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# A Sieve Bootstrap approach to constructing Prediction Intervals for Long Memory Time series

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## Abstract

*This paper is concerned with the construction of bootstrap prediction intervals for autoregressive fractionally integrated moving-average processes which is a special class of long memory time series. For linear short-range dependent time series, the bootstrap based prediction interval is a good nonparametric alternative to those constructed under parameter assumptions. In the long memory case, we use the AR-sieve bootstrap which approximates the data generating process of a given long memory time series by a finite order autoregressive process and resamples the residuals. A simulation study is conducted to examine the performance of the AR-sieve bootstrap procedure. For the purpose of illustration a real data example is also presented.*

**Keywords:** Sieve bootstrap, prediction intervals, long memory time series.

## Introduction

Forecasting is one of the fundamental objectives in the statistical analysis of time series. A forecast or prediction interval usually consists of an upper and a lower limit between which the future value of the given time series is expected to lie with a prescribed probability. Parametric approaches to constructing forecast intervals for a linear time series assume that the series follows a linear finite dimensional model with known error distributions. As a consequence, these methods fail to get satisfactory coverage when the error distribution is misspecified. For example, the most commonly used Box-Jenkins procedure assumes a normal error distribution and it performs poorly for a skewed bimodal distribution[1].

Since the work by the above authors, some bootstrap procedures have been proposed as a distribution free alternative to parametric methods. Bootstrap methods for constructing prediction intervals for processes with a finite unknown $p$, where $p$ is the order of *AR* model, under the assumption that a consistent estimator of $p$ is available have been proposed[2,3]. Some further developments on bootstrap prediction intervals can be found in the literature[4-6]. Recently, some bootstrap methods for constructing prediction intervals for a general class of linear processes have been proposed[7,8]. Different ways of introducing model uncertainty in the time series bootstrap have been introduced and a comparison of these methods with existing alternatives in the case of prediction intervals have also been studied[9]. However, all the work mentioned above seems to exclude the long memory time series. In the last two decades we have witnessed a rapid development in the field of long memory time series. Many observed time series exhibits long range dependence. A review of the application and analysis of the long memory time series models can be found in the literarure[10,11]. For long memory time series one can extend the Box-Jenkins procedure but the drawback of the Box-Jenkins procedure is its lack of robustness to model miss-specification and error distribution. For long memory *ARFIMA* processes, a model based bootstrap method to construct prediction intervals has been proposed[12]. Their method identifies the model and estimates its parameter by Whittle estimator. The identification and estimation of the *AR* model is simpler as compared to *ARFIMA* model. This article considers the construction of prediction intervals for long memory time series using the *AR* sieve-bootstrap method. We use the residual based *AR*($p$)-sieve bootstrap resembling scheme by approximating the given long memory model by an *AR*($p$) model. The *AR* sieve-bootstrap method for the construction of prediction intervals of *ARFIMA* model was considered by Amjad et al.[13]. Later on the same problem was also independently studied by Rupasinghe el al.[14]. The paper is organized as follows: in Section 2 we describe the sieve-bootstrap procedure to construct prediction intervals for long memory time series. Section 3 presents the simulation results. Section 4 contains a real data example. Finally, we conclude in section 5.

## Sieve Bootstrap Prediction Intervals

The method of sieve bootstrap approximates the data generating mechanism of a linear process by an *AR*($p$) model where

$$p = p(n) \to \infty \text{ as } n \to \infty \text{ and } p(n) = o(n)$$

and $n$ is the sample size[15,16]

The bootstrap samples are then drawn from the fitted *AR*($p$) model. Recently, the consequences of applying the sieve bootstrap for fractionally integrated processes which is a special class of long memory time series has been investigated[17]. The work done by the above authors motivates us to consider its use in constructing forecast intervals for long memory models.

