Research Journal of Recent Sciences ISSN 2277-2502
Vol. **4(9),** 50-59, September **(2015)** *Res.J.Recent Sci.*

Love Wave Propagation in a Functionally Graded Magneto-Electro-Elastic Half-Space with Quadratic Variations

Masoud Nobahar* and Mehrdad Pourvosoughi Sarcheshme
Department of Civil Engineering, Sharif University of Technology, International Branch, Kish Island, IRAN

Available online at: www.isca.in, www.isca.me
Received 1st March 2014, revised 8th June 2014, accepted 12th November 2014

Abstract

The propagation behavior of Love wave in functionally graded magneto-electro-elastic half-spaces with a quadratic variation is addressed. The coupling factor which is magneto-electromechanical, the dispersion relations, electrical as well as magnetic potential and displacement are provided in an analytical method in conditions that are the open and short electric magnet. Phase velocity, group velocity and magneto-electromechanical coupling factor are influenced by the gradient coefficient, and are estimated and discussed. Research on the wave propagation in magneto-electro-elastic materials is still very limited. This work provides us with a theoretical foundation to design and practically apply SAW devices with high performance.

Keywords: Love wave, piezoelectric, piezomagnetic, functionally graded piezoelectric materials, surface acoustic wave.

Introduction

In physics waves are generally divided into progressive and standing waves. Progressive seismic waves transmit away from seismic sources. Standing seismic waves which are known as free oscillations of the earth represent vibrations of the earth as a whole. These oscillations are created by strong earthquake. From the perspective of the spatial concentration of energy, waves can be divided into body waves and surface waves. Body waves can propagate into the interior of the corresponding medium, whereas surface waves are concentrated along the surface of the medium[1].

Love wave is a kind of surface elastic waves. It can propagate only if the S-wave (secondary waves) velocity generally increases with the distance from the surface of the medium. The particle motion in these waves is transverse and parallel to the surface[1]. Love waves cannot propagate in homogeneous half-space.

Currently, a new subject for many researchers in applied and engineering mechanics is Love wave propagation which is in a solid including an extent that is half infinite. Surface waves appearance in a homogeneous half space that is decorated by a layer of the first love[2]. One can refer to the reference list which is located in Ref3 to observe isotropic material and to Ref4 for seeing anisotropic material, to explore this problem.

Piezoelectric and piezomagnetic materials are highly employed in various engineering structures in recent times, especially in smart or intelligent systems as intelligent sensors, damage detectors, surface acoustic wave (SAW) devices, etc. The propagation of surface wave's propagation in homogeneous structures that are layered and piezoelectric5-6 has been paid attention to by many scholars.

In this work, Love wave propagation in a functionally graded piezo-magneto-elastic semi-infinite medium is considered. The dispersion relation is analytically gained for different surface boundary conditions. The functionally graded materials (FGM) are important in the most advanced integrated systems for vibration control and health monitoring. FGM is a material with material composition and properties varying continuously along specific directions. The intelligent devices made of FGMs have no distinguishable internal boundaries and do not produce stress concentration while they are loaded.

The devices made of FGMs can adapt to severe environments, have high reliability, and play essential roles in most advanced smart structures of vibration control and health monitoring[7-8]. On the other hand, smart or intelligent materials, such as piezoelectric and piezomagnetic currently studied intensively, due to their ability to convert energy from one form to another. (In energy of magnetic, electrical and mechanical). Magneto-electro-elastic materials which concurrently have piezoelectric, piezomagnetic, and especially magnetoelectric coupling effects between the mechanical/thermal, electrical and magnetic fields, have been widely used in engineering structures, particularly in smart structures and intelligent structure systems. These materials have been widely employed in ultrasonic imaging devices, sensors, actuators, transducers and many other emerging components[9].

If we assume an isotropic that is elastic transversely, function graded magneto-electro-elastic half-space as shown in figure-1, the origin of the Cartesian coordinate system (x, y, z) is laid on the surface where the y-axis pointsinto the half-space, and it is estimated to be the medium symmetry axis. The piezoelectic and piezomagnetic material polling direction is parallel to the z-axis (the piezoelectricity and piezomagneticity is polarized in the y-axis direction), and the material properties change gradually with depth. Magneto-electro-elastic material for which the linear equation (equation of motion) elasticity without body forces and initial stress is[10]:

$$\sigma_{ij,j} = \rho \frac{\partial^2 u_i}{\partial t^2} \tag{1-a}$$

The magneto-electrostatics Gauss law which is devoid of free charge, initial electric displacement and initial magnetic induction are given as follows [10]:

$$D_{i,i} = 0 \tag{1-b}$$
$$B_{i,i} = 0 \tag{1-c}$$

Withi, j = x, y, z and the subscript comma denotes a partial derivative with respect to the coordinates.

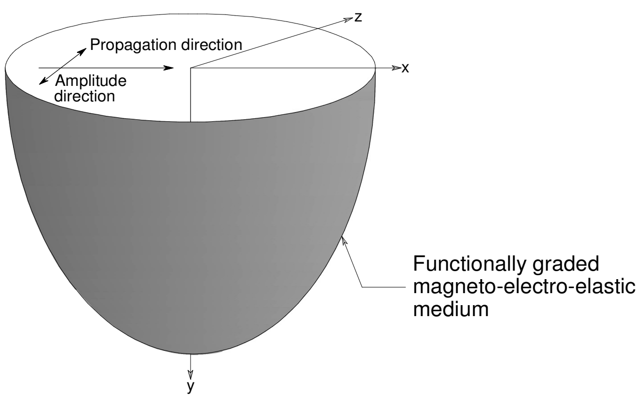

Figure-1
A functionally graded magneto-electro-elastic half-space with quadratic variation

For an elastic transversely isotropic, the Magneto-Electro elastic half-space that is Functionally Graded, the coupled relations that are constitutive are as

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{yz}\\ \sigma_{xz}\\ \sigma_{xy}\end{Bmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0\\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0\\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & c_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & c_{44} & 0\\ 0 & 0 & 0 & 0 & 0 & c_{66}\end{pmatrix}\begin{Bmatrix}\bar{\varepsilon}_{xx}\\ \bar{\varepsilon}_{yy}\\ \bar{\varepsilon}_{zz}\\ 2\bar{\varepsilon}_{yz}\\ 2\bar{\varepsilon}_{xz}\\ 2\bar{\varepsilon}_{xy}\end{Bmatrix} - \begin{pmatrix} 0 & 0 & e_{31}\\ 0 & 0 & e_{31}\\ 0 & 0 & e_{33}\\ 0 & e_{15} & 0\\ e_{15} & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix}E_x\\ E_y\\ E_z\end{Bmatrix} - \begin{pmatrix} 0 & 0 & f_{31}\\ 0 & 0 & f_{31}\\ 0 & 0 & f_{33}\\ 0 & f_{15} & 0\\ f_{15} & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix}H_x\\ H_y\\ H_z\end{Bmatrix} \tag{2}$$

$$\begin{Bmatrix}D_x\\ D_y\\ D_z\end{Bmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & e_{15} & 0\\ 0 & 0 & 0 & e_{15} & 0 & 0\\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0\end{pmatrix}\begin{Bmatrix}\bar{\varepsilon}_{xx}\\ \bar{\varepsilon}_{yy}\\ \bar{\varepsilon}_{zz}\\ 2\bar{\varepsilon}_{yz}\\ 2\bar{\varepsilon}_{xz}\\ 2\bar{\varepsilon}_{xy}\end{Bmatrix} + \begin{pmatrix}\varepsilon_{11} & 0 & 0\\ 0 & \varepsilon_{11} & 0\\ 0 & 0 & \varepsilon_{33}\end{pmatrix}\begin{Bmatrix}E_x\\ E_y\\ E_z\end{Bmatrix} + \begin{pmatrix} g_{11} & 0 & 0\\ 0 & g_{11} & 0\\ 0 & 0 & g_{33}\end{pmatrix}\begin{Bmatrix}H_x\\ H_y\\ H_z\end{Bmatrix}, \tag{3}$$

