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# Non-Destructive Evaluation of Stress Field in Billets by Ultrasonic Technique

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## Abstract

*The equations of particle motion in an anisotropic elastic media in the presence of the stress have been derived. These equations are nonlinear and in the form of twenty seven variable of displacement gradients and hence they are not applicable in the practical works. Due to the fact that the particle displacement is composed of two parts, the static part caused by the applied stress and the dynamic part due to the propagating stress wave, to linearize the equations of motion, a Taylor series expansion about the static deformation state is used. Three linearized components of the equations of motion have the form of an eigenvalue problem and its solution gives the wave velocities in the billet in the presence of stress. The analytical results in time delay for one dimensional stress field in a billet are compared with the experimental results and there is a reasonable agreement between them.*

**Keywords:** Non destructive testing, acoustics, stress measurement, ultrasonic investigation.

## Introduction

Design, protecting and confinable maintenance of structure and machine elements demands the highest possible strength per mass ratio, among other parameters. This goal cannot be achieved unless a through knowledge of stresses within the components body is at engineer's disposal. But the way of destructive evaluations of stress in structure or machine elements have been far under question and gradually will be abolished because destructive methods can weaken parts and elements.

There has always been a need for test methods to measure the in-place stresses within the structure and machine elements. Ideally these methods should be nondestructive because destructive methods can weaken parts and elements. Between non destructive stress measuring methods, using ultrasonic waves in acoustoelasticity has been attended by many researchers[1-5]. In this method high frequency sound waves are launched into a test object under specific angles by sender transducer and reflected waves will be received by another transducer. Based on velocity and round trip time of flight through the material, we can obtain useful information about characteristic of the stress field in the body. In this research, the characteristics of stress field is obtained in the machine element with rectangular cross sections such as billets, due to compression, tension, bending or a partial torsion in each arbitrary cross section of the part. In the elastic and isotropic body, the wave propagation equations have been obtained both in tensorial[6] and matrix[7] forms. The equations of wave propagation are presented by Murnagahan[8] for the first time. In this research we used matrix formulation of Green[7]. In the case of infinitesimal deformation, the initial and final coordinates of a material point in the undeformed and deformed states respectively, cannot be interchanged. Therefore, the equations of motion may be derived either in Lagrangian formulation (undeformed coordinates) or in Eulerian formulation (deformed coordinates). Here, we use Lagrangian formulation. The motion equations obtained in this way, are nonlinear partial differential equations which we use a method of perturbation to linearize them.

## Equations of Motion

Taking the point $\xi_j$ , ( j = 1, 2, 3 ) as the Lagrangian coordinate , the motion equations in these coordinates, without body forces, can be obtained as:

$$\sigma_{ij,j} = \rho_0 \ddot{u}_i \quad ; \quad i = 1, 2, \tag{1}$$

The comma (,) denotes partial derivative with respect to $\xi_j$`s, where summation convention on repeated index is intended, and:

$$u_i = x_i - \xi_i \quad ; \quad \ddot{u}_i = \frac{\partial^2 u_i}{\partial t^2} \tag{2}$$

Where $x_i = x_i\ (\xi_1, \xi_2, \xi_3)$ denotes the coordinates in the deformed state.

The stress and infinitesimal strain is defined by Murnagahan as:

$$\sigma_{ij} = (\delta_{ik} + u_{i,k}) \frac{\partial \varphi}{\partial E_{kj}} \quad ; \quad i = 1, 2, 3 \quad ; \quad j = 1, 2, 3 \tag{3}$$

$$E_{kj} = \frac{1}{2}\left[\left(\delta_{kl} + u_{l,k}\right)\left(\delta_{jl} + u_{l,j}\right) - \delta_{kj}\right] \quad ;$$
$$j = 1, 2, 3 \quad ; \quad k = 1, 2, 3 \qquad (4)$$

Murnagahan has shown that the elastic energy density $\Phi$ for an isotropic solid is a function of the three strain invariants $I_1$, $I_2$, $I_3$ of the Lagrangian strains $E_{ij}$, without considering initial strains, could be written as:

$$\Phi = \frac{\lambda + 2\mu}{2} I_1^2 - 2\mu I_2 + \frac{l + 2m}{3} I_1^3 - 2m I_1 I_2 + n I_3 \qquad (5)$$

Where:

$$I_1 = E_{jj} \, ; \; I_2 = \frac{1}{2}(E_{jj}E_{kk} - E_{jk}E_{jk}) \; ; \quad I_3 = \det(E_{jk}) \qquad (6)$$