The autoregressive fractionally integrated moving-average processes is a well known class of long memory models[18,19]. It is defined as

$$\phi(B)X_t = \theta(B)(1-B)^{-d}u_t$$

Where: $\phi(B) = \sum_{i=0}^{p} \phi_i B^i$ and $\theta(B) = \sum_{j=0}^{q} \theta_j B^j$

$\theta_0 = 1$, $\phi_0 = 1$ ( and $B$ is the backward shift operator )

are the autoregressive and moving-average operators respectively; $\phi(B)$ and $\theta(B)$ have no common roots, $(1-B)^{-d}$ is fractionally differencing operator defined by the binomial expansion

$$(1-B)^{-d} = \sum \frac{\Gamma(j+d)}{\Gamma(j+1)} B^j, \; j = 0,1,2,\dots\dots\dots \;\dots..$$

for $d < 0.5$, $d \neq 0, -1, -2, \dots\dots\dots$ and $u_t$ is a white noise sequence with finite variance. If $d > 0$ then the series exhibits long memory. *ARFIMA* models have proven useful tools in the analysis of long range dependence processes.

The algorithm to construct *AR*-sieve bootstrap prediction intervals is described as follows.

For a given sample of size *n*, select the order *p*(*n*) of the autoregressive approximation. Any order selection criterion can be used. We use the AIC criterion and fix $p_{\max} = n/10$ here[20].

Fit an *AR*(*p*) model to the given series to get the estimates $\hat{\phi} = (\hat{\phi}_1, \hat{\phi}_2, \dots\dots\dots \; \dots\dots\dots \hat{\phi}_p)'$ of the autoregressive approximation and compute the residuals. For model fitting we use the Yule-Walker method.

Draw an iid sample $\hat{G}_{\tilde{u}}$ denoted by $\hat{u}_t^*$ where $\hat{G}_{\tilde{u}}$ is the empirical distribution function of the centered residuals defined as

$$\hat{G}_{\tilde{u}}(x) = (n-p)^{-1} \sum_{t=p+1}^{n} I(\tilde{u}_t \le x)$$

where $\tilde{u}_t = \hat{u}_t - \bar{\hat{u}}_t$ and $\bar{\hat{u}}_t = (n-p)^{-1} \sum_{t=p+1}^{n} \hat{u}_t$

The bootstrap series is constructed by the recursion

$$\sum_{j=0}^{p} \hat{\phi}_j (X^*_{t-j} - \bar{X}) = \hat{u}_t^*$$

Where $\hat{\phi}_0 = 1$ and the starting *p* observations are set to be $\bar{X}$. Note that we generate *n+100* observations by the above recursion and then discard the first *100* observations.

Compute the order $p^* = p^*(n)$ as in step (1) and the autoregressive coefficients $\hat{\phi}^* = (\hat{\phi}^*_1, \hat{\phi}^*_2, \dots\dots\dots \; \dots\dots\dots \hat{\phi}^*_p)$ as in step (2) for the bootstrap sample $(X^*_1, X^*_2, \dots\dots\dots \; \dots\dots X^*_T)$ given from the previous step.

Compute the h-step ahead predicted future observations by the recursion

$$X^*_{T+h} - \bar{X} = \sum_{j=1}^{p^*} \hat{\phi}_j (X^*_{T+h-j} - \bar{X}) + \hat{u}_t^*$$

where $h \ge 0$ and $X_t^* = X_t$, for $t \le T$

Repeat the steps 3 to 6 *B* times to obtain the bootstrap future values $(X^{*1}_{T+h}, X^{*2}_{T+h}, \dots\dots\dots \; \dots\dots X^{*B}_{T+h})$, the prediction limits are defined as the quantiles of the bootstrap distribution function of $X^*_{T+h}$. The monte Carlo estimate of the bootstrap distribution function of $X^*_{T+h}$ is given by $\hat{G}_{X^*+h,B}(x) = \#\{X^{*b}_{T+h} \le x\}/B$, $b = 1,2,3,\dots\dots\dots, B$. The $100\,\beta$ percent prediction interval for $X_{T+h}$ is constructed as

$$\left[Q^*\left(\frac{1-\beta}{2}\right), Q^*\left(\frac{1+\beta}{2}\right)\right] \text{ where } Q^* = \hat{G}^{-1}_{X^*+h}$$

## Simulation Studies

To assess the finite sample performance of the sieve bootstrap prediction intervals we perform a simulation study. We use the following four models with two different values of the long memory parameter $d$, where *d*=0.2 (moderate long memory) and *d*=0.4 (strong persistence) are entertained.