$$\begin{Bmatrix}B_x\\ B_y\\ B_z\end{Bmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & f_{15} & 0\\ 0 & 0 & 0 & f_{15} & 0 & 0\\ f_{31} & f_{31} & f_{33} & 0 & 0 & 0\end{pmatrix}\begin{Bmatrix}\bar{\varepsilon}_{xx}\\ \bar{\varepsilon}_{yy}\\ \bar{\varepsilon}_{zz}\\ 2\bar{\varepsilon}_{yz}\\ 2\bar{\varepsilon}_{xz}\\ 2\bar{\varepsilon}_{xy}\end{Bmatrix} + \begin{pmatrix} g_{11} & 0 & 0\\ 0 & g_{11} & 0\\ 0 & 0 & g_{33}\end{pmatrix}\begin{Bmatrix}E_x\\ E_y\\ E_z\end{Bmatrix} + \begin{pmatrix}\mu_{11} & 0 & 0\\ 0 & \mu_{11} & 0\\ 0 & 0 & \mu_{33}\end{pmatrix}\begin{Bmatrix}H_x\\ H_y\\ H_z\end{Bmatrix}, \tag{4}$$

Where $c_{66} = (c_{11} - c_{12})/2, \quad \bar{\varepsilon}_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}),$ (5-a)

$$E_i = -\phi_{,i}, \tag{5-b}$$
$$H_i = -\psi_{,i}. \tag{5-c}$$

In Equations-1-a–5-c, σ_{ij} , $\bar{\varepsilon}_{ij}$, u_i, E_i,D_i,B_i andH_i are the components of stress, strain, displacement, electric field, electric displacement, magnetic induction (i.e., magnetic flux) and magnetic field, respectively. ϕ is the electric potential, and ψ is the magnetic potential, and ρ is the mass density, and c_{ij} , e_{ij}, ε_{ij}, f_{ij} , g_{ij} and μ_{ij} are the elastic moduli, piezoelectric, dielectric, piezomagnetic, magneto-electric and magnetic permeability constants, respectively. Based on the idea that Love wave emits in the x- direction, we can formulate that

$$u = v = 0, \tag{6-a}$$
$$w = w(x, y, t), \tag{6-b}$$
$$\phi = \phi(x, y, t), \tag{6-c}$$
$$\psi = \psi(x, y, t), \tag{6-d}$$

Here u, v, and w are respectively the displacements in the x, y, and z-directions. We believe that the constant mass density of transversely isotropic functionally graded magneto-electro-elastic medium equals ρ, while the other coefficients c_{ij} , e_{ij} , ε_{ij}, fij, gij and μij have a quadratic variation in the y-direction

$$c_{ij}(y) = c_{ij}^{0}(1 + by)^2, \tag{7-a}$$
$$e_{ij}(y) = e_{ij}^{0}(1 + by)^2 \tag{7-b}$$
$$\varepsilon_{ij}(y) = \varepsilon_{ij}^{0}(1 + by)^2, \tag{7-c}$$
$$f_{ij}(y) = f_{ij}^{0}(1 + by)^2, \tag{7-d}$$
$$g_{ij}(y) = g_{ij}^{0}(1 + by)^2, \tag{7-e}$$
$$\mu_{ij}(y) = \mu_{ij}^{0}(1 + by)^2, \tag{7-f}$$

Where b>0 is the gradient factor. c_{ij}^0 , e_{ij}^0 , ε_{ij}^0 , f_{ij}^0 , g_{ij}^0 , μ_{ij}^0 ,is, respectively, the value of c_{ij} , e_{ij} , ε_{ij} , f_{ij} , g_{ij} , μ_{ij} on the surface. Obviously that b = 0 corresponds to a homogeneous condition. Homogeneous condition bears no crucial interpretation in the Love wave emission context. Employing equations-2–7-f into 1-a)-1-c, The main equations for this medium can be written as

$$\left(\nabla^2 + \frac{2b}{(1+by)}\frac{\partial}{\partial y}\right)(c_{44}^0 w + e_{15}^0\phi + f_{15}^0\psi) = \frac{\rho}{(1+by)^2}\frac{\partial^2 w}{\partial t^2}, \quad (8\text{-a})$$
$$\left(\nabla^2 + \frac{2b}{(1+by)}\frac{\partial}{\partial y}\right)(e_{15}^0 w - \varepsilon_{11}^0\phi - g_{11}^0\psi) = 0, \quad (8\text{-b})$$
$$\left(\nabla^2 + \frac{2b}{(1+by)}\frac{\partial}{\partial y}\right)(f_{15}^0 w - g_{11}^0\phi - \mu_{11}^0\psi) = 0, \quad (8\text{-c})$$

Where $\nabla^2 = \left(\frac{\partial^2}{\partial x^2}\right) + \left(\frac{\partial^2}{\partial y^2}\right)$is the Laplacian with two dimensions (or Laplace operator).

Substituting equations-5-a to 5-c into equations-1-a to 1-c and making use of equations-6-a to 6-d, we can to reach coupled wave equations and the structural equations as follows

$$c_{44}\nabla^2 w + e_{15}\nabla^2\phi + f_{15}\nabla^2\psi = \rho\frac{\partial^2 w}{\partial t^2}, \quad (9)$$

$$e_{15}\nabla^2 w - \varepsilon_{15}\nabla^2\phi - g_{11}\nabla^2\psi = 0, \quad (10\text{-a})$$
$$f_{15}\nabla^2 w - g_{11}\nabla^2\phi - \mu_{11}\nabla^2\psi = 0, \quad (10\text{-b})$$

$$\sigma_{xz} = c_{44}w_{,x} + e_{15}\phi_{,x} + f_{15}\psi_{,x}, \quad (11\text{-a})$$
$$\sigma_{yz} = c_{44}w_{,y} + e_{15}\phi_{,y} + f_{15}\psi_{,y}, \quad (11\text{-b})$$

$$D_x = e_{15}w_{,x} - \varepsilon_{11}\phi_{,x} - g_{11}\psi_{,x}, \quad (12\text{-a})$$
$$D_y = e_{15}w_{,y} - \varepsilon_{15}\phi_{,y} - g_{11}\psi_{,y}, \quad (12\text{-b})$$

$$B_x = f_{15}w_{,x} - g_{11}\phi_{,x} - \mu_{11}\psi_{,x}, \quad (13\text{-a})$$
$$B_y = f_{15}w_{,y} - g_{11}\phi_{,y} - \mu_{11}\psi_{,y}, \quad (13\text{-b})$$

By assuming

$$\xi = e_{15}^0 w - \varepsilon_{11}^0\phi - g_{11}^0\psi, \quad (14\text{-a})$$
$$\zeta = f_{15}^0 w - g_{11}^0\phi - \mu_{11}^0\psi, \quad (14\text{-b})$$

Equations (9)–(10-b) can be reduced as follows:

$$c_{44}^*\left(\nabla^2 + \frac{2b}{1+by}\frac{\partial}{\partial y}\right)w = \frac{\rho}{(1+by)^2}\frac{\partial^2 w}{\partial t^2}, \quad (15\text{-a})$$
$$\nabla^2\xi + \frac{2b}{1+by}\xi_{,y} = 0, \quad (15\text{-b})$$
$$\nabla^2\zeta + \frac{2b}{1+by}\zeta_{,y} = 0, \quad (15\text{-c})$$

wherec_{44}^* is given by

$$c_{44}^* = c_{44}^0 + \frac{(e_{15}^0\mu_{11}^0 - f_{15}^0 g_{11}^0)}{\varepsilon_{11}^0\mu_{11}^0 - {g_{11}^0}^2}e_{15}^0 + \frac{(e_{15}^0 g_{11}^0 - f_{15}^0\varepsilon_{11}^0)}{{g_{11}^0}^2 - \varepsilon_{11}^0\mu_{11}^0}f_{15}^0, \quad (16)$$

