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Homotopy Type Methods for Numerical Solution of Non Linear Riccati Equation

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Abstract

In this paper, we apply various homotopy type methods for the approximate solution of nonlinear Riccati differential equations, such as Optimal Homotopy Asymptotic Method (OHAM), Homotopy Perturbation Method (HPM) andHomotopy Analysis Method (HAM). We also comparethe results of each of them with the exact solution of the given problem. Conclusions reveal that the method OHAM is more effective and the results obtained by this method are in good agreement with the exact solution.

Keywords: Nonlinear Riccati differential equations; OHAM; HPM; HAM.

Introduction

A nonlinear differential equation which is useful in the dynamics of a system is given below and which is called Riccati equation

$$\frac{dv}{dz}-a(z)-b(z)v-c(z)v^2=0 \qquad (1)$$

Where $v(z)$ is unknown function and $a(z)\neq 0$, $c(z)\neq 0$. This equation is named after the Italian mathematician, Riccati (1676-1754), the recent applications of the given equation involve such type of areas as financial mathematics[1,2]. The solution of this equation is found using the classical numerical methods such as the forward Euler and Runge-Kutta techniques. A scheme which was presented by Saidiet al[3] was stable. The Riccati equation solution is independent of the solution obtained by the techniques that produce solutions to linear differential equations. The analysis is less efficient as well. Moreover because of similarity of the Riccati equation to the equation analyzed in chaos theory, there is further investigation that involves the concept of chaos theory but there exists simple analysis of Riccati equation. If one solution of Riccati equation is known then a whole family of solutions can be found. For solving this equation no analytical method exists. A method for solving this equation numerically is discretization of it in time domain. Such type of equations can be solved by other methods e.g. Adomian Decomposition Method[4--7] and Homotopy Perturbation Method[8-11], but we use OHAM, HPM and HAM for the solution of Riccati nonlinear differential equations, results are compared with their exact solutions. The OHAM results are reliable and show excellent agreement with the exact solutions.

Basic Idea of Oham: We consider the following differential equation.

$$S(v(z))+f(z)+E(v(z))=0,\ D\left(v,\frac{dv}{dz}\right)=0 \qquad (2)$$

Where S is a linear operator, $v(z)$ is an unknown function. $f(z)$ is a known function, E is a nonlinear operator and D is a boundary operator. According to OHAM, a homotopy is constructed as
$Y(\theta(z,p),p):R\times[0,1]\rightarrow R$ that satisfies

$$(1-p)\begin{bmatrix} S(\theta(z,p)) \\ +f(z)\end{bmatrix}=h(p)[S(\theta(z,p)) + f(z)+E(\theta(z,p))], \qquad (3)$$
$$D\left(\theta(z,p),\frac{\partial\theta(z,p)}{\partial z}\right)=0$$

Where $z\in R$ and $p\in[0,1]$, $h(p)$ is a non zero auxiliary function for $p\neq 0$, $\mathrm{h}(0)=0$ and $\theta(z,p)$ is an unknown function. Obviously, when $p=0$ and $p=1$ it holds that $\theta(z,0)=v_0(z)$ and $\theta(z,1)=v(z)$. As p varies from 0 to 1, the solution $\theta(z,p)$ approaches from $v_0(z)$ to $v(z)$ where $v_0(z)$ is obtained from equation (3) for $p=0$ and we have

$$S(v_0(z))+f(z)=0,\ D\left(v_0,\frac{dv_0}{dz}\right)=0 \qquad (4)$$

$$h(p) = pC_1 + p^2 C_2 + \cdots \quad (5)$$

Where $C_1, C_2, \ldots$ are constants to be determined. By using least square method. $h(p)$ can be expressed in many forms as suggested by V. Marinca et al[12--16]. For getting an approximate solution, we expand $\theta(z, p, C_i)$ in Taylor's series about p in the following pattern

$$\theta(z, p, C_i) = v_0(z) + \sum_{k=1}^{\infty} v_k(z, C_1, C_2, \cdots, C_k) p^k \quad (6)$$

Using equation (6) into equation (3) and equating the coefficient of like powers of p we obtain the following linear equations. Zeroth order problem is given by equation (4) and the first order problem is given by equation (7)

$$L(v_1(z)) + g(z) = C_1 N_0(v_0(z)), \quad B\left(v_1, \frac{dv_1}{dz}\right) = 0 \quad (7)$$

