: In PC loading matrix, whose Eigen value >1, indicates significant PC loading.

The 13 geo-morphological parameters extracted from selected watersheds are tabulated in Table-2.

Correlation Matrix: The procedure carried out to inter-correlation matrix among geomorphic parameters is as follow:
i. The standardization of the parameters:

$$X = (x_{ij} - x_j) / S_j$$

Where: *x* = Standardized parameters Matrix, $x_{ij} = i^{\text{th}}$ observation of j^{th} parameter, $i = 1, 2, 3, \dots, N$ (Number of observations), $j = 1, 2, 3, \dots, P$ (Number of parameters), $x_j =$ mean of j^{th} parameter, $S_j =$ standard deviation of j^{th} parameter.

Table-1
Empirical formulae

Geo-morphological parameter	Formula
Watersheds average slope (S_a) ² .	$S_a = H L_{ca}/(10A)$ H=Maximum watershed relief (m) A = Drainage area (km ²) L _{ca} = Contours average length (km) $L_{ca} = \sum_{i=1}^n \frac{L_{ci}}{n}$ L _{ci} = Individual contours length (km) n = Identifiable number of contours,
Relief ratio (R_r) ²	$R_r = \frac{H}{L}$ L = Main stream length (m)
Relative relief (R_R)	$R = \frac{H}{P} \times 100$ P = Periphery of watershed
Main stream channel slope (S_c) ^{2,6}	$S_c = \frac{\text{Area Underlaid by the curve}}{5(L)^2}$ L= Main stream length expressed in km
Elongation ratio (R_e) ²	$R_e = \frac{D_c}{L}$ D _c = Dia. of the circle having the same area as of basin L = Basins max. length (m)
Basin shape factor (S_b) ²	$S_b = \frac{(\text{Maximum length of basin})^2}{\text{area of the basin}}$
Ration of Length–width (L_b/L_w) ²	$L_b/L_w = \frac{\text{Maximum length of watershed}}{\text{Width of the watershed}}$
Stream-length ratio (R_l) ⁹	$R_l = \frac{\bar{L}_u}{\bar{L}_u - 1}$ L _u = Mean length of stream of u order and L _{u+1} = Mean length of stream of the next lower order
Bifurcation ratio (R_b) ^{9,10}	$R_b = \frac{N_u}{N_u + 1}$ N _u = No. of stream of the order u, and N _{u + 1} = No. of stream of order u + 1.
Hypsometric analysis of drainage basin ^{8,11}	The curve is developed by plotting the h/H and a/A.
Circulatory ratio (R_c) ¹²	$R_c = \frac{A_u}{A_c}$ A _u = Watershed area and A _c = Area of circle having the same length equal to the perimeter.
Ruggedness number (R_N) ²	$R_N = \frac{HD_d}{1000}$ H = Watershed's relief and D _d = Watershed's drainage density
Drainage factor (D_f) ²	$D_f = \frac{F_s}{D_d^2}$ Stream frequency (F _s), D _d = Watershed's drainage density.

ii. The parameters correlation matrix is nothing but the minor product moment of standardized predictor measures divided by N ,

$$R = (X' - X)/N$$

Where: X' is the transpose of the standardized matrix for the predictor parameters.

PC Loading Matrix: The PC loading matrix, showed, that maximum how much any one of the particular parameter is correlated with the other factor. It is calculated by pre-multiplying the characteristic vector with the squared root of correlation matrix's characteristic values.

$$A = Q \times D^{0.5}$$

Where: $A = PC$ loading matrix, $Q =$ Correlation matrix of characteristic vector. $D =$ Correlation matrix's characteristic value.

Results and Discussion

Thirteen geomorphic parameter extracted from 11 selected watersheds in upper and middle sub basin in Godavari basin are tabulated in Table-2. The PCA is carried out using the SPSS 18.0 software package. The SPSS package stands for Statistical Package for the Social Sciences. The SPSS package is mostly used in social sciences for various statistical analyses. This package is having its utility in health researchers, market researchers as well as in the governmental and many more users.