Correspondingly, the electric and magnetic potential are obtained as follows

$$\phi = \frac{1}{\mu_{11}^0\varepsilon_{11}^0 - {g_{11}^0}^2}\left[(\mu_{11}^0 e_{15}^0 - g_{11}^0 f_{15}^0)w - \mu_{11}^0\xi + g_{11}^0\zeta\right], \quad (17)$$
$$\psi = \frac{1}{{g_{11}^0}^2 - \varepsilon_{11}^0\mu_{11}^0}\left[(g_{11}^0 e_{15}^0 - \varepsilon_{11}^0 f_{15}^0)w - g_{11}^0\xi + \varepsilon_{11}^0\zeta\right], \quad (18)$$

The constitutive equations can be written as

$$D_x = (1+by)^2\xi_{,x}, \quad (19\text{-a})$$
$$D_y = (1+by)^2\xi_{,y}, \quad (19\text{-b})$$
$$B_x = (1+by)^2\zeta_{,x}, \quad (20\text{-a})$$
$$B_y = (1+by)^2\zeta_{,y}, \quad (20\text{-b})$$

$$\sigma_{xz} = m_1 w_{,x} + m_2\xi_{,x} + m_3\zeta_{,x}, \quad (21\text{-a})$$
$$\sigma_{yz} = m_1 w_{,y} + m_2\xi_{,y} + m_3\zeta_{,y}, \quad (21\text{-b})$$

where

$$m_1 = (c_{44}^0 + a_1 e_{15}^0 + a_2 f_{15}^0)(1+by)^2, \quad (22\text{-a})$$
$$m_2 = (b_1 e_{15}^0 + b_2 f_{15}^0)(1+by)^2 \quad (22\text{-b})$$
$$m_3 = (c_1 e_{15}^0 + c_2 f_{15}^0)(1+by)^2 \quad (22\text{-c})$$

$$a_1 = \frac{(\mu_{11}^0 e_{15}^0 - g_{11}^0 f_{15}^0)}{(\mu_{11}^0\varepsilon_{11}^0 - {g_{11}^0}^2)} \quad (23\text{-a})$$
$$b_1 = \frac{-\mu_{11}^0}{(\mu_{11}^0\varepsilon_{11}^0 - {g_{11}^0}^2)} \quad (23\text{-b})$$
$$c_1 = \frac{g_{11}^0}{(\mu_{11}^0\varepsilon_{11}^0 - {g_{11}^0}^2)}, \quad (23\text{-c})$$

$$a_2 = \frac{(g_{11}^0 e_{15}^0 - \varepsilon_{11}^0 f_{15}^0)}{({g_{11}^0}^2 - \varepsilon_{11}^0\mu_{11}^0)} \quad (24\text{-a})$$
$$b_2 = \frac{-g_{11}^0}{({g_{11}^0}^2 - \varepsilon_{11}^0\mu_{11}^0)}, \quad (24\text{-b})$$
$$c_2 = \frac{\varepsilon_{11}^0}{({g_{11}^0}^2 - \varepsilon_{11}^0\mu_{11}^0)}, \quad (24\text{-c})$$

The dielectric constant of air is very less than the constant of the piezoelectric medium and this is the reason why the air is regarded as vacuum. The Laplace equation is quenched by the electric potential and magnetic potential in the vacuum

$$\nabla^2\phi^a = 0 \quad (25\text{-a})$$
$$\nabla^2\psi^a = 0 \quad (25\text{-b})$$

Therefore, the electric potential and magnetic field equations in vacuum are

$$\phi^a(x,y,t) = Ge^{ik(x-iy-ct)}, \quad (26\text{-a})$$
$$\psi^a(x,y,t) = He^{ik(x-iy-ct)} \quad (26\text{-b})$$

Where G and H are unknown coefficients estimated through the relevant boundary conditions. In vacuum, dielectric constant (dielectric permittivity) is $\varepsilon_0 = 8.85 \times 10^{-12}\,\frac{F}{m}$, and the magnetic permeability coefficient is $\mu_0 = 4\pi \times 10^{-7}\,\frac{H}{m}$. The magnetic and electric potential in vacuum are nil in those areas that are verydistant from the free surface

$$Y \to -\infty, \phi^a = 0, \quad (27\text{-a})$$
$$Y \to -\infty, \psi^a = 0, \quad (27\text{-b})$$

Considering the free condition of the traction at the surface, it is suggested that

$$\sigma_{yz}(x,0,t) = 0, \quad (28)$$

The conditions that are mangeto-electrically open at free surface lead to

$$\phi(x,0,t) = \phi^a(x,0,t), \quad D_y(x,0,t) = D^a(x,0,t) \quad (29\text{-a})$$
$$\psi(x,0,t) = \psi^a(x,0,t), \quad B_y(x,0,t) = B^a(x,0,t) \quad (29\text{-b})$$

The condition that ismangeto-electrically short at the free surface bears

$$\phi(x,0,t)=0, \quad (30\text{-a})$$

$$\psi(x,0,t)=0, \quad (30\text{-b})$$

we also have to satisfy these regularity conditions:

$$\lim_{y\to-\infty}\phi^a(x,y,t)=0, \quad (31\text{-a})$$

$$\lim_{y\to-\infty}\psi^a(x,y,t)=0, \quad (31\text{-b})$$

$$\lim_{y\to\infty}w(x,y,t)=0, \quad (31\text{-c})$$

$$\lim_{y\to\infty}\phi(x,y,t)=0, \quad (31\text{-d})$$

$$\lim_{y\to\infty}\psi(x,y,t)=0, \quad (31\text{-e})$$

Analytical solution

To provide a solution for the coupled system of partial equations (15-a)-(15-c), the following forms, will be taken as:

$$w(x,y,t)=w(y)e^{ik(x-ct)}, \quad (32\text{-a})$$

$$\xi(x,y,t)=\xi(y)e^{ik(x-ct)}, \quad (32\text{-b})$$

$$\zeta(x,y,t)=\zeta(y)e^{ik(x-ct)}, \quad (32\text{-c})$$

these ordinary differential equations are attained:

$$w''(y)+\frac{2b}{(1+by)}w'(y)-k^2\left(1-\frac{\rho c^2}{(1+by)^2c_{44}^*}\right)w(y)=0, \quad (33\text{-a})$$

$$\xi''(y)+\frac{2b}{(1+by)}\xi'(y)-k^2\xi(y)=0, \quad (33\text{-b})$$

$$\zeta''(y)+\frac{2b}{(1+by)}\zeta'(y)-k^2\zeta(y)=0, \quad (33\text{-c})$$

The precise solutions by these equations are

$$w(y)=\frac{1}{\sqrt{1+by}}\left[\left(A\mathcal{J}_s\left(k\frac{(1+by)}{b}\right)\right)+\left(B\mathcal{K}_s\left(k\frac{(1+by)}{b}\right)\right)\right], \quad (34\text{-a})$$

$$\xi(y)=\frac{1}{\sqrt{1+by}}\left[\left(C\mathcal{J}_{\frac{1}{2}}\left(k\frac{(1+by)}{b}\right)\right)+\left(D\mathcal{K}_{\frac{1}{2}}\left(k\frac{(1+by)}{b}\right)\right)\right], \quad (34\text{-b})$$

$$\zeta(y)=\frac{1}{\sqrt{1+by}}\left[\left(E\mathcal{J}_{\frac{1}{2}}\left(k\frac{(1+by)}{b}\right)\right)+\left(F\mathcal{K}_{\frac{1}{2}}\left(k\frac{(1+by)}{b}\right)\right)\right], \quad (34\text{-c})$$

where

$$s=\sqrt{\frac{1}{4}-\frac{\rho k^2c^2}{b^2c_{44}^*}}=\sqrt{\frac{1}{4}-\frac{k^2c^2}{b^2\frac{c_{44}^*}{\rho}}}=\sqrt{\frac{1}{4}-\alpha^2\beta^2}, \quad (35)$$