*M1* : *ARFIMA* (*0,d,0*)
*M2* : *ARFIMA* (*1,d,0*)
*M3* : *ARFIMA* (*0,d,1*)
*M4*: *ARFIMA* (*1,d,1*)

We use two sample sizes 100 and 300 and different autoregressive and moving-average parameters for each model. We only present the results for autoregressive parameter $\phi_1 = 0.5$ and moving-average parameter $\theta_1 = 0.3$. We use three different error distributions, the standard normal, the *t*-distribution with 5 degrees of freedom (i.e. leptokurtic one) and the exponential distribution with scale parameter equal to one (i.e. the asymmetric one). The exponential and *t*-distributions are centered and scaled to have zero mean and unit variance. We construct *h*= 1, 3, 5, 10 step ahead forecast intervals at the nominal coverage level of 95 percent. To evaluate the performance of the intervals we calculate the empirical coverage and length of the intervals with corresponding standard errors. The number of simulations *S* and the number of bootstrap resamples *B* are set to be 1000. For each combination of the model, parameters, sample size and innovation distribution we perform the following steps.

Generate a realization of size $n$ and generate $R$=1000 future values of $X_{T+h}$. These future values are generated conditional on the past $n$ values of the generated realization, the true error distribution and the true values of the parameters.

Calculate the bootstrap forecast interval $\left[Q^*\left(\frac{1-\beta}{2}\right), Q^*\left(\frac{1+\beta}{2}\right)\right]$ based on $B$=1000 bootstrap resamples where $\left[Q^*\left(\frac{1-\beta}{2}\right)\right]$ and $\left[Q^*\left(\frac{1+\beta}{2}\right)\right]$ is the $\left(\frac{1-\beta}{2}\right)th$ and $\left(\frac{1+\beta}{2}\right)th$ is percentile of the 1000 bootstrap predicted values.

Using the $R$ true future values we calculate the empirical coverage of the interval. The empirical coverage ($\beta^*$) is obtained as the percentage of $R$ future values which lie in-between $\left[Q^*\left(\frac{1-\beta}{2}\right)\right]$ and $\left[Q^*\left(\frac{1+\beta}{2}\right)\right]$. The length of the interval is calculated as $L^* = Q^*\left(\frac{1+\beta}{2}\right) - Q^*\left(\frac{1-\beta}{2}\right)$.

We repeat the above steps $S$=1000 times and obtain the empirical mean length ($\overline{L^*}$) and the empirical mean coverage $\overline{\beta}^*$ with corresponding standard errors for each of the forecast intervals as follows.

$$\overline{L^*} = S^{-1}\sum_{i=1}^{S} L_i^* ,$$

$$SE(\overline{L^*}) = (S^{-1}(S-1)^{-1}\sum_{i=1}^{S}(L_i^* - \overline{L^*})^2)^{1/2}$$

$$\overline{\beta}^* = S^{-1}\sum_{i=1}^{S}\beta_i^* ,$$

$$SE(\overline{\beta}^*) = (S^{-1}(S-1)^{-1}\sum_{i=1}^{S}(\beta_i^* - \overline{\beta}^*)^2)^{1/2}$$

The results for the four models are displayed in tables 1-4. For a particular sample size and parameters each model has almost the same performance for all the three error distributions which shows the robustness of the bootstrap forecast intervals to the error distribution. As expected the coverage increases as we increase the sample size from 100 to 300. Another notable feature is that a slight decrease occurs in the coverage as the model become more persistent. It might be due to the fact that the quality of $AR(p)$ approximation deteriorates for strongly persistent long memory processes.