The general equations for $v_k(z)$ are given by

$$L(v_k(z)) - L(v_{k-1}(z)) = C_k N_0(v_0(z)) + \sum_{i=1}^{k-1} C_i \begin{bmatrix} L(v_{k-i}(z)) \\ +N_{k-i}(v_0(z), v_1(z), \ldots, v_{k-1}(z)) \end{bmatrix} \quad (8)$$

$$k = 2,3,\ldots, B\left(v_k, \frac{dv_k}{dz}\right) = 0$$

Where $N_m(v_0(z), v_1(z), \ldots, v_m(z))$ is the coefficient of p^m in the expansion of $N(\theta(z, p))$ about the embedding parameter p.

$$N(\theta(z, p, C_i)) = N_0(v_0(z)) + \sum_{m=1}^{\infty} N_m(v_0, v_1 \cdots, v_m) p^m \quad (9)$$

It suggests that the convergence of the series (6) depends upon the constants $C_1, C_2, \cdots$ If it is convergent at $p = 1$, one has

$$\theta(z, C_i) = v_0(z) + \sum_{k=1}^{\infty} v_k(z, C_1, \cdots, C_k) \quad (10)$$

The result of themth order approximations are given

$$\tilde{v}(z, C_1, C_2, \ldots, C_m) = v_0(z) + \sum_{i=1}^{m} v_i(z, C_1, C_2, \ldots, C_i) \quad (11)$$

Using equation (11) into equation (2), we have the following residual:

$$X(z, C_1, C_2 \cdots, C_m) = S(\tilde{v}(x, C_1, C_2 \cdots, C_m)) + f(z) + E(\tilde{v}(z, C_1, C_2 \cdots, C_m)) \quad (12)$$

If $X = 0$ is taken, then $\tilde{v}$ will be the exact solution. In order to find the values of C_i, $i = 1, 2, 3, \cdots$ We first construct the result

$$T(C_1, C_2 \cdots, C_m) = \int_c^d X^2(z, C_1, C_2 \cdots, C_m) dz \quad (13)$$

and then minimizing it, we have

$$\frac{\partial T}{\partial C_1} = 0, \frac{\partial T}{\partial C_2} = 0, \cdots, \frac{\partial T}{\partial C_m} = 0 \quad (14)$$

With these constants known, the approximate solution (of order m) is found.

Numerical Problems: Problem 1: Considering the following first order differential equation on domain[0,1]

$$\frac{dv}{dz} - v^2(z) + 1 = 0, v(0) = 0 \quad (15)$$

The exact solution of the problem is

$$v(z) = -Tanh\, z \quad (16)$$

Applying the technique,OHAM theZeroth order problem is

$$v_0'(z) + 1 = 0, v_0(0) = 0 \quad (17)$$

$$v_0(z) = -z \quad (18)$$

First order problem is

$$v_1'(z, C_1) = 1 + C_1 - C_1 v_0^2(z) + (1 + C_1) v_0'(z), \quad v_1(0) = 0 \quad (19)$$

$$v_1(z, C_1) = \frac{-1}{3} z^3 C_1 \quad (20)$$

Second order problem is

$$v_2'(z, C_1) = -2C_1 v_0(z) v_1(z) + (1 + C_1) v_1'(z), \quad v_2(0) = 0 \quad (21)$$

$$v_2(z, C_1) = \frac{1}{15}\left(-5z^3 C_1 - 5z^3 C_1^2 - 2z^5 C_1^2\right) \quad (22)$$

Third order problem is

$$v_3'(z, C_1) = -C_1 v_1^2 - 2C_1 v_0(z) v_2(z) + (1 + C_1) v_2'(z), \quad v_3(0) = 0 \quad (23)$$

$$v_3(z, C_1) = \frac{1}{315}\begin{pmatrix} -105z^3 C_1 - 210z^3 C_1^2 - 84z^5 C_1^2 \\ -105z^3 C_1^3 - 84z^5 C_1^3 - 17z^7 C_1^3 \end{pmatrix} \quad (24)$$

Fourth order problem is

$$v_4'(z, C_1) = -2C_1 v_1(z) v_2(z) - 2C_1 v_0(z) v_3(z) + (1 + C_1) v_3'(z), v_4(0) = 0 \quad (25)$$

$$v_4(z, C_1) = \frac{1}{2835}\begin{pmatrix} -945z^3 C_1 - 2835z^3 C_1^2 - 1134z^5 C_1^2 \\ -2835z^3 C_1^3 - 2268z^5 C_1^3 - 459z^7 C_1^3 - 945z^3 C_1^4 \\ -1134z^5 C_1^4 - 459z^7 C_1^4 - 62z^9 C_1^4 \end{pmatrix} \quad (26)$$