Wherec_{sh} is the high speed in the homogeneous magneto-electro elastic medium α and β are constants and they are given by

$$c_{sh}=\sqrt{\frac{c_{44}^*}{\rho}}, \quad (36\text{-a})$$

$$\alpha=\frac{k}{b}, \quad (36\text{-b})$$

$$\beta=\frac{c_{44}^*}{\rho}. \quad (36\text{-c})$$

In equations-34, A, B, C, D, E and F are not recognized and are proper for the boundary conditions, and $\mathcal{J}_n(y)$ and $\mathcal{K}_n(y)$are respectively the nth-order moderated Bessel functions of the first land and the second land. Evidently, either s is real in low-phase speeds$\left({}^{c}/_{c_{sh}}\leq{}^{b}/_{2k}\right)$or unreal and imaginary for high-phase speeds$\left({}^{c}/_{c_{sh}}>{}^{b}/_{2k}\right)$.

By making use of the modified Bessel function asymptotic expansion for large arguments[11], we can posit the displacementof asymptotic behavior and the unknown functions $\xi(y)$ and $\zeta(y)$ within the limit of $y\to+\infty$ as

$$w(y)\sim\sqrt{\frac{k}{2\pi b}}\left(Ae^{kz}-\pi Be^{-kz}\right), \quad (37\text{-a})$$

$$\xi(y)\sim\sqrt{\frac{k}{2\pi b}}\left(Ce^{kz}-\pi De^{-kz}\right), \quad (37\text{-b})$$

$$\zeta(y)\sim\sqrt{\frac{k}{2\pi b}}\left(Ee^{kz}-\pi Fe^{-kz}\right). \quad (37\text{-c})$$

As this analysis shows, the regularity conditions (31-a)-(31-b) would require A=C=E=0. Therefore, the solution for overcoming the displacement and the unknown functions $\xi(y)$ and $\zeta(y)$ in eqation-34 Can be reduced to

$$w(x,y,t)=\frac{B}{\sqrt{1+by}}\mathcal{K}_s\left(k\frac{(1+by)}{b}\right)e^{ik(x-ct)}, \quad (38\text{-a})$$

$$\xi(x,y,t)=\frac{D}{\sqrt{1+by}}\mathcal{K}_{\frac{1}{2}}\left(k\frac{(1+by)}{b}\right)e^{ik(x-ct)}, \quad (38\text{-b})$$

$$\zeta(x,y,t)=\frac{F}{\sqrt{1+by}}\mathcal{K}_{\frac{1}{2}}\left(k\frac{(1+by)}{b}\right)e^{ik(x-ct)}. \quad (38\text{-c})$$

In addition, considering the modified Bessel function integral representation of the second type.[11]

$$\mathcal{K}_n(\xi)=\int_0^\infty e^{-\xi\cosh(t)}\cosh(nt)\,dt,\ |\arg(\xi)|<\frac{\pi}{2}, \quad (39)$$

It follows that $\mathcal{K}_n(\xi)$ equals a function that is real-valued for both real and unreal order n. In equation-38-a-38-c it concerns the phase velocity cases that are respectively higher and lower. This is in line with the idea that there is no net wave propagation in the Y direction there.

Substituting equation-38-a-38-c in equation-17 and 18 and using equation-32-a-32-c the electric potential and magnetic potential are obtained in terms of unknown coefficients B, D, and F:

$$\phi(x,y,t)=\frac{1}{(\mu_{11}^0\varepsilon_{11}^0-{g_{11}^0}^2)\sqrt{1+by}}\left[B(\mu_{11}^0e_{15}^0-g_{11}^0f_{15}^0)\mathcal{K}_s\left(k\frac{(1+by)}{b}\right)-D\mu_{11}^0\mathcal{K}_{\frac{1}{2}}\left(k\frac{(1+by)}{b}\right)+Fg_{11}^0\mathcal{K}_{\frac{1}{2}}\left(k\frac{(1+by)}{b}\right)\right]e^{ik(x-ct)}, \quad (40)$$

$$\psi(x,y,t)=\frac{1}{({g_{11}^0}^2-\varepsilon_{11}^0\mu_{11}^0)\sqrt{1+by}}\left[B(g_{11}^0e_{15}^0-\varepsilon_{11}^0f_{15}^0)\mathcal{K}_s\left(k\frac{(1+by)}{b}\right)-Dg_{11}^0\mathcal{K}_{\frac{1}{2}}\left(k\frac{(1+by)}{b}\right)+F\varepsilon_{11}^0\mathcal{K}_{\frac{1}{2}}\left(k\frac{(1+by)}{b}\right)\right]e^{ik(x-ct)}. \quad (41)$$

The Equations of phase velocity

In this part, we will find the phase velocity equation for conditions that are both magneto-electrically open and short at the free surface. Replacing equations-40 and 41 into the equation-2, and using equations-5-a-5-c and 7-a-7-f, the elastic, electrical and magnetic medium fields are attained with respect to the unknown coefficients B, D, F, G and H. Using boundary conditions can help us to find these unknown coefficients analytically. They are written as

$$\sigma_{yz} = c_{44}^0 w_{,y} + e_{15}^0 \phi_{,y} + f_{15}^0 \psi_{,y}, \tag{42}$$

The following equation is produced as a result of the traction free boundary condition (28):

$$\Bigg\{c_{44}^0 \frac{\partial}{\partial y}\left[\frac{B}{\sqrt{1+by}}\mathcal{K}_s\left(k\frac{(1+by)}{b}\right)e^{ik(x-ct)}\right] + e_{15}^0 \frac{\partial}{\partial y}\left[\frac{1}{\left(\mu_{11}^0\varepsilon_{11}^0 - {g_{11}^0}^2\right)}\left(\frac{B\left(\mu_{11}^0 e_{15}^0 - g_{11}^0 f_{15}^0\right)}{\sqrt{1+by}}\mathcal{K}_s\left(k\frac{(1+by)}{b}\right) - \frac{D\mu_{11}^0}{\sqrt{1+by}}\mathcal{K}_{\frac{1}{2}}\left(k\frac{(1+by)}{b}\right) \frac{Fg_{11}^0}{\sqrt{1+by}}\mathcal{K}_{\frac{1}{2}}\left(k\frac{(1+by)}{b}\right)\right)e^{ik(x-ct)}\right] + f_{15}^0 \frac{\partial}{\partial y}\left[\frac{1}{({g_{11}^0}^2 - \varepsilon_{11}^0\mu_{11}^0)}\left(\frac{B\left(g_{11}^0 e_{15}^0 - \varepsilon_{11}^0 f_{15}^0\right)}{\sqrt{1+by}}\mathcal{K}_s\left(k\frac{(1+by)}{b}\right) - \frac{Dg_{11}^0}{\sqrt{1+by}}\mathcal{K}_{\frac{1}{2}}\left(k\frac{(1+by)}{b}\right) + \frac{F\varepsilon_{11}^0}{\sqrt{1+by}}\mathcal{K}_{\frac{1}{2}}\left(k\frac{(1+by)}{b}\right)\right)e^{ik(x-ct)}\right]\Bigg\} = 0, \tag{43}$$