Expanding $\Phi$ as a function of $E_{kj}$ by Mathematica software and obtaining the nines derivatives $\frac{\partial \Phi}{\partial E_{kj}}$ and replacing them in the equation (3), we obtain $\sigma_{ij}$ , then differentiating it with respect to $\xi_j$ and replacing into equation (1) for i =1 to write the motion equation in the $\xi_1$ direction, we obtain:

$$\rho_0 \ddot{u}_1 = ( 2\mu+\lambda )[ u_{1,1} + u_{1,11}+ u_{3,13} + u_{2,12} + (u_{1,22} +3u_{1,11} + u_{1,33} + u_{3,13} + u_{2,12}) + u_{2,11}u_{2,1} + u_{3,11}u_{3,1} + u_{1,2} (2u_{1,12} +u_{2,22} + u_{3,23} ) + u_{2,2} (u_{1,22} +u_{1,11} + u_{1,33} + u_{2,12} ) + u_{3,12}u_{3,2} + u_{1,3} (2u_{1,13} + u_{2,23} +u_{3,33} ) + u_{2,13}u_{2,3} + u_{3,3} (u_{1,22} +u_{1,11} + u_{1,33} + u_{3,13} )] + \mu [- u_{1,1} ( u_{2,12} + u_{3,13} )+ u_{1,22} + u_{1,33} - u_{3,13}- u_{2,12} + u_{2,1} (2u_{1,12}+u_{2,22} + u_{2,33} ) + u_{3,1} (2u_{1,13}+ u_{3,22} + u_{3,33} ) + u_{1,2} (u_{2,33} + u_{2,11} -u_{3,23} ) + u_{2,2} ( -2u_{1,33} - 2u_{1,11} -u_{2,12} ) + u_{3,2} ( 2u_{1,23} - u_{3,12} ) + u_{1,3} (u_{3,22}+ u_{3,11} - u_{2,23} ) + u_{2,3} ( 2u_{1,23} - u_{2,13} ) + u_{3,3} ( -2u_{1,22} -2u_{1,11} -u_{3,13} )] + 2( 1 + 2m )[(u_{1,1}+u_{2,2}+u_{3,3} )(u_{1,11} +u_{2,12} +u_{3,13} )] + m[u_{1,1} (u_{1,22} + u_{1,33} - 3u_{2,12} - 3u_{3,13} ) + ( u_{2,1} + u_{1,2})( 2u_{1,12} + u_{2,11} + u_{2,22} + u_{3,32} ) + u_{3,1} (2u_{1,13} +u_{3,11} + u_{3,33} + u_{2,23} ) + u_{2,2} (u_{1,22} - 4u_{1,11} + u_{1,33} - 3u_{2,12}- 5u_{3,13} ) + u_{3,2} (u_{3,12} + u_{2,13} ) + u_{1,3} ( u_{3,11} +2u_{1,13} + u_{3,33} + u_{2,23} ) + u_{2,3} (u_{3,12}+ u_{2,13}) + u_{3,3} (u_{1,22}- 4u_{1,11} + u_{1,33} - 3u_{3,13} - 5u_{2,12} )] + \frac{1}{4} n [(u_{1,2} +u_{2,1} )(u_{2,33} - u_{3,23}) + ( u_{3,1} + u_{1,3} )( u_{3,22} - u_{2,23} ) + (u_{3,2} + u_{2,3} )( 2u_{1,23} - u_{3,12}- u_{2,13} ) + 2u_{2,2} ( u_{3,13} - u_{1,33} ) + 2u_{3,3} ( u_{2,12} - u_{1,22} )] \qquad (7)$$

The equations of motion in the other two directions $\xi_2$ and $\xi_3$ may be obtained by a circular permutation on the subscripts 1, 2 and 3 in equation-7. These are three nonlinear partial differential equations with respect to 27 terms $u_{i,j}$ and $u_{i,jk}$ , that $u_{i,kj} = u_{i,jk}$

**Linearization Process:** Considering a low-amplitude plane wave, the components of displacement can be obtained as:

$$u_n = \varepsilon_n \xi_n + m_n A \exp\{i(\omega t - k_j x_j)\} \quad ;$$
$$n = 1, 2, 3 \qquad (8)$$