Fifth order problem is

$$v_5'(z,C_1) = -C_1 v_2^{2} - 2C_1 v_1(z) v_3(z) - 2C_1 v_0(z) v_4(z) + (1+C_1) v_4'(z), v_5(0) = 0 \tag{27}$$

$$v_5(z,C_1) = \frac{1}{155925}\begin{pmatrix} -51975z^3C_1 - 207900z^3C_1^2 - 83160z^5C_1^2 \\ -311850z^3C_1^3 - 249480z^5C_1^3 - 50490z^7C_1^3 \\ -207900z^3C_1^4 - 249480z^5C_1^4 - 100980z^7C_1^4 \\ -13640z^9C_1^4 - 51975z^3C_1^5 - 83160z^5C_1^5 \\ -50490z^7C_1^5 - 13640z^9C_1^5 - 1382z^{11}C_1^5 \end{pmatrix} \tag{28}$$

Using equations (18), (20), (22), (24), (26) and (28) the fifth order approximate solution for $p = 1$ *is*

$$\tilde{v}(z,C_1) = v_0(z) + v_1(z,C_1) + v_2(z,C_1) \quad v_3(z,C_1) + v_4(z,C_1) + v_5(z,C_1) \tag{29}$$

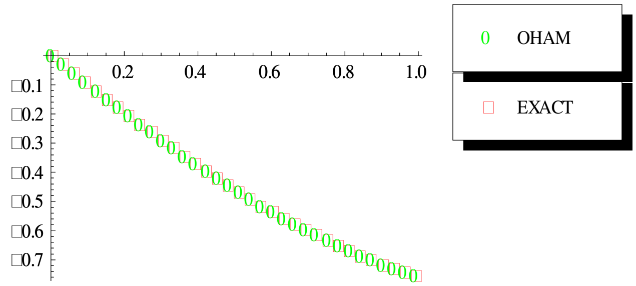
Following the technique, OHAM 1 on the domain $[0,1]$ we get

$$X = \tilde{v}' - \tilde{v}^2 + 1 \tag{30}$$

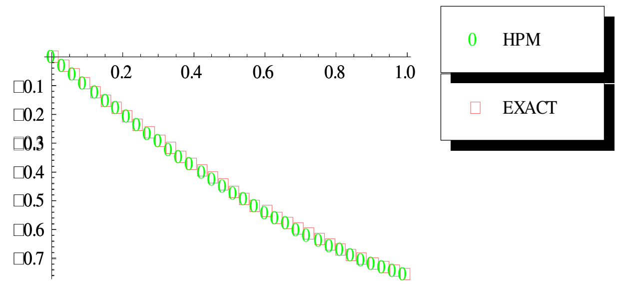
The value of $C_1 = -0.7736625644632285$ is obtained, and our approximate solution is

$$\tilde{v}(z) = -z + 0.333135z^3 - 0.131901z^5 + 0.0496428z^7 \quad 0.0149286z^9 + 0.00245669z^{11} + O(z^{12}) \tag{31}$$

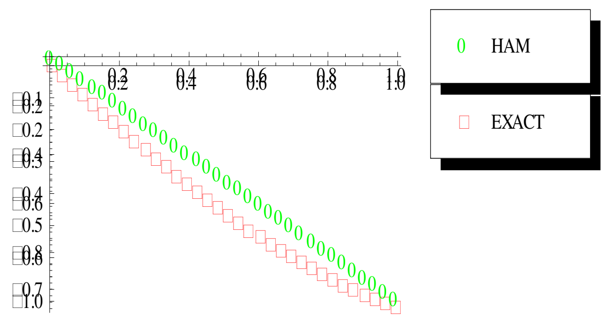
The following table-1 exhibits values of the variablez exact solution, OHAM solution, HPM Solution, HAM solution and their absolute errors. Also we investigate that there exists relation between their values. From the figure-1 we conclude that the two graphs1 (a) OHAM and exact1 (b) HPM and exact are coincident but in 1(c) HAM and exact are not coincident. Red curves of each graph show exact solutions while the green curves of each graph exhibit series solutions however, in general the method OHAM is more effective than the others.



(a) OHAM and exact solutions graph



(b) HPM and exact solutions graph



(c) HAM and exact solutions graph

Figure-1

The variable Z is represented along the horizontal line and the function v(z) is represented along the vertical line

Table-1

Values of the variablez exact solution, OHAM solution, HPM Solution, HAM solution and their absolute errors