The correlation matrix (Table-3) shows strong correlations (coefficient of correlation more than 0.9) exist in bet. Watershed average slope (S_a) and Elongation ratio (R_e), Relief ratio (R_r), between Circulatory ratio (R_c) and Elongation ratio (R_e), Relief ratio (R_r) between Relative Relief (R_R) and Basin shape factor (S_b), Drainage factor (D_f). Also, very good correlations (coefficient of correlation is more than 0.75) exist between S_a , R_e and R_c and between R_e , S_b and R_r and R_r and R_N , S_c and R_v , between S_a , R_R , R_r and H_{si} , D_f and R_r also between R_R , R_r , S_c and R_i . Also, some more moderately correlated parameters (having coefficient of correlation greater than 0.60) exist among S_b and S_a , R_N and S_a , between R_e , R_r and S_c , between R_e , R_c and H_{si} between R_e , R_c and D_f also R_i and S_a . The parameters like R_b and L_b/L_w are not showing correlation with any other parameters So, it's very difficult for grouping the parameters into components. Hence next step of PCA is applied for getting correlation matrix from the PCA.

The PC loading matrix is obtained from correlation matrix (Table-4) shows that the Ist three components having Eigen values > 1 , which account together about 91.80 % of the total explained variance. The Ist component is strongly correlated (loadings of > 0.8) with S_b , R_N , S_c , H_{si} , R_i and moderately

(loadings of > 0.6) with R_R and R_r , which is termed as steepness or slope component. The IInd component is showing strong correlation with S_a , R_e , R_c and moderate with R_R (termed as component of shape). Whereas IIIrd component is not showing any strong correlation with any geomorphic parameters. The results show that few parameters among them are highly correlated with other components whereas due to poor correlation some parameters could not be grouped with any of the components.

The parameters which are showing less significance while explain of component variance are screened out. Then, from the screened out parameters, correlation matrix and PC loading matrix was obtained. The less correlated parameters such as R_i and L_b/L_w were screen out and finally 11 parameters are selected.

The PC loadings matrix obtained from the correlation matrix of finally selected 11 parameters (Table 4) showed that, Ist three components now accounted 94.284 % of the total explained variance. The PC loadings are also improved considerably in all significant parameters. The Elongation ratio (R_e), Circulatory ratio (R_c) and Relief ratio (R_r) are highly correlated (loadings > 0.9) with Ist component. The Relative Relief (R_R) shows good correlations (loadings > 0.8) with the Ist component. The S_a and R_c are highly correlated (loadings > 0.9) with IInd component whereas Relief ratio (R_r) also shows good correlation. The results clearly showed, steepness component is the dominated component followed by shape and drainage components. Hence, it is clear that the hydrologic responses such as like runoff and soil loss in these watersheds is high.

The study shows that the parameters/ variables having less significance in grouping are screened out using the PCA. The runoff and sediment yield modelling in a watershed can predicted using the multivariate analysis technique.

Conclusion

The present study was carried out for 11 sub watersheds in upper and middle Godavari sub basin, Maharashtra (India). The analysis of the geomorphic parameters of the watersheds were performed on the basis 13 parameters of the selected watershed. The matrix of correlation for selected geo-morphic parameters revealed strong correlation (coefficient of correlation > 0.9) exist between Elongation ratio (R_e) and Watershed's average slope (S_a), between Average slope of watershed (S_a), Elongation ratio (R_e) and Circulatory ratio (R_c), between Relative Relief (R_R) and Basin shape factor (S_b) between Average slope of watershed (S_a), Circulatory ratio (R_c), Relative Relief (R_R) and Relief ratio (R_r), exist between Basin shape factor (S_b), Relative Relief (R_R) and Ruggdness ratio (R_N), between Main stream channel slope (S_c) and Basin shape factor (S_b), Ruggdness ratio (R_N), between Hypsometric integral (H_{si}) with Basin shape factor (S_b), Ruggdness ratio (R_N), Main stream channel slope (S_c) also exist in between Drainage factor (D_f) with Basin shape

factor (S_b), Relative Relief (R_R), Ruggedness ratio (R_N), Main stream channel slope (S_c) and Hypsometric integral (H_{si}), between stream length ratio (R_l) and Basin shape factor (S_b), Ruggedness ratio (R_N), Hypsometric integral (H_{si}) and Drainage factor (D_f). The PC loading matrix obtained from correlation matrix shows that the Ist three components, having Eigen values greater than 1, accounts 91.800 % of the total variance explained.

