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A Study of the Performance of the PID Controller and Nonlinear Controllers in Vehicle Suspension Systems Considering Practical Constraints

S. H. Hashemipour, M. Rezaei lasboei and M. Khaliji
Department of Electrical and Mechanical Engineering, Young Researcher Club, Roudsar and Amlash Branch, Islamic Azad University, Roudsar, IRAN

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Abstract

In this paper first an applicable PID controller has been designed for the vehicle suspension system with considering the nonlinear dynamics of the hydraulic actuator. In this method, linear sky-hook model was chosen as the reference model. In order to improve the characteristics of the reference model in terms of ride comfort and stability and also considering the practical constraints of the suspension system, an optimal LQR controller has been designed for the sky-hook model. To improve the reference model behavior in terms of ride comfort with considering the limitations of the suspension system working space and create a tradeoff between ride comfort and handling, an optimum LQR controller with adjustable weight matrices for the sky-hook model was designed. Of course, this controller had no suitable performance with regards to the system non-linear dynamics. Then, a non-linear controller was designed based on Lyapunov method. Simulation results indicate that this controller is successful in reducing the vertical acceleration to improve the ride comfort, but it cannot control the stability and stability of the vehicle. Meanwhile, the designed controller is not robust enough to system parameter perturbations. Therefore, the sliding mode control as a robust nonlinear control method has been adopted as an alternative way for the controller design. In this method, the sliding surfaces are selected in a way that the nonlinear system tracks a sky-hook model which has desirable behavior. Simulation results revealed that the non-linear model of the suspension system with a sliding mode controller could satisfactorily track the behavior of the new improved sky-hook reference model. Also, the sliding mode controller showed a good behavior when parameters changed.

Keywords: Suspension system, PID, Sliding Mode, Lyapunov method.

Introduction

One of the causes for vehicle vibrations and shakes is road roughness. Shocks due to bumps on the road are transferred to the vehicle body through the wheels and cause discomfort for passengers and reduce driving quality. The suspension system is responsible for absorbing these shocks and reducing vehicle shakes as much as possible thereby providing more comfort to passengers.

In passive suspension systems, the reduction in vertical acceleration results in an increase in suspension travel. The suspension travel has to be limited because the movement of vehicle spring-damper system is constrained. Since the reduction in vertical acceleration results in an increase in suspension travel, there's a limit on the maximum reduction of vertical acceleration. To overcome these limitations and increase ride comfort within system constraints, and also to increase the vehicle stability, active suspension systems have been considered[1, 2]. In these systems, by applying an extra force to the suspension system using a hydraulic actuator, the vertical acceleration exerted on passengers is reduced while also reducing suspension travel considerably and thus improving ride comfort and driving quality. The *LQR* control method and H_∞ robust control have been used widely for designing controllers for active suspension systems[2, 3]. Sam et al[4] designed a sliding mode PI controller for the quarter vehicle model and compared their results with those obtained by the *LQR* control method. The abovementioned methods are based on a linear model and neglect the dynamics of the hydraulic actuator which is highly nonlinear. However, the nonlinearity in the actuator dynamics has a large effect on the overall behavior of the system and cannot be neglected. Using nonlinear control methods, Kurimoto et al[5] designed a sliding mode controller for active suspension. They used a 4-DOF quarter model of vehicle which included a simplified model of nonlinear actuator dynamics. Yokoyama[6] designed a sliding mode controller and observer for a semi-active suspension system. State feedback control[7], fuzzy control[8], and optimal stochastic control[9] are some other control strategies which have been used for active suspension system design. In the present article, the studied model was a nonlinear one. First, a PID controller was designed for the nonlinear model. The controlled sky-hook linear model was chosen as the reference model here. In the method based on Lyapunov theory, a reverse procedure is applied where, in the first step, a candidate Lyapunov function is chosen considering the representation of system state equations and the direct relation between the decrease in vertical acceleration and the increase in ride comfort. Then, the controller is designed in a way that the candidate Lyapunov function be the true Lyapunov function of the controlled system.