**Table-1**
**Simulation results for M1**

| Step-ahead | Sample size | Distr. | d=0.2 $\overline{\beta}^*$ (se) $\overline{L^*}$ (se) | d=0.4 $\overline{\beta}^*$ (se) $\overline{L^*}$ (se) |
|---|---|---|---|---|
| 1 | 100 | N | 93.9(0.003) 3.991(0.0004) | 92.8(0.006) 4.390(0.0004) |
| | | t(5) | 94.1(0.003) 4.077(0.0005) | 93.1(0.006) 4.429(0.0006) |
| | | EXP. | 93.5(0.007) 3.867(0.0007) | 90.3(0.013) 4.318(0.0007) |
| | 300 | N | 94.7(0.003) 4.059(0.0002) | 93.7(0.006) 4.654(0.0004) |
| | | t(5) | 94.7(0.002) 4.118 (0.0003) | 94.0(0.006) 4.706(0.0004) |
| | | EXP. | 94.7(0.006) 3.925 (0.0004) | 92.6(0.011) 4.587(0.0005) |
| 3 | 100 | N | 93.9(0.002) 4.006(0.0004) | 92.5(0.004) 4.403(0.0005) |
| | | t(5) | 94.1(0.002) 4.095(0.0005) | 93.0(0.004) 4.481 (0.0006) |
| | | EXP. | 93.4(0.007) 3.867(0.0006) | 90.0(0.010) 4.348(0.0008) |
| | 300 | N | 94.6(0.003) 4.048(0.0002) | 93.5(0.004) 4.650(0.0004) |
| | | t(5) | 94.6(0.002) 4.119 (0.0003) | 94.0(0.003) 4.729(0.0004) |
| | | EXP. | 94.5(0.004) 3.941 (0.0004 | 92.4(0.008) 4.615(0.0005) |
| 5 | 100 | N | 94.0(0.004) 4.020(0.0004) | 92.3(0.003) 4.415(0.0005) |
| | | t(5) | 94.1(0.002) 4.093(0.0005) | 92.8(0.003) 4.486(0.0006) |
| | | EXP. | 93.4(0.005) 3.888(0.0007) | 89.9(0.008) 4.359 (0.0007) |
| | 300 | N | 94.6(0.003) 4.058(0.0002) | 93.4(0.003) 4.657(0.0004) |
| | | t(5) | 94.5(0.002) 4.104 (0.0003) | 93.8(0.003) 4.721(0.0004) |
| | | EXP. | 94.5(0.004) 3.939 (0.0004) | 92.3(0.007) 4.618(0.0005) |
| 10 | 100 | N | 94.0(0.002) 4.013 (0.0004) | 92.0(0.003) 4.410(0.0004) |
| | | t(5) | 94.1(0.002) 4.103(0.0006) | 93.5(0.003) 4.494(0.0006) |
| | | EXP. | 93.3(0.005) 3.884(0.0007) | 89.5(0.007) 4.346(0.0007) |
| | 300 | N | 94.6(0.003) 4.057(0.0002) | 93.2(0.003) 4.652(0.0004) |
| | | t(5) | 94.6(0.002) 4.119 (0.0003) | 93.7(0.002) 4.745(0.0004) |
| | | EXP. | 94.4(0.003) 3.956 (0.0004) | 92.0(0.006) 4.629(0.0005) |