Variable z	Exact Solution	OHAM Solution	HPM Solution	HAM Solution	Error OHAM	Error HPM	Error HAM
0	0.0000	0.0000	0.0000	0.0000	0.0×10^{0}	0.0×10^{0}	0.0×10^{0}
0.1	-0.099668	- 0.0996682	- 0.099668	-0.0996882	1.8×10^{-7}	3.6×10^{-16}	1.8×10^{-7}
0.2	-0.197375	- 0.197376	- 0.197375	-0.197376	1.2×10^{-6}	2.9×10^{-12}	1.2×10^{-6}
0.3	-0.291313	- 0.291315	- 0.291313	-0.291315	2.7×10^{-6}	5.5×10^{-10}	2.7×10^{-6}
0.4	-0.379979	- 0.379952	- 0.379949	-0.379952	3.5×10^{-6}	2.3×10^{-8}	3.5×10^{-6}
0.5	-0.462117	- 0.46212	- 0.462117	-0.46212	2.9×10^{-6}	4.0×10^{-7}	2.9×10^{-6}
0.6	-0.53705	- 0.537051	- 0.537045	-0.537051	1.6×10^{-6}	4.1×10^{-6}	1.6×10^{-6}
0.7	-0.604368	- 0.604369	- 0.604339	-0.604369	8.7×10^{-7}	2.9×10^{-5}	8.7×10^{-7}
0.8	-0.664037	- 0.664038	-0 .66388	-0.664038	9.2×10^{-7}	1.5×10^{-4}	9.2×10^{-7}
0.9	-0.716298	- 0.716299	- 0.71561	-0.716299	1.1×10^{-6}	6.8×10^{-4}	1.1×10^{-6}
1.0	-0.761594	- 0.761594	- 0.759038	-0.761594	1.8×10^{-7}	2.6×10^{-7}	1.8×10^{-7}

Problem 2: Considering the following nonlinear differential equation on domain $[0,1]$

$$\frac{dv}{dz}-10-3v+v^2=0, v(0)=0 \quad (32)$$

The exact solution is

$$v(z)=-2+\frac{14e^{7z}}{5+2e^{7z}} \quad (33)$$

We apply the technique, OHAM Zeroth order is

$$v_0'(z)=10, v_0(0)=0 \quad (34)$$

$$v_0(z)=10z \quad (35)$$

First order problem is

$$v_1'(z,C_1)=-10-10C_1-3C_1v_0(z)$$
$$+C_1v_0^2(z)+(1+C_1)v_0'(z), v_1(0)=0 \quad (36)$$

$$v_1(z,C_1)=\frac{5C_1}{3}\left(-9z^2+20z^3\right) \quad (37)$$

Second order problem is

$$v_2'(z,C_1)=-3C_1v_1(z)+2C_1v_0(z)v_1(z)$$
$$+(1+C_1)v_1'(z),\ v_2(0)=0 \quad (38)$$

$$v_2(z,C_1)=\frac{5}{3}\begin{pmatrix} C_1[-9z^2+20z^3] \\ +C_1^2[-9z^2+29z^3-60z^4+80z^5] \end{pmatrix} \quad (39)$$

Third order problem is

$$v_3'(z,C_1)=C_1v_1^2(z)-3C_1v_2(z)$$
$$+2C_1v_0(z)v_2(z)+(1+C_1)v_2'(z), v_3(0)=0 \quad (40)$$

$$v_3(z,C_1)=\frac{5C_1}{252}\begin{pmatrix} -756z^2+1680z^3 \\ +C_1\begin{bmatrix}-1512z^2+4872z^3 \\ -10080z^4+13440z^5\end{bmatrix} \\ +C_1^2\begin{bmatrix}-756z^2+3192z^3 10647z^4 \\ +21756z^5-28560z^6+27200z^7\end{bmatrix} \end{pmatrix} \quad (41)$$

Fourth order problem is

$$v_4'(z,C_1)=2v_1(z)v_2(z)-3C_1v_3(z)$$
$$+2C_1v_0(z)v_3(z)+(1+C_1)v_3'(z), v_4(0)=0 \quad (42)$$

$$v_4(z,C_1)=$$
$$\frac{C_1}{2268}\begin{pmatrix} -34020z^2+75600z^3+C_1[-102060z^2 \\ +328860z^3-680400z^4+907200z^5] \\ +C_1^2\begin{bmatrix}-102060z^2+430920z^3 \\ -1437345z^4+2937060z^5 \\ -3855600z^6+3672000z^7\end{bmatrix} \\ +C_1^3\begin{bmatrix}177660z^3-756945z^4 \\ +2045169z^5-4297860z^6 \\ +6588000z^7-6696000z^8+4960000z^9\end{bmatrix} \\ +O(z^{10}) \end{pmatrix} \quad (43)$$