The sliding mode control method is then applied for the design of a nonlinear and robust controller for the active suspension system. A novel innovative reference model has been used in the present work for choosing sliding surfaces. In the literature on sliding mode control for active suspension systems, a linear sky-hook model with a fairly desirable behavior has been used alone as the reference model to be tracked by the controlled system [9]. However, in the present work, in order to obtain greater reduction in vertical acceleration, while taking into account practical system constraints, a controlled sky-hook model is chosen as the reference model. Since reduction in vertical acceleration results in an increase in suspension travel, in order to make a trade-off, first the *LQR* optimal control method is used for designing a controller for the sky-hook model. In this method, by adjusting weight matrices, the sky-hook model is controlled in a way that maximum reduction in vertical acceleration is attained while remaining within allowable displacement limits of the suspension system workspace. The sliding surfaces are designed in a way that the controlled system follows the behavior of this optimally controlled sky-hook model.

In the next section of the article, the vehicle suspension system model has been introduced along with the hydraulic actuator and the sky-hook reference model. Then, controlling theories are introduced with their simulation results presented. The last section of the article includes comparison and conclusion.

Quarter Model including Hydraulic Actuator Dynamics

One of the most widely used models in active suspension system design is the vehicle quarter model. This model encompasses the main components of the suspension system including sprung and unsprung masses, spring and the main damper. The actuator dynamics can also be included in this model with ease. Figure 1 shows the vehicle quarter model.

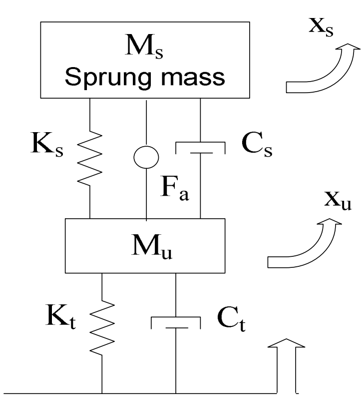


Figure-1
Quarter- car Model for Active Suspension Control Design

The hydraulic actuator in this model is a four-way valve-piston system. The actuator force is calculated using $u_a=AP_L$ where A is the area of the piston and P_L is the pressure drop across the piston. The time derivative of P_L can be expressed as in equation (1) [11].

$$\frac{V_t}{4\beta_e}\dot{P}_L = C_{tp}P_L - A(\dot{x}_s - \dot{x}_u) + Q \tag{1}$$

In the above equation, V_t is the total actuator volume, P_L is the pressure drop, β_e is the effective bulk modulus, C_{tp} is the total leakage coefficient of the piston, and Q is the hydraulic load flow which can be calculated using Equation (2):

$$Q = C_d w x_v \sqrt{\frac{1}{\rho}\left[P_s - \mathrm{sgn}(x_v)P_L\right]} \tag{2}$$

where C_d is the discharge coefficient, x_v is the spool valve displacement, w is the spool valve area gradient, P_s is the supply pressure and ρ is the hydraulic fluid density. Using Newton's second law and actuator equations, the following relations can be obtained:

$$m_s\ddot{x}_s = k_s\,(x_u - x_s) + C_s\,(\dot{x}_u - \dot{x}_s) + F_a \tag{3}$$

$$m_u\ddot{x}_u = -K_s\,(x_u - x_s) - C_s\,(\dot{x}_u - \dot{x}_s) + C_t\,(\dot{r} - \dot{x}_u) + k_t\,(r - x_u) - F_a \tag{4}$$

$$F_a = AP_L \tag{5}$$

$$\dot{P}_L = -\beta P_L + \alpha A(x_2 - x_4) + \gamma x_v\sqrt{P_s - P_L sign(x_v)} \tag{6}$$

$$\dot{x}_v = \frac{1}{\tau}(-x_v + u) \tag{7}$$

Choosing the state variables as in Equation (8) the state space representation of the system can be written as Equations (9)-(14):

$$X = \left(r - x_u, \dot{x}_u, x_u - x_s, \dot{x}_s, P_L, x_v\right) \tag{8}$$

$$\dot{x}_1 = \dot{r} - x_2 \tag{9}$$

$$\dot{x}_2 = \frac{1}{M_u}\left[K_t x_1 - C_s\left(x_2 - x_4\right) - K_s x_3 + C_t(\dot{r} - x_2) - Ax_5\right] \tag{10}$$

$$\dot{x}_3 = x_2 - x_4 \tag{11}$$

$$\dot{x}_4 = \frac{1}{M_s}\left[C_s\left(x_2 - x_4\right) + K_s x_3 + Ax_5\right] \tag{12}$$

$$\dot{x}_5 = -\beta x_5 + \alpha A\left(x_2 - x_4\right) + \gamma x_6\sqrt{P_s - sign(x_6)x_5} \tag{13}$$