Where convention of summation is not intended on n, but is intended on j., $x_i$ can be written in terms of the initial coordinates $\xi_n$ as:

$$x_j = (1 + \varepsilon_j)\xi_j \quad ; \quad j = 1, 2, 3 \qquad (9)$$

Where convention of summation is not intended on j. The component of the wave vector $k_j$ may also be written in terms of the wave number k and the wave normal direction cosines $l_j$ as:

$$k_j = \frac{2\pi}{X} l_j = K l_j \quad ; \quad j = 1, 2, 3 \qquad (10)$$

replacing from (9) and (10) into (8), the three displacement components ($u_n$) may be obtained as:

$$u_n = \varepsilon_n \xi_n + m_n e^{iwt}.F \quad , \; n = 1, 2, 3 \qquad (11)$$

Where:

$$F = A \exp\left\{-ik\left[\left(1+\varepsilon_1\right)\xi_1 l_1 + \left(1+\varepsilon_2\right)\xi_2 l_2 + \left(1+\varepsilon_3\right)\xi_3 l_3\right]\right\} \qquad (12)$$

Which: ($\varepsilon_n$) are strains, ($l_n$) are the wave normal directional cosines and ($m_n$) are the directional cosines of the polarisation vector. Equations (11) then represent the final form of the infinitesimal dynamic displacements superimposed upon the finite static displacements:

$$u_j = u_j^s + u_j^d \; ; \; u_j^d << u_j^s ; \quad j = 1, 2, 3 \qquad (13)$$

Having the proper form of the displacement components ready to be replaced into the motion equations-7, the next step is the linearization process. A Taylor series expansion about the static deformation state will be used to accomplish this. The motion equation in the direction of $\xi_1$ is seen from equation-7 to be a function of 27 variables. That is:

$$\rho_0 \ddot{u}_1 = f( u_{1,1} , u_{1,2} , .... , u_{1,11} , .... , u_{3,33} ) \qquad (14)$$

The function f in equation-14 is analytical and may be written as a series of Taylor about the static-deformation-values as:

$$\rho_0 \ddot{u}_1 = f( u_{1,1} , u_{1,2} , .... , u_{1,11} , .... , u_{3,33} )$$
$$= f( u_{1,1}^s, u_{1,2}^s, ...., u_{3,3}^s, u_{1,11}^s, u_{1,12}^s, ...., u_{3,33}^s )$$
$$+ [( d_{11} \frac{\partial}{\partial u_{1,1}} + d_{12} \frac{\partial}{\partial u_{1,2}} + .... + d_{33} \frac{\partial}{\partial u_{3,3}}$$
$$+ d_{111} \frac{\partial}{\partial u_{1,11}} + .... + d_{333} \frac{\partial}{\partial u_{3,33}} ) f ] \Big|_{static} + H.O.T \qquad (15)$$

Where with respect to equation-13 we can write:

$$d_{ij} = u_{i,j} - u_{i,j}^s = u_{i,j}^d \quad ;$$
$$d_{ijk} = u_{i,jk} - u_{i,jk}^s = u_{i,jk}^d \qquad (16)$$

And the notation $\Big|_{static}$ implies the evaluation of the partial derivatives at static values. Equation-15 is then reduced to:

$$\rho_0 \ddot{u}_1 = f(u_{i,jk}, u_{i,j}) = f( u_{i,j}^s, u_{i,jk}^s )$$

$$+ \left[\left( u^{d}_{i,j} \frac{\partial}{\partial u_{i,j}} + u^{d}_{i,jk} \frac{\partial}{\partial u_{i,jk}} \right) f \right]_{static} + H.O.T \qquad (17)$$

Where summation convention is intended on repeated indices. The first expression of equation-17 indicates the static element of the deformation $u_n = \varepsilon_n \xi_n$. Since $u^{s}_{i,jk} = 0$ and the terms containing first-derivatives in (7) are multiplied by the second-derivatives, it can be seen that the first expression of equation (17) equals to zero. That is:

$$f(u^{s}_{i,j}, u^{s}_{i,jk}) = 0$$

The dynamic deformation gradients in equation-17 may be obtain by using equation-11 and substituting them into equation-17 , discarding the quadratic and higher or higher order terms of the strains as being negligibly small and taking $\frac{\omega}{K} = V$ the linearized term of the motion equation is obtained as:

$$m_1 \{ -\rho_0 V^2 + l_1^2 [ \lambda + 2\mu + ( 4\lambda + 10\mu + 4m ) \varepsilon_1 + ( \lambda + 2l ) \theta ] + l_2^2 [ \mu + 2\mu\varepsilon_2 - ( 2\mu + \frac{1}{2} n ) \varepsilon_3 + ( \lambda + 2\mu + m ) \theta ] + l_3^2 [ \mu - ( 2\mu + \frac{1}{2} n ) \varepsilon_2 + 2\mu \varepsilon_3 + ( \lambda + 2\mu + m ) \theta ]\} + m_2 \{ l_1 l_2 [ \lambda + \mu + 2( \lambda + \mu ) ( \varepsilon_1 + \varepsilon_2 ) + ( \frac{1}{2} n - 2m ) \varepsilon_3 + ( 2l + m ) \theta ]\} + m_3 \{ l_1 l_3 [ \lambda + \mu + 2 ( \lambda + \mu )( \varepsilon_1 + \varepsilon_3 ) + ( \frac{1}{2} n - 2m ) \varepsilon_2 + ( 2l + m ) \theta ] \} = 0 \qquad (18)$$

Where: $\theta = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$

Also the other terms of linearized motion equations may be written by a permutation of 1, 2 and 3 in subscripts of equation-18. These three equations are the most general form of the equations of motion.

**Evaluation of Stress-Strain Field in the Billet**: Taking billet as a one-dimensional body, previous equations can be simpler. Assuming one dimensional stress in billet and using Poisson's ratio, strains can be obtained as: figure-1.

$$\varepsilon_1 = \varepsilon \quad ; \quad \varepsilon_2 = \varepsilon_3 = -\upsilon \varepsilon \qquad (19)$$

In this case, waves propagate in (1 – 3) plane only and the cosines of direction of the $l_i$ have values as:

$$l_2 = 0 \quad ; \quad l_1^2 + l_3^2 = 1 \qquad (20)$$

Using above equations, the equations of motion in 1, 2 and 3 directions can be obtained for one dimensional stress in the billet. Rearranging these equations as an eigenvalue problem:

$$\begin{bmatrix} \lambda_{11} - \rho_0 V^2 & 0 & \lambda_{13} \\ 0 & \lambda_{22} - \rho_0 V^2 & 0 \\ \lambda_{31} & 0 & \lambda_{33} - \rho_0 V^2 \end{bmatrix} \begin{Bmatrix} m_1 \\ m_2 \\ m_3 \end{Bmatrix} = 0 \qquad (21)$$

This must hold true for any vector $\begin{Bmatrix} m_1 \\ m_2 \\ m_3 \end{Bmatrix}$ and $\lambda_{ij}$ can be written as:

$\lambda_{11} = l_1^2 [ \lambda + 2\mu + ( 5\lambda + 10\mu + 4m + 2l) \varepsilon - \upsilon ( 2\lambda + 4l ) \varepsilon ] + l_3^2 [ \mu + ( \lambda + 2\mu + m ) \varepsilon - \upsilon ( 2\lambda + 4\mu + 2m - \frac{1}{2} n ) \varepsilon ]$

$\lambda_{13} = \lambda_{31} = l_1 l_3 [\lambda + \mu + (2\lambda + 2\mu + 2l + m) \varepsilon - \upsilon (2\lambda + 2\mu + 4l + \frac{1}{2} n ) \varepsilon ]$

$\lambda_{22} = l_1^2 [ \mu + ( 4\mu + \lambda + m ) \varepsilon - \upsilon ( 2\lambda + 2\mu + 2m - \frac{1}{2} n ) \varepsilon ] + l_3^2 [ \mu + ( \lambda + m - \frac{1}{2} n ) \varepsilon - \upsilon ( 2\lambda + 6\mu + 2m ) \varepsilon ]$

$\lambda_{33} = l_1^2 [ \mu + ( 4\mu + \lambda + m ) \varepsilon - \upsilon ( 2\lambda + 2\mu + 2m - \frac{1}{2} n ) \varepsilon ] + l_3^2 [ \lambda + 2\mu + ( \lambda + 2l ) \varepsilon - \upsilon ( 6\lambda + 10\mu + 4m + 4l ) \varepsilon ]$ (22)