**Table-2**
**Simulation results for *M2***

| Step-ahead | Sample size | Distr. | d=0.2 $\bar{\beta}^*$ (se) $\bar{L}^*$ (se) | d=0.4 $\bar{\beta}^*$ (se) $\bar{L}^*$ (se) |
|---|---|---|---|---|
| 1 | 100 | N | 94.0(0.003) 4.188(0.0004) | 93.3(0.004) 4.127(0.0004) |
| | | t(5) | 94.3(0.003) 4.274(0.0006) | 93.8(0.004) 4.185(0.0007) |
| | | EXP. | 94.1(0.010) 4.204(0.0007) | 92.8(0.009) 4.072(0.0007) |
| | 300 | N | 94.6(0.003) 4.240(0.0002) | 94.1(0.004) 4.258(0.0003) |
| | | t(5) | 94.7(0.003) 4.284(0.0004) | 94.3(0.003) 4.301(0.0004) |
| | | EXP. | 94.7(0.009) 4.265(0.0004) | 93.4(0.010) 4.189(0.0004) |
| 3 | 100 | N | 94.2(0.002) 4.217(0.0004) | 93.4(0.003) 4.151(0.0004) |
| | | t(5) | 94.3(0.002) 4.331(0.0006) | 93.8(0.003) 4.225(0.0006) |
| | | EXP. | 94.2(0.004) 4.256(0.0008) | 92.8(0.007) 4.095(0.0007) |
| | 300 | N | 94.6(0.002) 4.246(0.0003) | 94.0(0.002) 4.261(0.0003) |
| | | t(5) | 94.7(0.002) 4.304(0.0004) | 94.3(0.002) 4.302(0.0004) |
| | | EXP. | 94.9(0.002) 4.284(0.0004) | 93.6(0.006) 4.212(0.0004) |
| 5 | 100 | N | 94.2(0.002) 4.223(0.0004) | 93.4(0.003) 4.156(0.0004) |
| | | t(5) | 94.4(0.002) 4.337(0.0006) | 93.7(0.003) 4.227(0.0006) |
| | | EXP. | 94.2(0.004) 4.255(0.0008) | 92.7(0.006) 4.101(0.0007) |
| | 300 | N | 94.7(0.002) 4.256(0.0003) | 94.0(0.002) 4.263(0.0003) |
| | | t(5) | 94.6(0.002) 4.301(0.0004) | 94.2(0.002) 4.300(0.0004) |
| | | EXP. | 94.9(0.002) 4.300(0.0004) | 93.5(0.006) 4.214(0.0004) |
| 10 | 100 | N | 94.3(0.002) 4.226(0.0004) | 93.3(0.003) 4.154(0.0004) |
| | | t(5) | 94.4(0.002) 4.340(0.0006) | 93.6(0.003) 4.244(0.0006) |
| | | EXP. | 94.3(0.003) 4.271(0.0008) | 92.5(0.006) 4.114(0.0007) |
| | 300 | N | 94.6(0.002) 4.295(0.0004) | 94.1(0.002) 4.267(0.0003) |
| | | t(5) | 94.6(0.002) 4.295(0.0004) | 94.2(0.002) 4.312(0.0004) |
| | | EXP. | 95.1(0.002) 4.321(0.0005) | 93.4(0.005) 4.229(0.0004) |