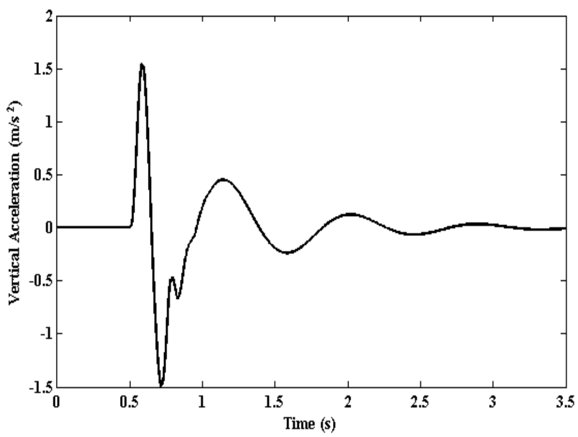
$$\dot{x}_6 = \frac{1}{\tau}\left(-x_6 + u\right) \tag{14}$$

Where $\alpha = \frac{4\beta_e}{V_t}, \beta = \alpha C_{tp}$ and $\gamma = \alpha C_d w\sqrt{\frac{1}{\rho}}$. In equation (8) the state variables are, respectively, the unsprung mass displacement, the unsprung mass velocity, the sprung mass displacement, the sprung mass velocity, the pressure drop across the piston, and the valve displacement. The parameters used for the simulation of this model are given in table I [11]:

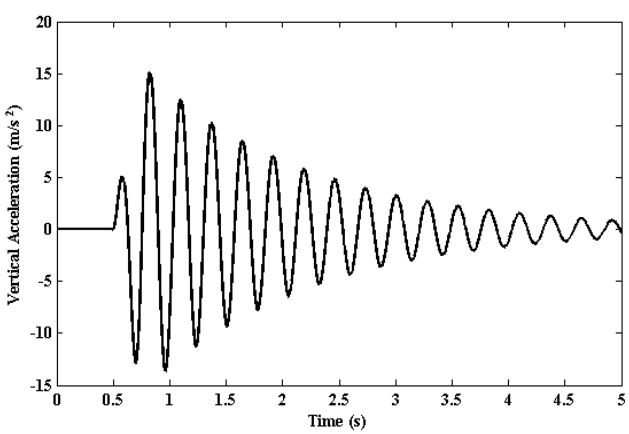
Table-1
Parameters of the quarter car model

$M_s = 290\ [Kg]$	$C_s = 1000\ [N\ s/m]$	$\gamma = 1.545e9\ [N/m^{\frac{5}{2}} Kg^{\frac{1}{2}}]$
$M_u = 59\ [Kg]$	$C_t = 500\ [N\ s/m]$	$A = 3.35e-4\ [m^2]$
$K_s = 16812\ [N/m]$	$\tau = 1/30\ [\sec]$	$\alpha = 4.515e13\ [N/m^5]$
$K_t = 190000\ [N/m]$	$P_s = 10342500\ [Pa]$	$\beta = 1.00\ [1/Sec]$

In figure 2.a, the vertical acceleration response of the linear suspension system without the nonlinear hydraulic actuator dynamics has been plotted. The same input has been applied to the nonlinear system including the actuator dynamics and the response has been plotted. It can be observed that nonlinearity, arising from actuator dynamics, drastically affects the behavior of the system and is not negligible. It should be noted that in the present work, the controller has been designed for the suspension system while taking the ill-behaved actuator dynamics as depicted in figure 2.b into account.



a) Vertical acceleration without considering actuator dynamic



b) Vertical acceleration with considering actuator dynamic
Figure-2
Impact of hydraulic actuator dynamic on system behavior

The Optimally Controlled Sky-Hook Model

System Equations: The sky-hook model is a virtual model that can be used as a reference model in suspension system controller design for increasing ride comfort [10]. In this paper, a controlled sky-hook model is used as the reference model. The purpose is to improve the reference model so that the trade-off between the reduction in vertical acceleration and the increase in suspension travel is improved. The model is shown in Figure 3 and its state equations can be written as:

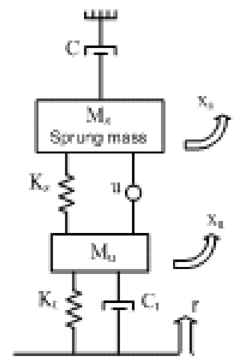


Figure-3
The sky-hook model

$$\begin{cases} \dot{x}_1 = \dot{r} - x_2 \\ \dot{x}_2 = \dfrac{1}{M_u}\left[K_t x_1 - K_s x_3 + C_t(\dot{r} - x_2) - u\right] \\ \dot{x}_3 = x_2 - x_4 \\ \dot{x}_4 = \dfrac{1}{M_s}\left[-C x_4 + K_s x_3 + u\right] \end{cases} \quad (15)$$

In Equation (24), the state variables are the unsprung mass displacement, the unsprung mass velocity, the sprung mass displacement, and the sprung mass velocity, respectively.