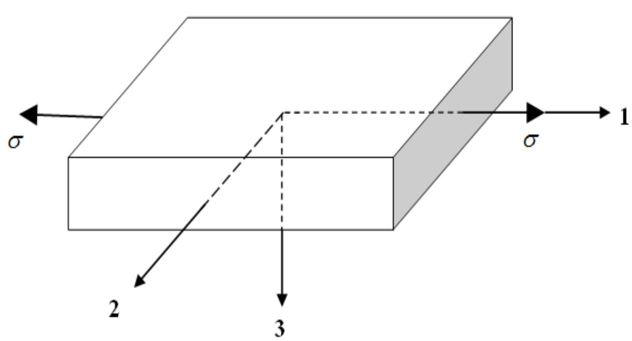


**Figure-1**
**Coordinate System**

The eigenvalues in equation-21 yield a pure shear horizontal (SH) wave mode, a quasi longitudinal mode and a quasi-shear-vertical mode respectively:

$$\rho_0 V_1^2 = \frac{1}{2}\left[ \lambda_{11} + \lambda_{33} + \sqrt{(\lambda_{11} - \lambda_{33})^2 + 4\lambda_{13}^2} \right] \qquad (23a)$$

$$\rho_0 V_2^2 = \lambda_{22} \qquad (23b)$$

$$\rho_0 V_3^2 = \frac{1}{2}\left[ \lambda_{11} + \lambda_{33} - \sqrt{(\lambda_{11} - \lambda_{33})^2 + 4\lambda_{13}^2} \right] \qquad (23c)$$

In figure-2 the launch angle φ measured from the 3-axis and the transducer-fixture rotation angle γ measured from 1-axis. The angles φ and α are related through the Snell's law:

$$\frac{\sin\phi}{V_W} = \frac{\sin\alpha}{V} \tag{24}$$

Where $V_W$ denotes the wave velocity in water and V is the velocity in the solid.

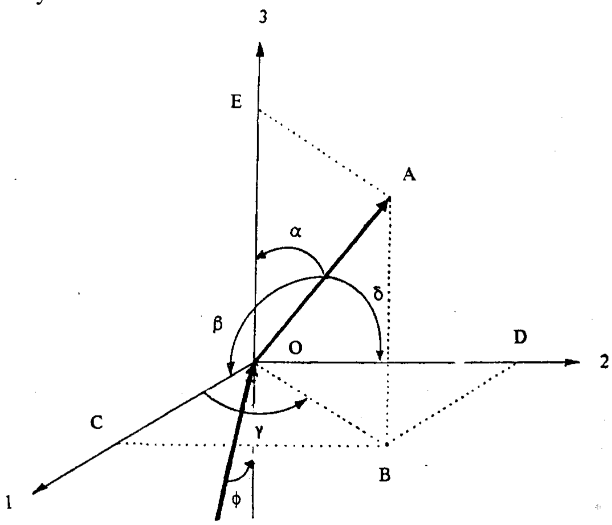


**Figure-2**
**The various angles defining the incident and the refracted wave path**

A careful inspection of the eigenvalue problem as represented by equation-21 with $\lambda_{ij}$ given in equations-22, along with the Snell's law given in equation-24 reveals the fact that the wave velocities are dependent on the direction cosines $l_1$, $l_2$ and $l_3$ of the wave normal and vice versa. In the other words, to determine the wave velocities one needs to know the direction cosines $l_1$, $l_2$ and $l_3$ of the wave vector. However, to obtain $l_1$, $l_2$ and $l_3$, the wave velocities are required to be known a priori. This double-headed difficulty can be solved by incorporating the Snell's law into the elements of the eigenvalue problem. The direction cosines of the wave normal may be formed as follows:

$$l_1 = \cos\beta = \sin\alpha.\cos\gamma = \frac{\sin\phi}{V_W} V.\cos\gamma \tag{25a}$$

$$l_3 = \cos\alpha = \sqrt{(1-\sin^2\alpha)} \quad = (1 - \frac{\sin^2\phi}{V_W^2} V^2)^{1/2} \tag{25b}$$

$$l_2 = \cos\delta = \sqrt{(1-l_1^2 - l_3^2)} = \frac{\sin\phi}{V_W} V.\sin\gamma \tag{25c}$$

The direction cosines of the wave vector are now in terms of the angles φ and γ may virtually vary from -90° to +90° and 0 to 360°, respectively. However, due to the geometric and loading symmetry, φ needs to be varied from 0 to 90° and γ from 0 to 180°. Equations (25 a) to (25 c) explicitly show the dependency of the velocity of wave on the wave path. Because of this, the elements $\lambda_{ij}$ of the eigenvalue problem (21) must undergo some changes to incorporate the Snell's law as explained above.