Fifth order problem is

$$v_5'(z,C_1)=C_1v_2^2(z)+2C_1v_1(z)v_3(z)-3C_1v_4(z)$$
$$+2C_1v_0(z)v_4(z)+(1+C_1)v_4'(z), v_5(0)=0 \quad (44)$$

$$v_5(z,C_1)=\frac{1}{49896}\begin{pmatrix} C_1[-748440z^2+1663200z^3] \\ +C_1^2[-2993760z^2+9646560z^3 \\ -19958400z^4+26611200z^5] \\ +C_1^3[-4490640z^2+18960480z^3 \\ -63243180z^4+129230640z^5 \\ -169646400z^6+161568000z^7] \\ +C_1^4[-2993760z^2+15634080z^3 \\ -66611160z^4+179974872z^5- \\ 378211680z^6+579744000z^7 \\ -589248000z^8+436480000z^9]+ \\ C_1^5[-748440z^2+4656960z^3 \\ -23326380z^4+77355432z^5 \\ -208733679z^6+427317660z^7 \\ -691891200z^8+860992000z^9- \\ 729696000z^{10}+442240000z^{11}] \end{pmatrix} \quad (45)$$

Utilizing equations (35), (37), (39), (41), (43) and (45) we find fifth order approximate series solution for $p=1$ is

$$\tilde{v}(z,C_1)=v_0(z)+v_1(z,C_1)+v_2(z,C_1)$$
$$+v_3(z,C_1)+v_4(z,C_1)+v_5(z,C_1) \quad (46)$$

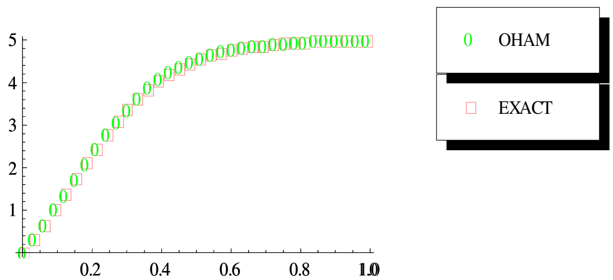
Following the technique, OHAM on domain $[0,1]$ we get the residual

$$X=\tilde{v}'-10-3\tilde{v}+\tilde{v}^2 \quad (47)$$

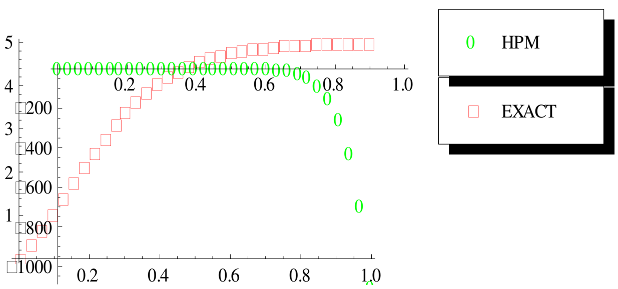
The value of $C_1=-0.3641329904497182$ is obtained. The approximate solution is

$$\tilde{v}(z)=10z+13.4407z^2-20.8922z^3-56.9436z^4$$
$$+37.7184z^5+133.797z^6-60.0512x^7-170.756z^8$$
$$+81.775z^9+93.6217z^{10}-56.7404z^{11}+O(z^{12}) \quad (48)$$

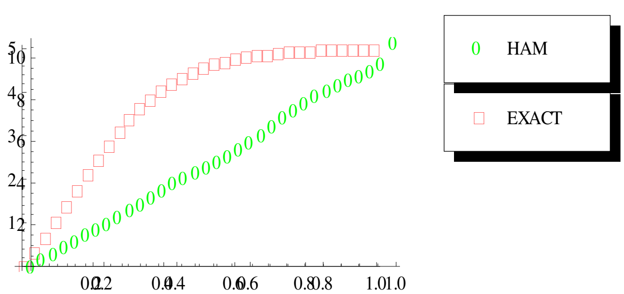
The following table 2 displays the values of the variable z, exact, OHAM, HPM and HAM solutions and errors. From the table 2 we conclude the absolute errors of OHAM and HAM are approximately the same while the errors of HPM are larger than the errors of the other methods i.e. OHAM and HAM. From figure 2 (a) we admit that the graphs of exact and OHAM solutions are coincident while in 2 (b) and 2(c) the graphs of exact and HPM, exact and HAM solutions are not coincident but generally we investigate that the method OHAM is more effective than HPM and HAM



(a) Graph of exact and OHAM solutions



(b) Graph of exact and HPM solutions



(c) Graph of exact and HAM solutions
Figure-2

Problem 3: Considering the following nonlinear differential equation on domain $0 \le x \le 1$

$$\frac{dv}{dz} - 4z^2 + 4zv - v^2 + 7 = 0, \quad v(0) = 0 \tag{49}$$

The exact solution is

$$v(z) = 2z + 3\left(\frac{1-e^{6z}}{1+e^{6z}}\right) \tag{50}$$

Applying the OHAM technique, mentioned above, Zeroth order problem is

$$v_0'(z) = 4z^2 - 7, v_0(0) = 0 \tag{51}$$

$$v_0(z) = \frac{1}{3}(-21z + 4z^3) \tag{52}$$

First order problem is

$$v_1'(z, C_1) = 7 - 4z^2 + 7C_1 - 4z^2C_1 + 4zC_1v_0 - C_1{v_0}^2 + (1+C_1)v_0', \quad v_1(0) = 0 \tag{53}$$

$$v_1(z, C_1) = \frac{1}{315}\left(\begin{array}{l} -8085z^3C_1 \\ +1512z^5C_1 - 80z^7C_1 \end{array}\right) \tag{54}$$