Optimal Controller Design for the Sky-Hook Model: In order to control and improve the behavior of the sky-hook reference model, state-space equations for the linear system were written as follows and an optimum LQR controller was designed for it.

$$\dot{x} = A_1 x + B_1 u + D_1 \dot{r} \quad (16)$$

Where

$$A_1 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ \frac{K_t}{M_u} & -\frac{C_t}{M_u} & -\frac{K_s}{M_u} & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & \frac{K_s}{M_s} & \frac{-C}{M_s} \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ -\frac{1}{M_u} \\ 0 \\ \frac{1}{M_s} \end{bmatrix}, \quad D_1 = \begin{bmatrix} 1 \\ \frac{C_t}{M_u} \\ 0 \\ 0 \end{bmatrix}$$

If the cost functional is chosen in the form of

$$J = \int_0^\infty (x^T Qx + u^T Ru)dt \tag{17}$$

The control law to minimize the above functional can be obtained as:

$$u = -k\,x \tag{18}$$

$$k = R^{-1}B_1^T P \tag{19}$$

Where P is obtained from the following algebraic Ricatti equation:

$$PA_1 + A_1^T P - PB_1R^{-1}B_1^T P + Q = 0 \tag{20}$$

By adjusting weight coefficients in this method, maximum reduction of vertical accelerations for admissible displacements in the suspension system's working space was achieved and it resulted in a good compromise between the vehicle's ride comfort and handling.

In the present work, these matrices were selected as in equation (30) by trial and error so that both a desirable decrease in vertical acceleration be obtained and suspension travel remain within limits:

$$R = 0.1\,,\; Q = diag\{1e3, 100, 1e5, 5e6\}$$

Through the above-mentioned choice, not only vertical acceleration was adequately reduced, but also suspension displacement remained in its admissible range. Thus, by tracking the above-said reference model's behavior, a suitable ride comfort and handling could be simultaneously achieved[1].

PID Controller

PID controller is one of the widely applicable linear controllers in science and industry[16]. Many complicated non-linear systems can be controlled by this kind of controller. In this article, the sky-hook controlled linear model has been chosen as the reference model and the difference between the system vertical acceleration and its desired vertical acceleration is applied to the PID controller as an error.

$$u = K_P e + K_D \frac{de}{dt} + K_I \int_0^t e\,dt \tag{30}$$

Simulation results in figure 4 show that the controller has a relatively good tracking ability, but the obtained control signal cannot be applied practically. Also, the more time passes after simulation, the higher the tracking error value would be.

Also to study the performance of the designed controller in the presence of parameter uncertainties, it is assumed that the vehicle mass and tire stiffness vary as large as 30% and 40%, respectively. Figure 4.g shows the vehicle acceleration responses with and without model uncertainties. The road input for the analysis is the one in figure 4.a.

The said results show that the controller is highly dependent on system parameters. Of course, these results are not unexpected. Due to the non-linear dynamic of the system, one should not expect a linear controller to cope with it. Thus, a non-linear controller is designed for the system in the next section of the article.

Then, the controller's performance was studied with the purpose of reducing maximum vertical acceleration irrespective of vehicle handling and stability.

Lyapunov Method

In general, two approaches exist to nonlinear controller design using Lyapunov direct method, both of which have a trial and error nature. In the first approach, after choosing a control law, a Lyapunov function is sought which justifies the law. Conversely, the second approach requires that a candidate Lyapunov function be chosen and then a suitable law be sought which turns the candidate Lyapunov function into a true Lyapunov function. The second method has been adopted in the present paper. The main objective in controller design is the minimization of the vertical acceleration transferred to passengers which has a direct relation with ride comfort. Considering the state space equations in equation (12) where $\dot{x}_4$ denotes the sprung mass and passengers' acceleration, $(\dot{x}_4)^2$ has been selected as the candidate Lyapunov function:

$$u = K_P e + K_D \frac{de}{dt} + K_I \int_0^t e\,dt \tag{31}$$

where

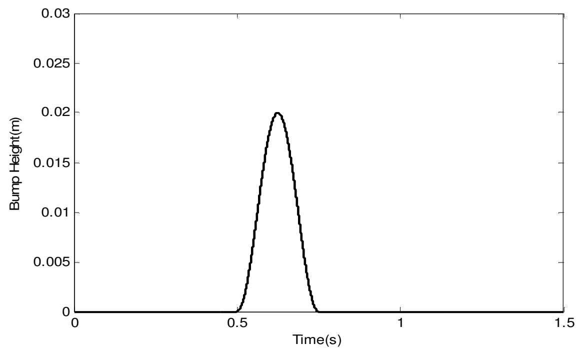
$$B = \left[C_s\left(x_2 - x_4\right) + K_s x_3 + Ax_5\right] \tag{32}$$

Constraining the derivative of the candidate function in Equation (31) to be negative, we have:

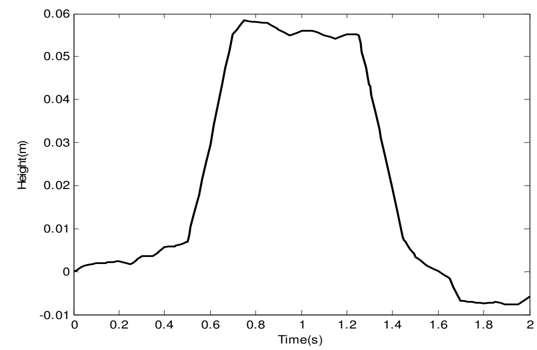
$$\dot{V} = \dot{x}_4\ddot{x}_4 = \frac{1}{M_s^2}B\dot{B} \le 0 \;\rightarrow\; B\dot{B} \le 0 \tag{33}$$

Substituting equation (32) into Equation (33) and using equations (9)-(13) one can write:

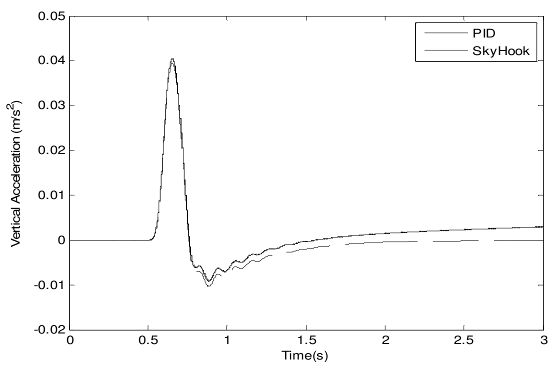
$$B\dot{B} = B\left\{\begin{array}{l} C_s\left(-\frac{B}{M_u} + \frac{k_t}{M_u}x_1 + \frac{C_t}{M_u}\left(\dot{r} - x_2\right) - \frac{B}{M_s}\right) + K_s(x_2 - x_4) - \\ A\beta x_5 + \alpha A^2(x_2 - x_4) + A\gamma x_6\sqrt{P_s - x_5 sign(x_6)} \end{array}\right\} \text{£}0 \tag{34}$$