Equation-43 can be reduced to

$$\Big\{B\big(2\alpha k_{s-1}(\alpha) + (2s+1)k_s(\alpha)\big)\left(c_{44}^0\varepsilon_{11}^0\mu_{11}^0 - c_{44}^0{g_{11}^0}^2 + {e_{15}^0}^2\mu_{11}^0 - 2e_{15}^0 f_{15}^0 g_{11}^0 + \varepsilon_{11}^0{f_{15}^0}^2\right) + \frac{\sqrt{2\pi}e^{-\alpha}(\alpha+1)(-De_{15}^0\mu_{11}^0 + Dg_{11}^0 f_{15}^0 + Fe_{15}^0 g_{11}^0 - F\varepsilon_{11}^0 f_{15}^0)}{\sqrt{\alpha}}\Big\} = 0. \tag{44}$$

From the conditions that are magneto-electrically open at the free surface in equations (29a) and (29b) the following equations can be obtained:

$$\frac{1}{\left(\mu_{11}^0\varepsilon_{11}^0 - {g_{11}^0}^2\right)}\left(B(\mu_{11}^0 e_{15}^0 - g_{11}^0 f_{15}^0)\mathcal{K}_s(\alpha) - D\mu_{11}^0\mathcal{K}_{\frac{1}{2}}(\alpha) + Fg_{11}^0\mathcal{K}_{\frac{1}{2}}(\alpha)\right) - G = 0, \tag{45-a}$$

$$\frac{1}{({g_{11}^0}^2 - \varepsilon_{11}^0\mu_{11}^0)}\left(B(g_{11}^0 e_{15}^0 - \varepsilon_{11}^0 f_{15}^0)\mathcal{K}_s(\alpha) - Dg_{11}^0\mathcal{K}_{\frac{1}{2}}(\alpha) + F\varepsilon_{11}^0\mathcal{K}_{\frac{1}{2}}\alpha\right) - H = 0, \tag{45-b}$$

$$\sqrt{2\pi}(\alpha+1)\big(D(\varepsilon_{11}^0 - 1)\mu_{11}^0 + g_{11}^0(F - Dg_{11}^0)\big) - e^{\alpha}\sqrt{\alpha}B\big(2\alpha k_{s-1}(\alpha) + (2s+1)k_s(\alpha)\big)(f_{15}^0 g_{11}^0 - e_{15}^0\mu_{11}^0) = 0, \tag{45-c}$$

$$e^{\alpha}\sqrt{\alpha}B\big(2\alpha k_{s-1}(\alpha) + (2s+1)k_s(\alpha)\big)(e_{15}^0 g_{11}^0 - \varepsilon_{11}^0 f_{15}^0) + \sqrt{2\pi}(\alpha+1)\left(F\left(-\varepsilon_{11}^0\mu_{11}^0 + \varepsilon_{11}^0 + {g_{11}^0}^2\right) - Dg_{11}^0\right) = 0, \tag{45-d}$$

Using Eqs. (44)–(45-d) the unknown coefficients B, D, F, G and H can be found. Therefore, the equation of phase velocity is found to be

$$\left(\frac{-1}{\left({g_{11}^0}^2 - \varepsilon_{11}^0{\mu_{11}^0}^2\right)^2}\right)\big(2\alpha\mathcal{K}_{s-1}(\alpha) + (2s+1)\mathcal{K}_s(\alpha)\big)\Big(c_{44}^0\Big(4{g_{11}^0}^4 + {g_{11}^0}^2(2\varepsilon_{11}^0\mu_0 - 8\varepsilon_{11}^0\mu_{11}^0 + \mu_0\varepsilon_0 + 2\mu_{11}^0\varepsilon_0) + \varepsilon_{11}^0\mu_{11}^0(\mu_0 - 2\mu_{11}^0)(\varepsilon_0 - 2\varepsilon_{11}^0)\Big) + 2{e_{15}^0}^2(\mu_0 - 2\mu_{11}^0)\left({g_{11}^0}^2 - \varepsilon_{11}^0\mu_{11}^0\right) + 2e_{15}^0 f_{15}^0 g_{11}^0\left(-4\varepsilon_{11}^0\mu_{11}^0 + 4{g_{11}^0}^2 + \mu_0\varepsilon_0\right) + 2{f_{15}^0}^2(\varepsilon_0 - 2\varepsilon_{11}^0)\left({g_{11}^0}^2 - \varepsilon_{11}^0\mu_{11}^0\right)\Big) = 0. \tag{46}$$

Replacing the conditions that are magneto-electrically short at the free surface equations. (30-a) with equations(40) leads to

$$\left(B(\mu_{11}^0 e_{15}^0 - g_{11}^0 f_{15}^0)\mathcal{K}_s(\alpha) - D\mu_{11}^0\mathcal{K}_{\frac{1}{2}}(\alpha) + Fg_{11}^0\mathcal{K}_{\frac{1}{2}}(\alpha)\right) = 0, \tag{47-a}$$

$$\left(B(g_{11}^0 e_{15}^0 - \varepsilon_{11}^0 f_{15}^0)\mathcal{K}_s(\alpha) - Dg_{11}^0\mathcal{K}_{\frac{1}{2}}(\alpha) + F\varepsilon_{11}^0\mathcal{K}_{\frac{1}{2}}\alpha\right) = 0. \tag{47-b}$$

To attain solutionsof the unknown coefficients B, D, and F that are nontrivial, Determinant of the coefficient matrix of equations. (2-44), (2-47-a) and (2-447 b) must be zero:

$$\mathcal{K}_s(\alpha)\left(c_{44}^0(2s+1)\left({g_{11}^0}^2 - \varepsilon_{11}^0\mu_{11}^0\right) + {e_{15}^0}^2\mu_{11}^0(2\alpha - 2s + 1) + 2e_{15}^0 f_{15}^0 g_{11}^0(-2\alpha + 2s - 1) + \varepsilon_{11}^0{f_{15}^0}^2(2\alpha - 2s + 1)\right) - 2\alpha\mathcal{K}_{s-1}(\alpha)\left(c_{44}^0\varepsilon_{11}^0\mu_{11}^0 - c_{44}^0{g_{11}^0}^2 + {e_{15}^0}^2\mu_{11}^0 - 2e_{15}^0 f_{15}^0 g_{11}^0 + \varepsilon_{11}^0{f_{15}^0}^2\right) = 0. \tag{48}$$

It can be easily seen that the phase velocity of the C44 eis related to lastic constant C, the coefficient of dielectric, piezoelectric constants E15, the coefficient F15 piezomagnetic, G11 magneto-electric coefficient, µ11 electromechanically magnetic permeability in both open-and short-term.

Results and Discussion

In previous sections the dispersion relations for the functionally graded magneto-electro-elastic medium with quadratic variation were attained under an analytic fashion under various boundary conditions. To prove the gradient coefficient impact on the phase and group velocity, a numerical example is proposed. A functionally graded magneto-electro-elastic medium is used to explain the result under this medium. In this paper we use a medium made of Cobalt Ferrite ($CoFe_2O_4$) and Terfenol-D as a piezomagnetic and use Barium Titanate ($BaTiO_3$) as a piezoelectric [12].$BaTiO_3$ is used for verification the result of other medium. The material properties used in numerical calculation are listed in Table (1). These results are made under this assumption that all material properties except mass density (ρ)

Phase velocity (c) estimated by numerical solution of the dispersion equation (46) and (48) for electrically open and short conditions that magnetism.Group velocity c_g is defined as equation (49). Where k Obtained from the equation (50)[13].

$$c_g = c + k\frac{dc}{dk}. \tag{49}$$

$$K^2 = 2\left(\frac{(c_{open} - c_{short})}{c_{open}}\right), \tag{50}$$

Figures-2 and 4 show the first phase and second phase speed mode for magneto-electrically open and short conditions, respectively.