**Table-3**
**Simulation results for *M3***

| Step-ahead | Sample size | Distr. | d=0.2 $\bar{\beta}^*$ (se) $\bar{L}^*$ (se) | d=0.4 $\bar{\beta}^*$ (se) $\bar{L}^*$ (se) |
|---|---|---|---|---|
| 1 | 100 | N | 94.2(0.002) 3.938(0.0003) | 93.5(0.002) 4.005(0.0004) |
| | | t(5) | 94.2(0.002) 4.039(0.0006) | 93.8(0.003) 4.081(0.0006) |
| | | EXP. | 94.9(0.006) 3.835(0.0007) | 92.3(0.009) 3.921(0.0007) |
| | 300 | N | 94.7(0.002) 3.960(0.0002) | 94.1(0.003) 4.142(0.0003) |
| | | t(5) | 94.7(0.002) 4.037(0.0004) | 94.4(0.002) 4.191(0.0004) |
| | | EXP. | 95.5(0.005) 3.844(0.0004) | 93.4(0.009) 4.062(0.0004) |
| 3 | 100 | N | 94.3(0.002) 3.957(0.0003) | 93.4(0.003) 4.009(0.0004) |
| | | t(5) | 94.3(0.002) 4.056(0.0006) | 93.8(0.003) 4.099(0.0006) |
| | | EXP. | 94.8(0.004) 3.848(0.0007) | 92.2(0.008) 3.949(0.0007) |
| | 300 | N | 94.7(0.002) 3.968(0.0002) | 94.0(0.003) 4.146(0.0003) |
| | | t(5) | 94.7(0.002) 4.042(0.0004) | 94.4(0.002) 4.205(0.0004) |
| | | EXP. | 95.6(0.003) 3.847(0.0004) | 93.3(0.008) 4.078(0.0004) |
| 5 | 100 | N | 94.2(0.002) 3.962(0.0003) | 93.3(0.002) 4.016(0.0004) |
| | | t(5) | 94.3(0.002) 4.053(0.0006) | 93.7(0.003) 4.093(0.0006) |
| | | EXP. | 94.8(0.004) 3.848(0.0007) | 92.0(0.007) 3.938(0.0007) |
| | 300 | N | 94.7(0.002) 3.962(0.0002) | 94.0(0.002) 4.144(0.0003) |
| | | t(5) | 94.6(0.002) 4.024(0.0003) | 94.2(0.002) 4.189(0.0004) |
| | | EXP. | 95.6(0.003) 3.853(0.0004) | 93.2(0.007) 4.086(0.0004) |
| 10 | 100 | N | 94.2(0.002) 3.952(0.0003) | 93.1(0.002) 4.015(0.0004) |
| | | t(5) | 94.3(0.002) 4.059(0.0006) | 93.6(0.003) 4.117(0.0006) |
| | | EXP. | 94.7(0.004) 3.850(0.0007) | 91.6(0.006) 3.947(0.0007) |
| | 300 | N | 94.7(0.002) 3.963(0.0002) | 94.0(0.002) 4.145(0.0003) |
| | | t(5) | 94.7(0.002) 4.048(0.0004) | 94.2(0.002) 4.205(0.0004) |
| | | EXP. | 95.5(0.002) 3.867(0.0004) | 93.0(0.006) 4.093(0.0004) |

## Real data example

In this section, we apply the sieve bootstrap approach to construct forecast intervals for Nile river data for the years 622-1281, measured at the Roda Guage near the Cairo. The Nile river data is well known to have long memory behavior from previous studies[10]. The length of the series is 663 and we use a moving subsamples of size *n*= 100 and 300 to train a model, which is then used to construct 1,3,5 and 10 steps ahead forecast intervals. The window type subsampling proceeds as follows.

Take the first *n* observations as a sample and construct the sieve bootstarp forecast intervals for observations (n+1), (n+3), (n+5) and (n+10), which are one; three, five and ten steps ahead forecast intervals respectively.

Remove the first observation from the above subsamples and include $(n+1)^{th}$ observation to make a subsample of size *n* and construct forecast intervals for observations (n+2), (n+4), (n+6) and (n+11) as above.

We continue with the same fashion until we take from observation (654-n) to (653) as a sample and construct forecast intervals for observations 654, 656, 658 and 663, which are one, three, five and ten steps ahead forecast intervals respectively.