Substituting from equations (25 a) to (25 c) into all of the equations represented by (22), one obtains:

$$\lambda_{11} = a_{11} + f_{11}V^2$$

$$\lambda_{13} = f_{13} V \sqrt{1 - \frac{\sin^2\phi}{V_W^2} V^2} \tag{26}$$

$$\lambda_{22} = a_{22} + f_{22}V^2$$

$$\lambda_{33} = a_{33} + f_{33}V^2$$

Where: $a_{11} = \mu + (\lambda + 2\mu + m)\varepsilon - \upsilon(2\lambda + 4\mu + 2m - \frac{n}{2})\varepsilon$

$$a_{22} = \mu + (\lambda + m - \frac{n}{2})\varepsilon - \upsilon(2\lambda + 6\mu + 2m)\varepsilon \tag{27}$$

$$a_{33} = \lambda + 2\mu + (\lambda + 2l)\varepsilon - \upsilon(6\lambda + 10\mu + 4m + 4l)\varepsilon$$

$$f_{11} = \{[\lambda + 2\mu + (5\lambda + 10\mu + 4m + 2l)\varepsilon - \upsilon(2\lambda + 4l)\varepsilon]\cos^2$$

$$\gamma - [\mu + (\lambda + 2\mu + m)\varepsilon - \upsilon(2\lambda + 4\mu + 2m - \frac{n}{2})\varepsilon]\} \frac{\sin^2\phi}{V_W^2}$$

$$f_{13} = [\lambda + \mu + (2\lambda + 2\mu + 2l + m)\varepsilon - \upsilon(2\lambda + 2\mu + 4l + \frac{n}{2})\varepsilon]$$

$$\frac{\sin\phi}{V_W} \cos\gamma$$

$$f_{22} = \{[\mu + (4\mu + \lambda + m)\varepsilon - \upsilon(2\lambda + 2\mu + 2m - \frac{n}{2})\varepsilon]\cos^2\gamma$$

$$+[\mu + (\lambda + m - \frac{n}{2})\varepsilon - \upsilon(2\lambda + 6\mu + 2m)\varepsilon]\} \frac{\sin^2\phi}{V_W^2}$$

$$f_{33} = \{[\mu + (4\mu + \lambda + m)\varepsilon - \upsilon(2\lambda + 2\mu + 2m - \frac{n}{2})\varepsilon]\cos^2\gamma$$

$$+[\lambda + 2\mu + (\lambda + 2l)\varepsilon - \upsilon(6\lambda + 10\mu + 4m + 4l)\varepsilon]\} \frac{\sin^2\phi}{V_W^2}$$

Having the λij defined as in the succession of equations-26 to 27, the eigenvalue problem-21 may now be rearranged to:

$$\begin{bmatrix} a_{11} + (f_{11} - \rho)V^2 & 0 & f_{13}V\sqrt{1 - \frac{\sin^2\phi}{V_W^2}V^2} \\ 0 & a_{22} + (f_{22} - \rho)V^2 & 0 \\ f_{13}V\sqrt{1 - \frac{\sin^2\phi}{V_W^2}V^2} & 0 & a_{33} + (f_{33} - \rho)V^2 \end{bmatrix} \begin{Bmatrix} m_1 \\ m_2 \\ m_3 \end{Bmatrix} = 0 \tag{28}$$

The determinant of this symmetric matrix, with all of the elements containing the wave velocity V, must vanish for any vector $m_i$ (i = 1, 2, 3) to obtain a non trivial solution. Performing the operations, grouping the appropriate terms and simplifying, the following cubic characteristic equation in $V^2$ is obtained:

$$V^6 + a_1 V^4 + a_2 V^2 + a_3 = 0 \tag{29}$$

Where: $a_1 = \frac{b_1}{b_4}, a_2 = \frac{b_2}{b_4}, a_3 = \frac{b_3}{b_4}$

$$b_1 = a_{11}(f_{22}-\rho)(f_{33}-\rho) + a_{22}(f_{11}-\rho) + a_{33}(f_{11}-\rho)$$
$$(f_{22}-\rho) - f_{13}^2(f_{22}-\rho) + a_{22} f_{13}^2 \frac{\sin^2\phi}{V_W^2}$$

$$b_2 = a_{11}a_{22}(f_{33}-\rho) + a_{11}a_{33}(f_{22}-\rho) + a_{22}a_{33}$$
$$(f_{11}-\rho) - a_{22} f_{13}^2$$

$$b_3 = a_{11}a_{22}a_{33} \tag{30}$$

$$b_4 = (f_{11}-\rho)(f_{22}-\rho)(f_{33}-\rho) + (f_{22}-\rho) f_{13}^2 \frac{\sin^2\phi}{V_W^2}$$