First mode and a second mode for each mode refer to the two smallest roots dispersion relations. These findings clearly show that for each wave, Love wave phase velocity increases with increasing slope. It can be seen that when k/b tends to infinity, the phase velocity approaches to c_{sh}.

Figures-3 and 5 show the first and the second mode group speed for the conditions that are respectively magneto-electrically open and short. It is evident that for conditions that are magneto-electrically open annd short, the energy propagation rate c_g(propagation of energy in dispersion behaviors) goes beyond the wave propagation c as depicted in figures-6 and 7 for large wave numbers the group velocity tends to c_{sh} for magneto-electrically open and short cases, respectively.

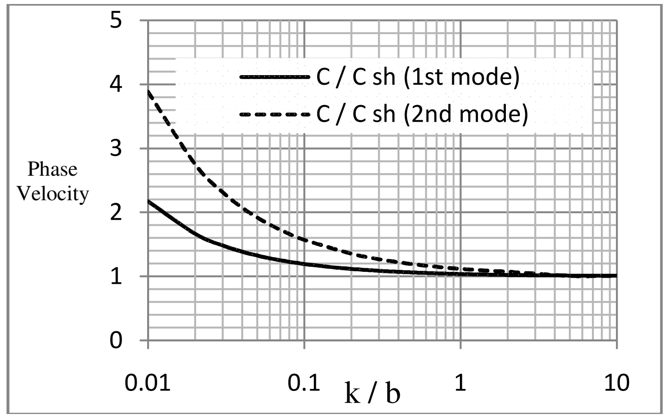

Figure-2
First and second mode phase speed for the casethat is magneto-electrically openfor Cobalt Ferrite material

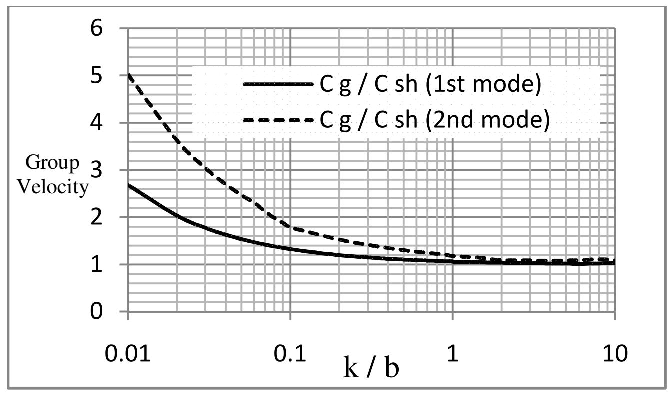

Figure-3
First and second mode group speed for the case that is magneto-electrically openfor Cobalt Ferrite material

Table-1
The properties of piezomagnetic and piezoelectric materials

Material constants	$CoFe_2O_4$	Terfenol-D	$BaTiO_3$
c^0_{44} (10^9 N/m^2)	45.3	5.99	44
ρ (10^3 kg/m^3)	5.3	9.23	5.7
ε^0_{11} (10^{-9} C^2/Nm)	0.08	15.04×10^7	9.82
μ^0_{11} (10^{-6} Ns^2/C^2)	157	3.97	5^a
e^0_{15} (C/m^2)	0	0	11.4
f^0_{15} (N/Am)	550	167.66	0

[a]Magnetic permeability μ^0_{11} (5×10^{-6} Ns^2/C^2) of $BaTiO_3$ is used in all literature available.

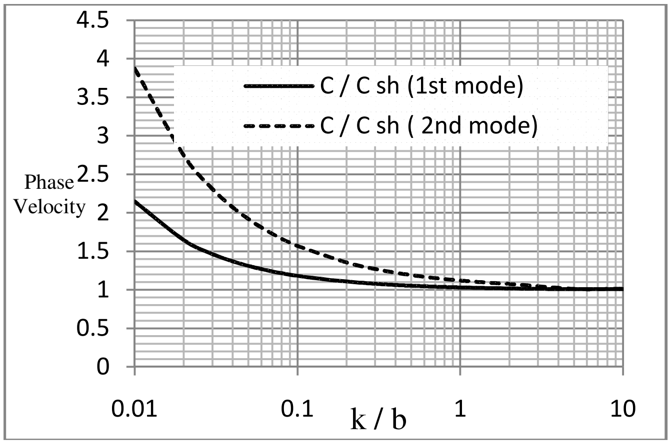

Figure-4
First and second mode phase speed for the case that is magneto-electrically shortfor Cobalt Ferrite material

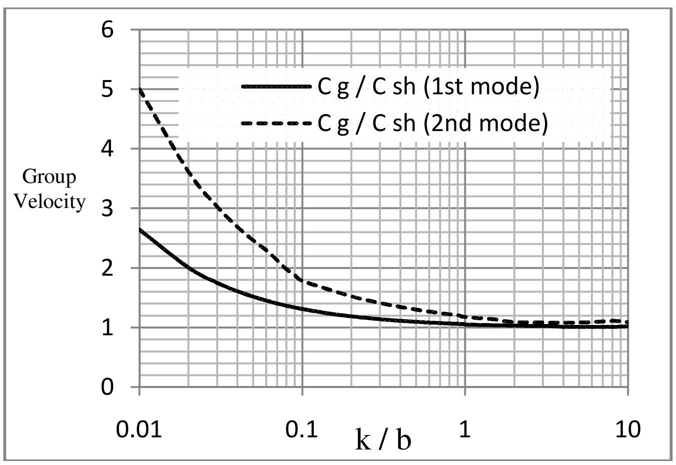

Figure-5
First and second mode group speed for the case that is magneto-electrically short for Cobalt Ferrite material

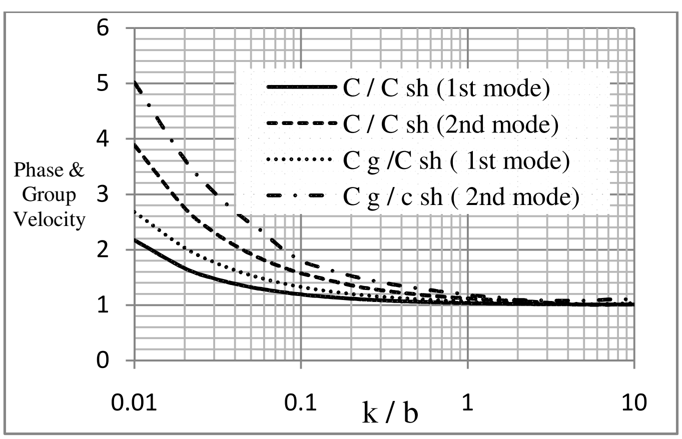

Figure-6
The first and the second modes group and phase speed for the case that is magneto-electrically open case for Cobalt Ferrite material

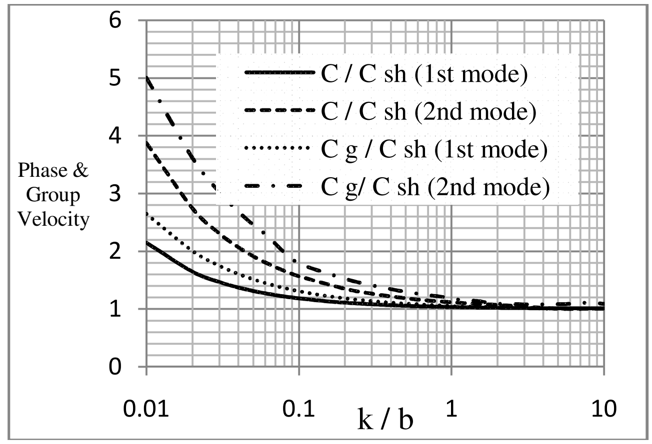

Figure-7
The first and the second modes group and phase speed for the case that is magneto-electrically short for Cobalt Ferrite material

Figure 8 depicts the relation between the coupled magneto-electromechanical factor and non-dimensional wave number. From this figure verify that at low wave number ,that is k/b≈0.01the magneto-electromechanical coupling factor of the first mode reaches to the maximum value of 0.025 and that of the second mode attains the maximum value of 0.005. The value of magneto-electromechanical coupling factor at its maximum value, represent the rate of sensitivity of the sensors. The magneto-electromechanical coupling the first and the second modes group and phase velocity for the case that is magneto-electrically short.