**Table-4**
**Simulation results for *M4***

| Step-ahead | Sample size | Distr. | d=0.2 $\bar{\beta}^*$ (se) $\bar{L}^*$ (se) | d=0.4 $\bar{\beta}^*$ (se) $\bar{L}^*$ (se) |
|---|---|---|---|---|
| 1 | 100 | N | 93.7(0.006) 4.668(0.0005) | 93.6(0.005) 4.350(0.0004) |
| | | t(5) | 93.6(0.009) 4.705(0.0006) | 93.9(0.005) 4.419(0.0006) |
| | | EXP. | 93.5(0.013) 4.801(0.0008) | 93.6(0.011) 4.393(0.0007) |
| | 300 | N | 94.7(0.006) 4.794(0.0003) | 94.5(0.004) 4.473(0.0003) |
| | | t(5) | 94.6(0.006) 4.815(0.0004) | 94.3(0.005) 4.513(0.0004) |
| | | EXP. | 93.8(0.013) 4.948(0.0005) | 94.0(0.011) 4.534(0.0005) |
| 3 | 100 | N | 93.7(0.003) 4.695(0.0005) | 93.6(0.003) 4.380(0.0004) |
| | | t(5) | 93.8(0.003) 4.761(0.0006) | 93.7(0.003) 4.449(0.0006) |
| | | EXP. | 93.5(0.003) 4.870(0.0008) | 93.5(0.004) 4.436(0.0008) |
| | 300 | N | 94.7(0.002) 4.814(0.0003) | 94.4(0.002) 4.500(0.0003) |
| | | t(5) | 94.6(0.002) 4.863(0.0004) | 94.4(0.002) 4.546(0.0004) |
| | | EXP. | 94.4(0.003) 4.993(0.0005) | 94.4(0.003) 4.583(0.0005) |
| 5 | 100 | N | 93.8(0.003) 4.701(0.0005) | 93.5(0.003) 4.378(0.0004) |
| | | t(5) | 93.8(0.003) 4.771(0.0007) | 93.6(0.003) 4.456(0.0007) |
| | | EXP. | 93.5(0.003) 4.898(0.0008) | 93.5(0.004) 4.454(0.0007) |
| | 300 | N | 94.6(0.002) 4.817(0.0003) | 94.4(0.002) 4.513(0.0003) |
| | | t(5) | 94.6(0.002) 4.859(0.0004) | 94.4(0.002) 5.551(0.0004) |
| | | EXP. | 94.4(0.002) 4.986(0.0005) | 94.4(0.003) 4.586(0.0005) |
| 10 | 100 | N | 93.7(0.003) 4.699(0.0005) | 93.6(0.003) 4.394(0.0004) |
| | | t(5) | 93.8(0.003) 4.768(0.0007) | 93.6(0.003) 4.470(0.0007) |
| | | EXP. | 93.5(0.013) 4.896(0.0008) | 93.5(0.004) 4.462(0.0007) |
| | 300 | N | 94.6(0.002) 4.811(0.0003) | 94.3(0.002) 4.504(0.0003) |
| | | t(5) | 94.5(0.002) 4.861(0.0004) | 94.4(0.002) 4.552(0.0004) |
| | | EXP. | 94.5(0.002) 5.002(0.0005) | 94.4(0.003) 4.585(0.0005) |

## Conclusion

A number of studies have been conducted to construct bootstrap prediction intervals for short memory time series models. In this study we investigate the *AR* sieve bootstrap to construct prediction intervals for long memory time series models. The investigation has been carried by conducting simulation study for a number of long memory models using three different error distributions the normal, the exponential and the t-distribution with five degrees of freedom. The results show that the bootstrap prediction interval has good coverage for all the error distributions. The bootstrap prediction interval also performs well for the real data example. Figures 1-4 show one, three, five and ten step-ahead sieve bootstrap prediction intervals respectively. The sample is fixed to be 300 for a nominal coverage of 95%.