Second order problem is

$$v_2'(z, C_1) = 4zC_1v_1 - 2C_1v_0v_1 + (1+C_1)v_1', \quad v_2(0) = 0 \tag{55}$$

Table-2
Values of the variable z, exact, OHAM, HPM and HAM solutions and errors

Variable z	Exact Solution	OHAM Solution	HPM Solution	HAM Solution	Error OHAM	Error HPM	Error HAM
0	0.0000	0.0000	0.0000	0.0000	0.0×10^{0}	0.0×10^{0}	0.0×10^{0}
0.1	1.12296	1.10832	1.12296	1.10832	1.5×10^{-6}	3.1×10^{-8}	1.4×10^{-6}
0.2	2.33036	2.29886	2.33035	2.29886	3.2×10^{-6}	9.1×10^{-6}	3.1×10^{-6}
0.3	3.3593	3.35125	3.35957	3.35125	8.0×10^{-7}	2.7×10^{-8}	8.0×10^{-7}
0.4	4.07626	4.10851	4.07582	4.10851	3.2×10^{-6}	4.3×10^{-8}	3.2×10^{-6}
0.5	4.50864	4.54624	4.50098	4.54624	3.7×10^{-6}	7.7×10^{-7}	3.7×10^{-6}
0.6	4.74706	4.75674	4.78838	4.75674	9.7×10^{-7}	4.1×10^{-6}	9.7×10^{-7}
0.7	4.87207	4.86157	4.20166	4.86157	1.0×10^{-6}	6.7×10^{-5}	1.0×10^{-6}
0.8	4.93588	4.9274	-9.61142	4.9274	8.5×10^{-7}	14.5×10^{-0}	8.5×10^{-7}
0.9	4.96801	4.9659	-113.087	4.9659	2.1×10^{-7}	118.1×10^{-0}	2.1×10^{-7}
1.0	4.98408	4.96981	-624.264	4.96981	1.4×10^{-6}	629.2×10^{-0}	1.4×10^{-6}

$$v_2(z,C_1)=\frac{1}{10395}\begin{pmatrix}-266805z^3C_1+49896z^5C_1\\ -2640z^7C_1-266805z^3C_1^2\\ -910602z^5C_1^2+227304z^7C_1^2\\ -20064z^9C_1^2+640z^{11}C_1^2\\ +O(z^{13})\end{pmatrix} \quad (56)$$

Third order problem is

$$v_3'(z,C_1)=-C_1v_1^2+4zC_1v_2 \\ -2C_1v_0v_2+(1+C_1)v_2',\, v_3(0)=0 \quad (57)$$

$$v_3(z,C_1)=$$

$$\frac{1}{14189175}\begin{pmatrix}-364188825z^3C_1+68108040z^5C_1\\ -3603600z^7C_1-728377650z^3C_1^2\\ -2485943460z^5C_1^2+620539920z^7C_1^2\\ -54774720z^9C_1^2\\ +1747200z^{11}C_1^2C_1^3\begin{bmatrix}-364188825z^3\\ -2554051500z^5\\ -4082563485z^7\\ +1349908560z^9\\ -165695712z^{11}\\ +9488640z^{13}\\ -216320z^{15}\end{bmatrix}\\ +O(z^{16})\end{pmatrix} \quad (58)$$

Fourth order problem is

$$v_4'(z,C_1)=-2C_1v_1v_2+4zC_1C_3 \\ -2C_1v_0v_3+(1+C_1)v_3',\, v_4(0)=0 \quad (59)$$

$$v_4(z,C_1)=$$

$$\frac{1}{68746552875}\begin{pmatrix}C_1\begin{bmatrix}-1764494857125z^3\\ +329983453800z^5\\ -17459442000z^7\end{bmatrix}\\ +C_1^2\begin{bmatrix}-5293484571375z^3\\ -18066594095550z^5\\ +4509773868600z^7\\ -39807527760z^9\\ +12697776000z^{11}\end{bmatrix}\\ +C_1^3\begin{bmatrix}-5293484571375z^3\\ -37123138552500z^5\\ -59340060254475z^7\\ +19620920919600z^9\\ -2408387173920z^{11}\\ +137917382400z^{13}\\ -3144211200z^{15}\end{bmatrix}\\ +C_1^4\begin{bmatrix}-1764494857125z^3\\ -18726561003150z^5\\ -63867293565075z^7\\ -61819955747550z^9\\ +26884144024920z^{11}\\ -4276552351680z^{13}\\ +347149691904z^{15}\\ -14675904000z^{17}\\ +260249600z^{19}\end{bmatrix}\end{pmatrix} \quad (60)$$