Here we consider another Function graded magneto-electro-elastic medium Made from Terfenol-D. Properties of substances used in the numerical calculations in table-1, where Terfenol-D is piezomagnetic.

Figures-9 and 11 demonstrate the first and the second modes phase velocity for the cases thatare magneto-electric open and short. First and second mode for In any case referring to the two smallest roots of the dispersion relation.The findings evidently show that for any wave number love wave phase velocity rises with an increase of the gradient factor. It can be seen that when k/b tends to infinity, the phase velocity approaches the shear velocity, c_{sh}.

Figures-10 and 12 show the first and the second modes group velocity for the cases that are respectively magneto-electrically open and short. It is apparent that for both magneto-electrically open and short conditions, the rate of energy propagation c_g (propagation of energy in dispersion behaviors) goes beyond the wave propagation c as depicted in figures-13 and 14. For large wave numbers the group velocity tends to c_{sh} for the cases that are respectively magneto-electrically open and short.

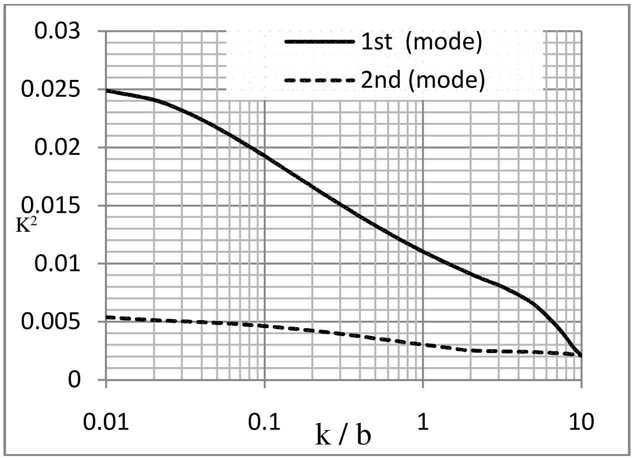

Figure-8
The first and the second modes magneto-electromechanical coupling factor modes for Cobalt Ferrite material

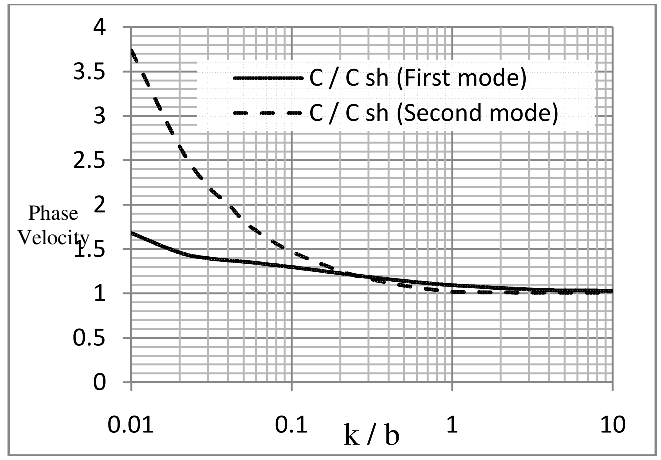

Figure-11
The first and the second modes phase velocity for the cases that are magneto-electrically short for Terfenol-D material

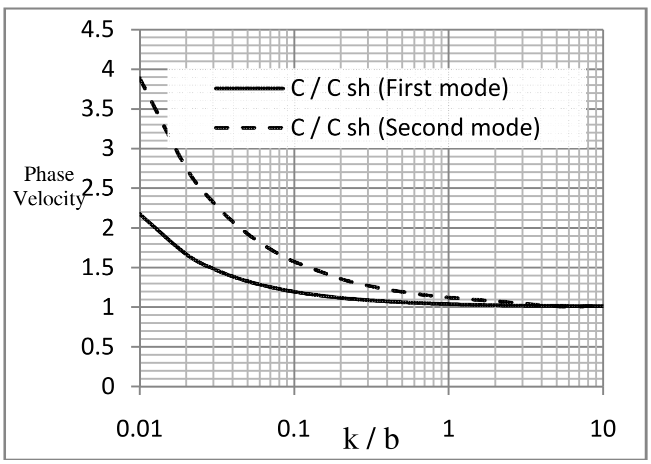

Figure-9
The first and the second modes phase velocity for the cases that are magneto-electrically open for Terfenol-D material.

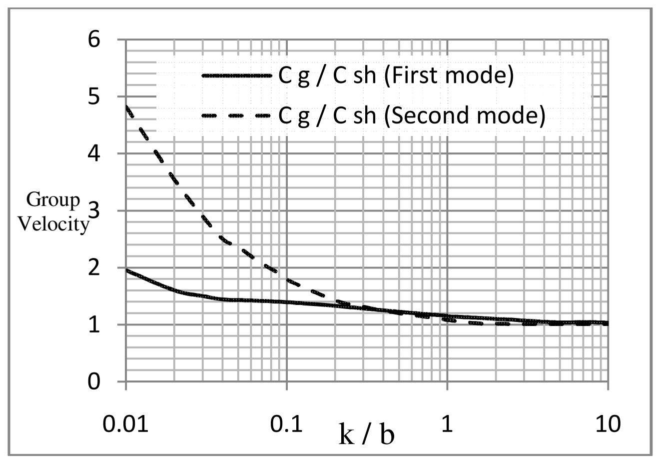

Figure-12
The first and the second modes group velocity for the case that is magneto-electrically short for Terfenol-D material

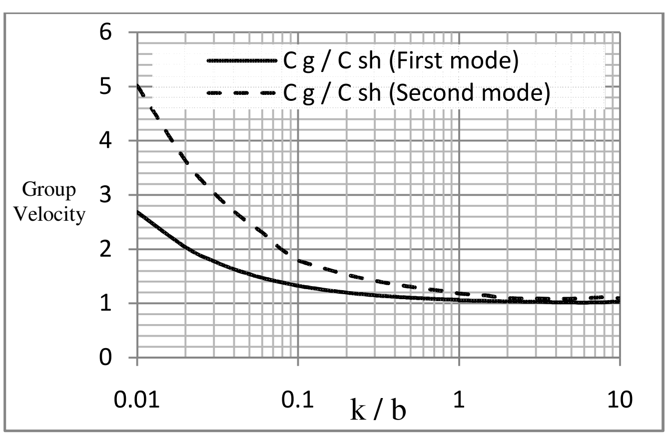

Figure-10
The first and the second modes group velocity for the cases that are magneto-electrically open for Terfenol-D material

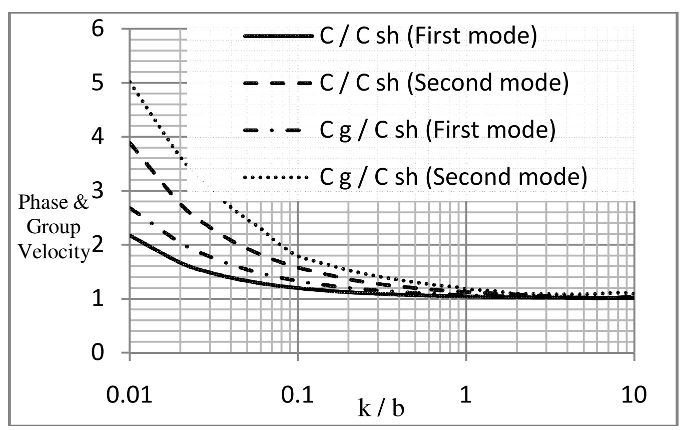

Figure-13
The first and the second modes group and phase velocity for the case that is magneto-electrically open for magneto-electrically open case for Terfenol-D material