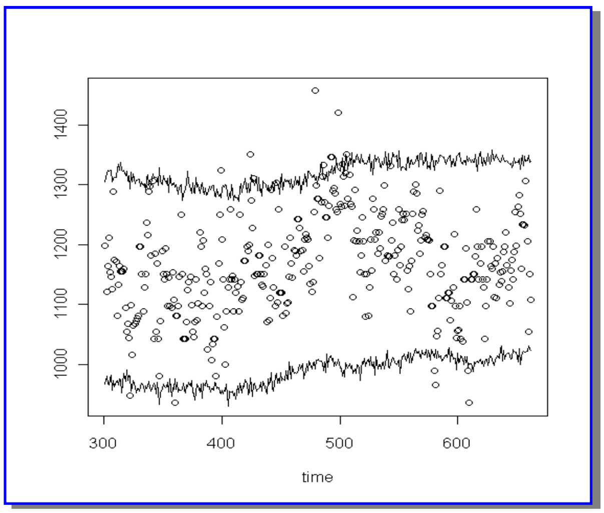


**Figure-1**
**One Step-Ahead 95% Prediction Intervals**

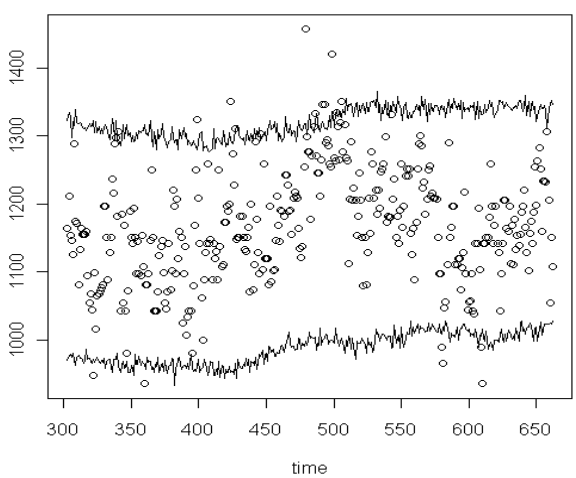


**Figure-2**
**Three Step-Ahead 95% Prediction Intervals**

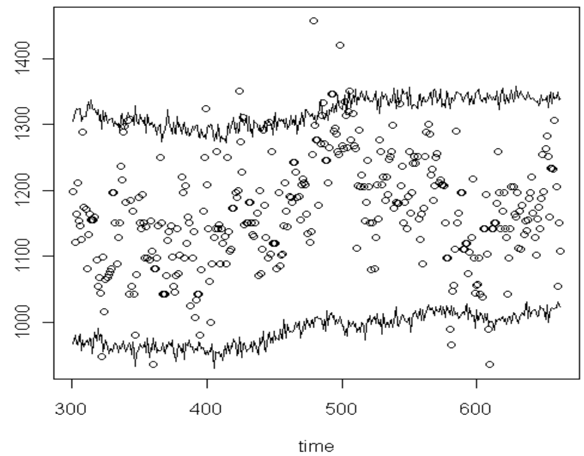


**Figure-3**
**Five Step-Ahead 95% Prediction Intervals**

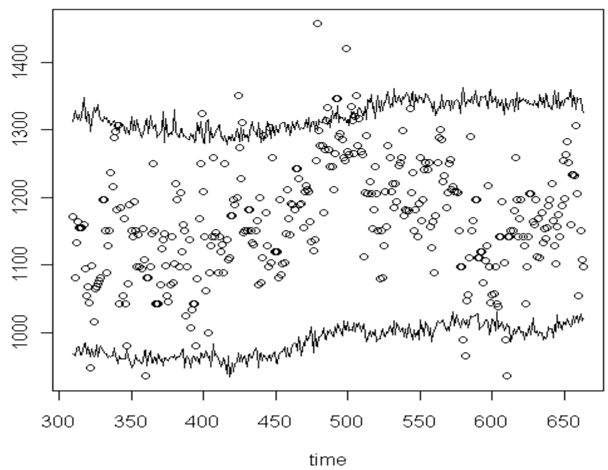


**Figure-4**
**Ten Step-Ahead 95% Prediction Intervals**

Note: (o) is real data point and zig-zag lines show the upper and lower limits of the sieve bootstrap prediction intervals.

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