The solutions to equation (29) are as follows:

$$V_1^2 = 2\sqrt{-Q}\cos\frac{\theta}{3} - \frac{1}{3}a_1 \tag{31a}$$

$$V_2^2 = 2\sqrt{-Q}\cos(\frac{\theta+2\pi}{3}) - \frac{1}{3}a_1 \tag{31b}$$

$$V_3^2 = 2\sqrt{-Q}\cos(\frac{\theta+4\pi}{3}) - \frac{1}{3}a_1 \tag{31c}$$

Where: $Q = \frac{1}{9}(3a_2 - a_1^2)$

$$R = \frac{1}{54}(9a_1a_2 - 27a_3 - 2a_1^3) \tag{32}$$

$$\theta = \cos^{-1}\frac{R}{\sqrt{-Q^3}}$$

It is important to note that wave velocities in equations (23) and (31) are different. The latter, given in equations (31), incorporate the Snell's law, while the former, represented by equations (23) do not. Changes in QSV velocity as a function of strain for different launch angels in accordance with equation (31c) have shown in figure-3 for aluminum. Where φ is launch angle and ε is strain in 1 direction. With respect to relation between stress and strain as:

$$\sigma_1 = E\varepsilon = \frac{\mu(3\lambda+2\mu)}{\lambda+2\mu}\varepsilon \tag{33}$$

Now one dimensional stress in every section of billet can be obtained.