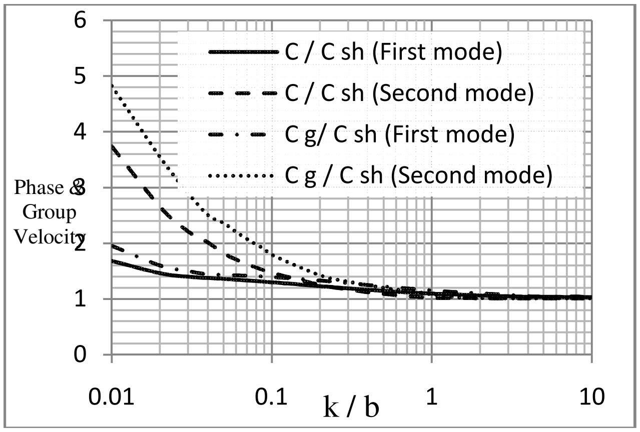

Figure-14
The first and the second modes phase and group velocity for the case that is magneto-electrically short for magneto-electrically short case for Terfenol-D material

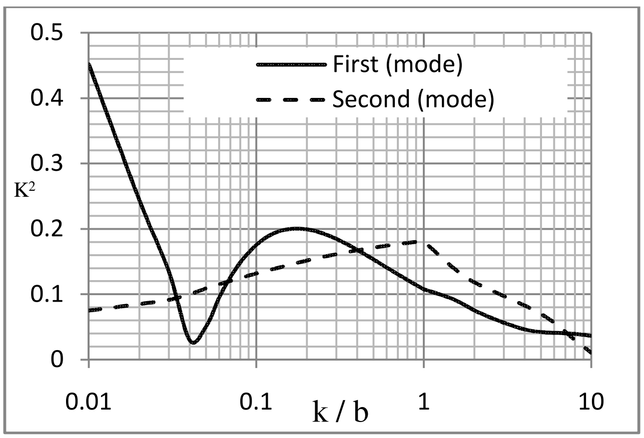

Figure-15
The first and the second modes magneto-electromechanical coupling factor for Terfenol-D material

Figure-15 depicts the relation between the coupled magneto-electromechanical factor and non-dimensional wave number. In the first mode, the factor that is associated with and magneto-electromechanical appears a maximum value of 0.45 at very low wave numbers, that is k/b ≈ 0.01, and attains the minimum value of about 0.02 for 0.02< k/b < 0.06. With respect to the second mode, the factor that is coupled and magneto-electromechanical reaches the utmost value of about 0.18 for 0.6 < k/b <1.3 and reaches the minimum value of 0.01 at very large wave numbers, that is k/b ≈ 10. The aforementioned peak of 0.18 is an important value for the factor that is coupled and magneto-electromechanical and can enhance the ability of SAW devices.

Conclusion

Magneto-electro-elastic Love waves behave in a semi-infinite medium with a graded Function in the second degree is a variety that has been studied analytically.

The gradient coefficient effects on phase velocity, Electromagnetic coupling factor and group velocity which are the basic elements to be taken into account in the design of SAW devices are plotted and discussed. The rise of the gradient factor causes an increase in phase and The group velocity in First and second mode for cases that are magneto-electrically open and short for Cobalt Ferrite material, and also the increase of the gradient factor raises the phase and group velocity of first and the second mode for the cases that are magneto-electrically open and short for Terfenol-D material. The magneto-electromechanical coupling factor rises when the gradient factor in the first and the second modes for Cobalt Ferrite material increases. In the first mode for Terfenol-D material, the coupled magneto-electromechanical factor attains a maximum value of 0.45 at very low wave numbers, and also in the second mode for Terfenol-D material, the coupled magneto-electromechanical factor attains a maximum value of about 0.18 for 0.6 < k/b < 1.3.This study assumes a theoretical basis for the development and use of SAW devices have high performance.

References

1. Novotny O., Seismic Surface Waves, Salvador, CPGG, *Universidade Federal da Bahia,*1-25 **(1999)**
2. Love A.E.H., Some Problems of Geodynamics, Cambridge University Press, London, **(1911)**
3. Ewing W.M. and Jardetzky W.S., Elastic Waves in Layered Media, McGraw-Hill, **(1957)**
4. Muyzert E. and Snieder R., An alternative parameterisation for surface waves in a transverse isotropic medium, *Physics of the Earth and Planetary Interiors*, **118,** 125–133 **(2000)**
5. Knap K.M. and Lenz J., Piezoelectric Love waves in non-classical elastic dielectrics, *International Journal of Engineering Science*, **27,** 879–893 **(1989)**
6. Danoyan Z.N. and Piliposian G.T., Surface electro-elastic Love waves in a layered structure with a piezoelectric substrate and a dielectric layer, *International Journal of Solids and Structures*, **44,** 5829–5847 **(2007)**
7. Huang D.J., Ding H.J. and Chen W.Q., Static analysis of anisotropic functionally graded magneto-electro-elastic beams subjected to arbitrary loading, *European journal of mechanics, European Journal of Mechanics A/Solids,* **29,** 356–369 **(2010)**
8. Huang D.J., Ding H.J. and Chen W.Q., Analytical solution for functionally graded magneto-electro-elastic plane beams, *International Journal of Engineering Science,* **45,** 467–485 **(2007)**
9. Xueli H, High-order derivatives of Green's functions in magneto-electro-elastic materials, *International Journal of Solids and Structures,* **46,** 3405–3411 **(2009)**
10. Liu H., Wang Z.K. and Wang T.J., Effect of initial stress

on the propagation behavior of Love waves in a layered piezoelectric structure, *Int. J. Solids Struct.*, **38,** 37–51 **(2001)**

11. Abramowitz M. and Stegun I.A., Handbook of Mathematical Functions, Dover, New York, **(1965)**

12. Feng W.J., Pan E., Wang X. and Jin J., Rayleigh waves in magneto-electro-elastic half planes, Acta Mech, **202,** 127-134 **(2009)**

13. Bernhard J. and Michael J.V., Properties of Love waves: applications in sensors, *Smart Materials and Structure,* **6,** 668-679 **(1997)**

14. Zakharenko A.A., Love-type waves in layered systems consisting of two cubic piezoelectric crystals, *Journal of Sound and Vibration*, **285,** 877–886 **(2005)**

15. Zakharenko A.A., Analytical studying the group velocity of three-partial Love (type) waves in both isotropic and anisotropic media, *Nondestructive Testing and Evaluation*, **4,** 237–254 **(2005)**

16. Collet B. and Destrade M., Piezoelectric love waves on rotated Y-cut mm2 substrates, IEEE Transactions on Ultrasonics, *Ferroelectrics, and Frequency Control*, **53,** 2132–2139 **(2006)**

17. Danoyan Z.N. and Piliposian G.T., Surface electro-elastic Love waves in a layered structure with a piezoelectric substrate and a dielectric layer, *International Journal of Solids and Structures*, **44,** 5829–5847 **(2007)**

18. Li X.Y., Wang Z.K. and Huang S.H., Love waves in functionally graded piezoelectric materials, *International Journal of Solids and Structures*, **41,** 7309–7328 **(2004)**

19. Du J., Jin X., Wang J. and Xian K., Love wave propagation in functionally graded piezoelectric material layer, *Ultrasonics*, **46,** 13–22 **(2007)**

20. Eskandari M. and Shodja H.M., Love wave's propagation in functionally graded piezoelectric materials with quadratic variation, *Journal of Sound and Vibration,* **313,** 195–204 **(2008)**