Fifth order problem is

$$v_5'(z,C_1)=C_1\begin{bmatrix}-v_2^2-2v_1v_3\\ +4zv_4-2v_0v_4\end{bmatrix}+(1+C_1)v_4',\, v_5(0)=0 \quad (61)$$

Table-3
OHAM solution, HPM solution, HAM solution and their absolute errors

z	Exact sol	OHAM sol	HPM sol	HAM sol	Error OHAM	Error HPM	Error HAM
0.0	0.000000	0.000000	0.000000	0.000000	0.0×10^{0}	0.0×10^{0}	0.0×10^{0}
0.1	-0.673938	0.677862	-0.673938	0.677862-	3.9×10^{-7}	1.5×10^{-9}	3.9×10^{-3}
0.2	-1.21115	-1.23345	-1.21114	-1.23345	2.2×10^{-6}	1.1×10^{-5}	2.2×10^{-2}
0.3	-1.54889	-1.59119	-1.54712	-1.59119	4.2×10^{-6}	1.8×10^{-3}	4.2×10^{-2}
0.4	-1.70096	-1.74404	-1.64022	-1.74404	4.3×10^{-6}	6.1×10^{-2}	4.3×10^{-2}
0.5	-1.71544	-1.74015	-0.834453	-1.74015	2.4×10^{-6}	8.8×10^{-1}	2.4×10^{-2}
0.6	-1.64042	-1.64509	5.79423	-1.64509	4.4×10^{-7}	7.4×10^{-0}	4.7×10^{-3}
0.7	-1.51136	-1.50596	41.7794	-1.50596	5.3×10^{-7}	43.2×10^{-0}	5.4×10^{-3}
0.8	-1.35102	-1.34184	191.103	-1.34184	9.1×10^{-7}	192.4×10^{-0}	9.2×10^{-3}
0.9	-1.17302	-1.16033	695.585	-1.16033	1.2×10^{-6}	696.7×10^{-0}	1.2×10^{-2}
1.0	-0.985164	-0.956882	2144.38	-0.956882	2.8×10^{-6}	2145.3×10^{-0}	2.8×10^{-2}

$$v_5(z,C_1)=\frac{1}{365250435424875}\begin{pmatrix} C_1\begin{bmatrix}-9374761175905125z^3\\+17532020900394 00z^5\\-92762015346000z^7\end{bmatrix}\\ +C_1^2\begin{bmatrix}-37499044703620500z^3\\-127983752572876200z^5\\+31947238085162400z^7\\-2819965266518400z^9\\+89951045184000z^{11}\end{bmatrix}\\ +C_1^3\begin{bmatrix}-56248567055430750z^3\\-394740470258865000z^5\\-630547480264051350z^7\\+208491905691669600z^9\\-25591522110073920z^{11}\\+1465510105382400z^{13}\\-33410388277200z^{15}\end{bmatrix}\\ +C_1^4\begin{bmatrix}-37499044703620500z^3\\-397976874438943800z^5\\-1357307722844973900z^7\\-1313797699546926 00z^9\\+571341828817599840z^{11}\\-90885290577903360z^{13}\\+7377625252343808 1z^{15}\\-311892311808000z^{17}\\+5530824499200z^{19}\end{bmatrix}\\ +C_1^5\begin{bmatrix}-9374761175905125z^3\\-133243358842994400z^5\\-694720242480414150z^7\\-1525109570505120600z^9\\-96337958711431437 0z^{11}\\+559528566702724080z^{13}\\-111008912869872854z^{15}\\+11668837527857664z^{17}\\-707211020533248z^{19}\\+23673704017920z^{21}\\-343484211200z^{23}\end{bmatrix}\\ +O(z^{25})\end{pmatrix} \quad (62)$$

Using equations (52), (54), (56),(58), (60) and (62) we get the following approximate solution

$$\begin{aligned}\tilde{v}(z,C_1) &= v_0(z)+v_1(z,C_1)+v_2(z,C_1)\\ &\quad + v_3(z,C_1)+v_4(z,C_1)+v_5(z,C_1)\end{aligned} \quad (63)$$

Following the technique, OHAM on domain $[0,1]$ we use the residual.

is more effective than HPM and HAM.