The results reveals that the, Ist component is having strong correlated with Basin shaped factor (S_b), Ruggedness ratio (R_N), Drainage factor (D_f) and stream length ratio (R_l). The IInd component is strongly correlated with Elongation ratio (R_e) and Circulatory ratio (R_c). Whereas, IIIrd component is not shows

any strong correlation with any parameters but indicates moderately (> 0.7) correlated with L_b/L_w . After screening out hypsometric integral (H_{si}) and slope of main stream channel (S_c) the PC loadings matrix of finally selected 11 geomorphic parameters shows that, Ist three components account approx. 94.283 % of total variance explained. On the basis of the properties of selected geomorphic parameters, 3 PC's are categorised as shape, steepness and drainage components. Finally on the basis of the study results it can be stated the PCA is good tool in screen out the insignificant parameters for the research in watersheds hydrologic behaviour like runoff and sediment yield modelling. The runoff and sediment yield modelling in watersheds can be predicted using the multivariate analysis technique.

Table-2(a)
Selected dimensionless geomorphic parameters

Ws No.	Area (km ²)	S_a	R_e	R_c	S_b	R_r	R_R	R_N
WS1	1445.1	0.7519	0.789	0.8121	0.609	0.5477	0.4974	0.6006
WS2	708.553	0.3364	0.545	0.5661	0.5599	1.0882	0.8116	0.2352
WS3	1153.6	0.0112	0.018	0.0212	0.019	0.0241	0.0103	0.0118
WS4	350.239	0.571	0.513	0.642	0.4781	0.5147	0.246	0.5889
WS5	429.752	1.2533	1.044	1.9315	3.4346	4.2467	3.1498	3.5313
WS6	914.128	0.4321	0.41	0.3542	0.4312	0.4919	0.4867	0.397
WS7	1567.43	1.5285	1.854	3.2959	0.7902	2.7392	2.9513	0.8979
WS8	684.461	0.0079	0.012	0.0135	0.0142	0.0163	0.0063	0.0116
WS9	357.361	0.0036	0.005	0.0067	0.0069	0.0101	0.0033	0.0048
WS10	2715.68	0.4875	0.456	0.3585	0.3187	0.4745	0.5328	0.5741
WS11	1537.51	0.1343	0.164	0.1296	0.1931	0.1559	0.124	0.1012

Table-2(b)
Selected dimensionless geomorphic parameters

Ws No.	Area (km ²)	S_c	H_{si}	D_f	R_l	R_b	L_b/L_w
WS1	1445.1	0.8892	0.8296	0.7211	0.6036	5.3457	1.8009
WS2	708.553	0.365	0.2457	0.7973	0.1601	4.4908	1.4885
WS3	1153.6	0.0125	0.0082	0.0065	0.0037	5.3352	1.5367
WS4	350.239	0.430	0.5899	0.5682	0.5518	3.0597	1.9138
WS5	429.752	1.6112	1.851	2.4499	3.4966	3.3433	1.8715
WS6	914.128	0.5947	0.4189	0.4824	0.4498	4.232	0.9432
WS7	1567.43	0.5611	0.7397	0.905	0.7382	3.2806	1.1441
WS8	684.461	0.0076	0.0057	0.0043	0.0012	2.4005	2.4442
WS9	357.361	0.0029	0.0024	0.0022	0.0014	6.5452	1.9967
WS10	2715.68	0.3075	0.6014	0.5952	0.996	4.0706	2.5720
WS11	1537.51	0.2117	0.0859	0.1339	0.2246	5.8987	1.4584