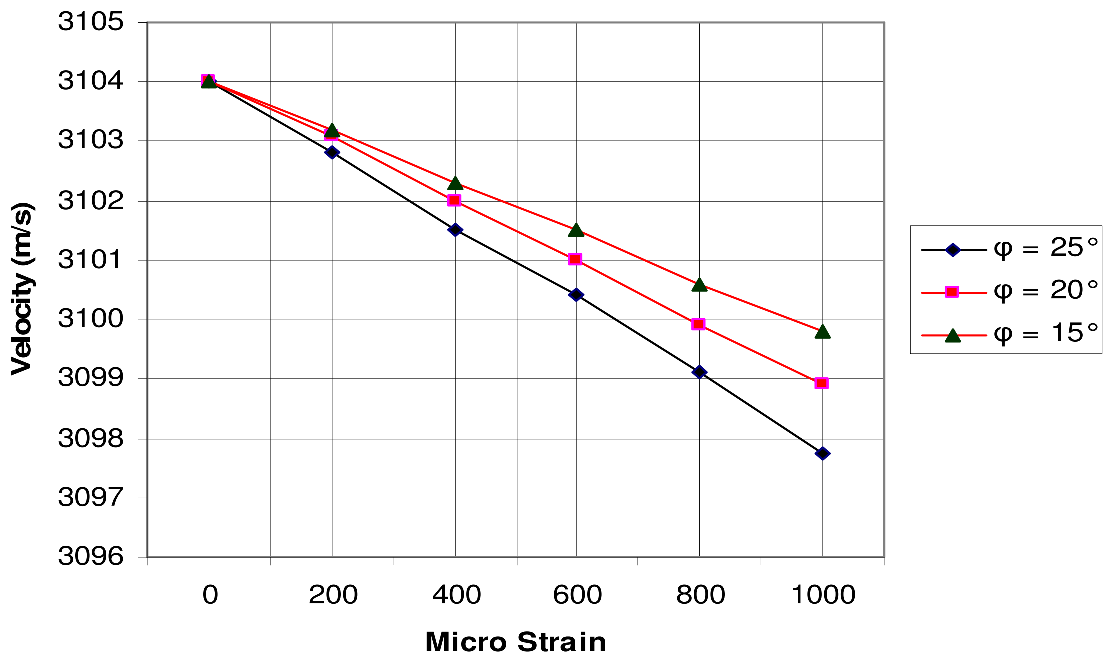


**Figure-3**
**Changes in QSV velocity as a function of strain**

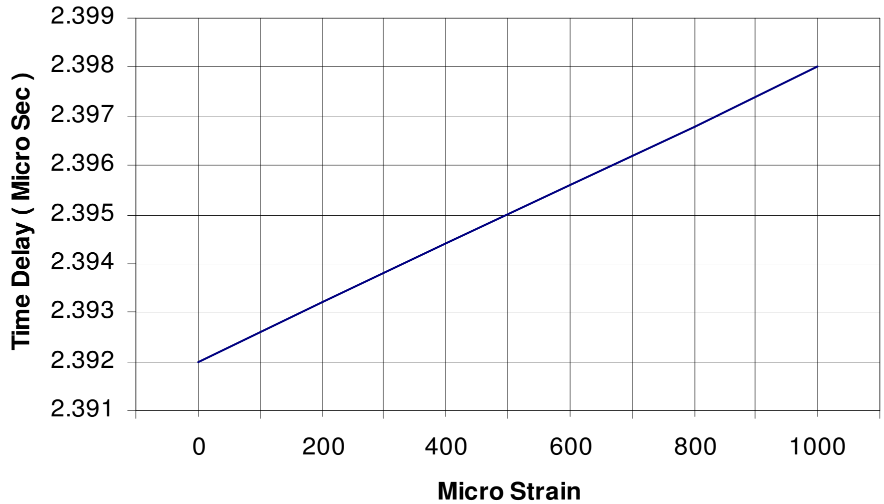


**Figure-4**
**Time Delay obtained from theoretical results**

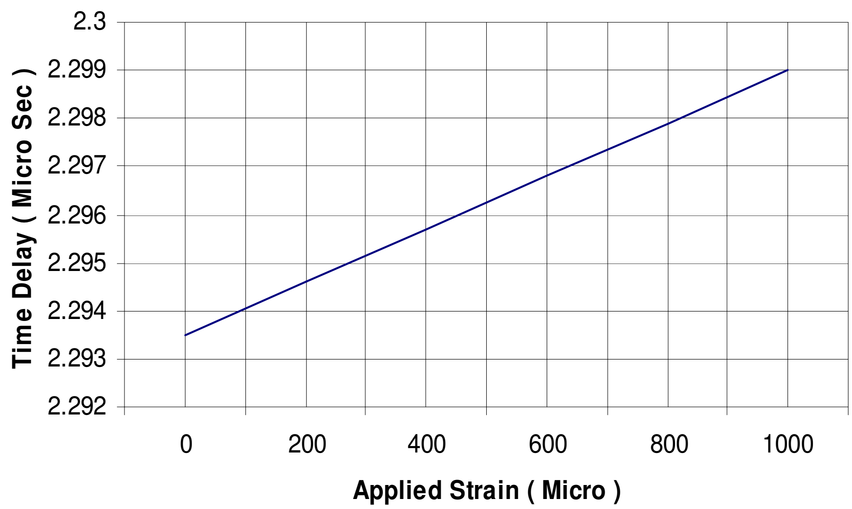


**Figure-5**
**Time Delay obtained from experimental works**

**Calculating Time Delay:** Since in an ultrasonic technique the quantity which is experimentally measured is time of flight and not the velocity of propagation, Egli and Koshti[9] defined a quantity called "time delay" as:

$$\Delta t = \frac{2hCos\alpha}{V} \tag{34}$$

Figure-4 shows the time delay $\Delta t$ as a function of strain for the QSV mode propagating in (1-3) plane and figure-5 shows the corresponding variation in time delay borrowed from the experimental works of Egli and Koshti[9]. Both figures are for aluminum specimens, the variations of time delay as a function of strains are the same in both figures, which represent a good agreement between analytical results obtained in this paper and the mentioned experimental results.

An aluminum property which is used in analytical result has shown in table-1.

**Table-1**
**Mechanical properties for aluminum**

| | Density ($kg/m^3$) | Elastic constants $*10^{10}$ (Pa) | | | | | Elasticity Module $*10^{10}$ (Pa) | Poisson's Ratio |
|---|---|---|---|---|---|---|---|---|
| | ρ | λ | μ | l | m | n | E | υ |
| Aluminum | 2730 | 5.93 | 2.65 | -31.1 | -40.1 | -40.8 | 7.1 | 0.34 |

## Conclusion

At first step the motion equations are obtained as function of deformation gradients $u_{i,j}$, $u_{i,jk}$ which are nonlinear. The nonlinear equations of motion are linearized by means of a method of perturbation. So that the final linearized results, as equation-18, comprise such variables that can be measured experimentally. These variables contain the wave velocity (V), the principal strains ($\varepsilon_1$ , $\varepsilon_2$ , $\varepsilon_3$ ), the material properties ( $\lambda$, $\mu$, l, m, n ) and the characteristic of the wave vector ( $l_1$ , $l_2$ , $l_3$ , $m_1$ , $m_2$ , $m_3$ ) and.

Considering one dimensional stress in the billet, Changes in wave propagating velocity as a function of strain for an aluminum specimen have shown in figure-3. Also the time delay as a function of strain is calculated from equation-34 and compared with experimental results obtained by Egli and Koshti[9] which show a good agreement between them.

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