$$X = \tilde{v}' + 7 - 4z^2 + 4z\tilde{v} - \tilde{v}^2 \quad (64)$$

The value of C_1 is -0.29729358104456594 and the approximate solution is

$$\tilde{v}=\frac{1}{3}\left(-21z+4z^3\right)+\frac{1}{315}\begin{pmatrix}2403.62z^3\\-449.508z^5\\+23.7835z^7\end{pmatrix}$$

$$+\frac{1}{10395}\begin{pmatrix}55738.3z^3-95315.9z^5\\+20874.8z^7-1773.33z^9\\+56.5654z^{11}\end{pmatrix}$$

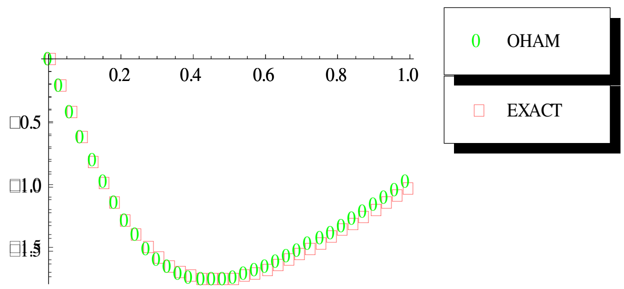
$$+\frac{1}{14189175}\begin{pmatrix}534.638\times10^7z^3-1.72855\times10^8z^5\\+1.6319\times10^8z^7-4.03112\times10^7z^9\\+4.50822\times10^6z^{11}-249322.0z^{13}\\+5683.99z^{15}+O(z^{16})\end{pmatrix}$$

$$+\frac{1}{68746552875}\begin{pmatrix}1.82024\times10^{11}z^3-8.65734\times10^{11}z^5\\+1.46408\times10^{12}z^7-1.03365\times10^{12}z^9\\+2.74414\times10^{11}z^{11}-3.70308\times10^{10}z^{13}\\+2.79442\times10^9z^{15}-1.14643\times10^8z^{17}\\+2.03298\times10^6z^{19}+O(z^{21})\end{pmatrix} \quad (65)$$

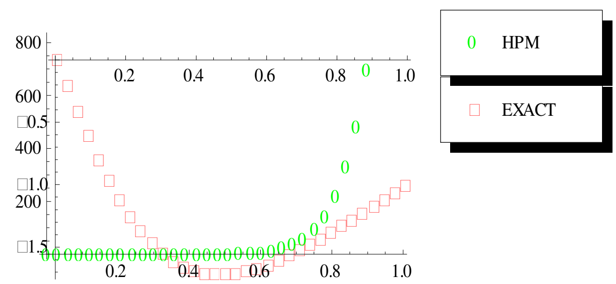
$$+\frac{1}{365250435424875}\begin{pmatrix}6.79581\times10^{14}z^3-4.26724\times10^{15}z^5\\+1.04299\times10^{16}z^7-1.24486\times10^{16}z^9\\+7.38081\times10^{15}z^{11}-2.044789\times10^{15}z^{13}\\+3.16311\times10^{14}z^{15}-2.95355\times10^{13}z^{17}\\+1.6856\times10^{12}z^{19}-5.49786\times10^{10}z^{21}\\+7.97691\times10^8z^{23}+O(z^{25})\end{pmatrix}$$

Table-3 displays the values of exact solution, OHAM solution, HPM solution, HAM solution and their absolute errors, the values due to OHAM and HAM are nearer to exact solution but HPM values do not agree with the exact values.

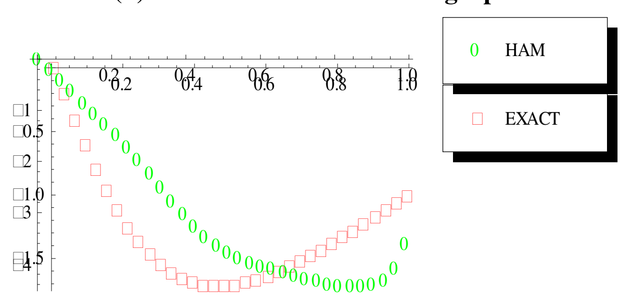
From the figure 3(a) we conclude that the exact and OHAM solution graphs are coincident exhibiting that OHAM, technique



(a) OHAM and exact solutions graph



(b) HPM and exact solutions graph



(c) HAM and exact solutions graph

Figure-3
The variable z and $v(z)$ are represented along the horizontal and the vertical axis

Conclusion

In this paper, Optimal Homotopy Asymptotic Method (OHAM), Homotopy Perturbation Method (HPM) and Homotopy Analysis Method (HAM) have been used for numerical solution of Riccati Equation. We have compared the results of each of them with the exact solution of the given problem. Conclusions reveal that the method OHAM is more effective and the results obtained by this method are in good agreement with the exact solution.

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