Table-3
Interco-relation matrix of selected geomorphic parameters

	S _a	R _e	R _c	S _b	R _R	R _r	R _N	S _c	H _{si}	D _f	R _l	R _b	L _b /L _w
S _a	1.000	0.969	0.950	0.682	0.858	0.913	0.711	0.765	0.821	0.779	0.673	-0.437	-0.266
R _e	0.969	1.000	0.974	0.527	0.781	0.876	0.544	0.621	0.668	0.650	0.495	-0.394	-0.353
R _c	0.950	0.974	1.000	0.569	0.834	0.924	0.588	0.589	0.652	0.650	0.527	-0.414	-0.343
S _b	0.682	0.527	0.569	1.000	0.913	0.807	0.990	0.917	0.926	0.970	0.968	-0.330	-0.038
R _R	0.858	0.781	0.834	0.913	1.000	0.974	0.908	0.827	0.867	0.930	0.865	-0.414	-0.192
R _r	0.913	0.876	0.924	0.807	0.974	1.000	0.812	0.748	0.800	0.848	0.766	-0.415	-0.259
R _N	0.711	0.544	0.588	0.990	0.908	0.812	1.000	0.910	0.950	0.961	0.989	-0.366	0.026
S _c	0.765	0.621	0.589	0.917	0.827	0.748	0.910	1.000	0.961	0.939	0.892	-0.281	-0.158
H _{si}	0.821	0.668	0.652	0.926	0.867	0.800	0.950	0.961	1.000	0.957	0.946	-0.382	0.009
D _f	0.779	0.650	0.650	0.970	0.930	0.848	0.961	0.939	0.957	1.000	0.944	-0.380	-0.063
R _l	0.673	0.495	0.527	0.968	0.865	0.766	0.989	0.892	0.946	0.944	1.000	-0.345	0.101
R _b	-0.437	-0.394	-0.414	-0.330	-0.414	-0.415	-0.366	-0.281	-0.382	-0.380	-0.345	1.000	-0.197
L _b /L _w	-0.266	-0.353	-0.343	-0.038	-0.192	-0.259	0.026	-0.158	0.009	-0.063	0.101	-0.197	1.000

a. This matrix is not positive definite. Figures shown in bold indicate strong correlations

Table-4
Finally selected 11 parameters PC loading matrix

Parameters	Principal Components										
	1	2	3	4	5	6	7	8	9	10	11
S _a	1.000	0.969	0.950	0.682	0.858	0.913	0.711	0.779	-0.437	0.673	-0.266
R _e	0.969	1.000	0.974	0.527	0.781	0.876	0.544	0.650	-0.394	0.495	-0.353
R _c	0.950	0.974	1.000	0.569	0.834	0.924	0.588	0.650	-0.414	0.527	-0.343
S _b	0.682	0.527	0.569	1.000	0.913	0.807	0.990	0.970	-0.330	0.968	-0.038
R _R	0.858	0.781	0.834	0.913	1.000	0.974	0.908	0.930	-0.414	0.865	-0.192
R _r	0.913	0.876	0.924	0.807	0.974	1.000	0.812	0.848	-0.415	0.766	-0.259
R _N	0.711	0.544	0.588	0.990	0.908	0.812	1.000	0.961	-0.366	0.989	0.026
D _f	0.779	0.650	0.650	0.970	0.930	0.848	0.961	1.000	-0.380	0.944	-0.063
R _b	-0.437	-0.394	-0.414	-0.330	-0.414	-0.415	-0.366	-0.380	1.000	-0.345	-0.197
R _l	0.673	0.495	0.527	0.968	0.865	0.766	0.989	0.944	-0.345	1.000	0.101
L _b /L _w	-0.266	-0.353	-0.343	-0.038	-0.192	-0.259	0.026	-0.063	-0.197	0.101	1.000
Eigen Value	7.720	1.636	1.015								
	94.284 %										

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