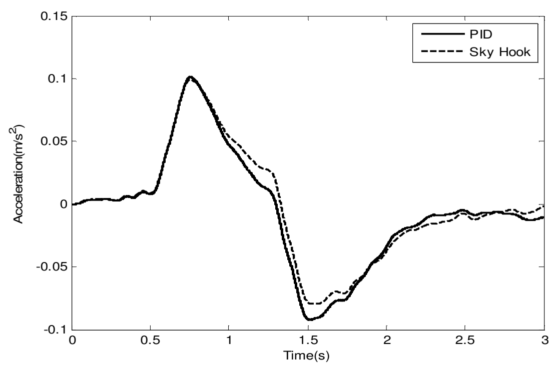
a) The first road input



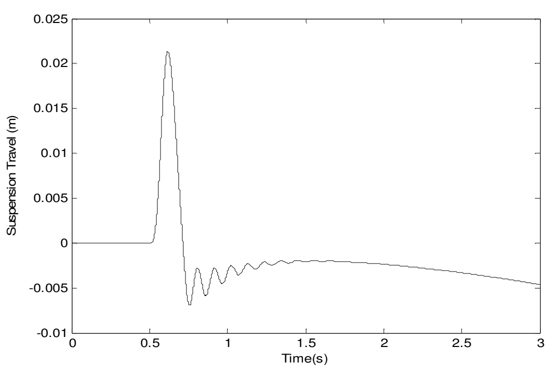
b) The second road input



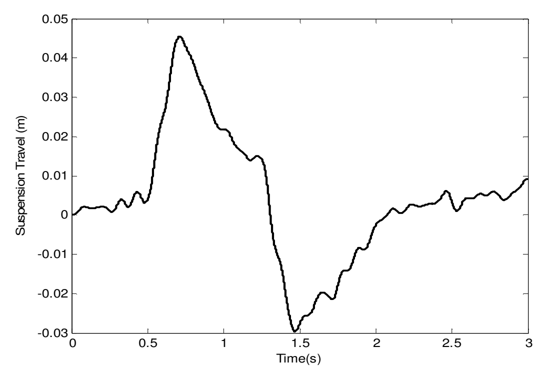
c) Vertical acceleration



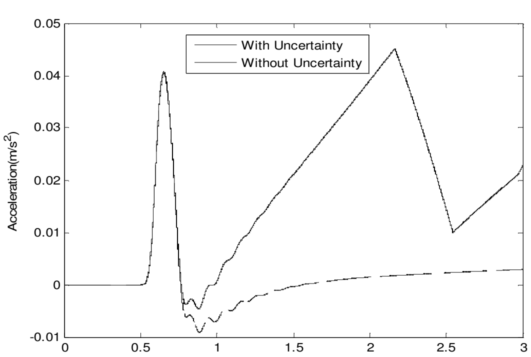
d) Vertical acceleration



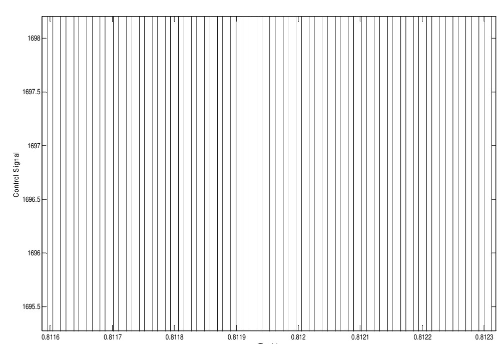
e) Suspension Travel



f) Suspension travel



g) Robustness analyze



h) Control Signal

Figure-4
PID controller for suspension system

The above equation can be written in the simplified form:

$$B\dot{B} = -\frac{C_s}{M_u}B^2 - \frac{C_s}{M_s}B^2 + V_2 \tag{35}$$

If $V_2 = -\lambda B^2$, the above equation will always be negative. Using this equation, substituting Equation (34) into the left hand side of equation (35) and simplifying the resulting equation, we have:

$$-\lambda B^2 = B\begin{bmatrix} \frac{K_t C_s}{M_u}x_1 + \frac{C_s C_t}{M_u}(\dot{r} - x_2) + k_s(x_2 - x_4) - A\beta x_5 + \\ \alpha A^2(x_2 - x_4) + A\gamma x_6\sqrt{P_s - x_5 sign(x_6)} \end{bmatrix} \tag{36}$$

Substituting B from Equation (32) into Equation (36), the following equation results:

$$A\gamma x_6\sqrt{P_s - x_5 sign(x_6)} = -\frac{K_t C_s}{M_u}x_1 - \frac{C_s C_t}{M_u}(\dot{r} - x_2) - \lambda k_s(x_3) - (A\lambda - A\beta)x_5 - (K_s + \alpha A^2 + \lambda C_s)(x_2 - x_4) \tag{37}$$

If $\lambda \geq -20$ is chosen, $\dot{V} \leq 0$ will result.

According to equation (14), the control signal, *u*, has a linear relation with the state variable, x_6. Thus, obtaining a control law for x_6 can be generalized to *u*.

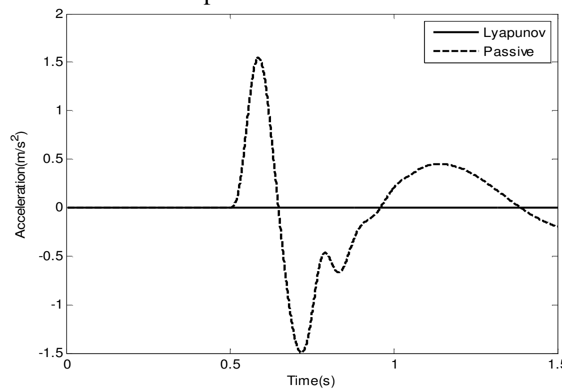
$$\dot{x}_6 = \frac{1}{\tau}(-x_6 + u) \tag{38}$$

Thus, a control rule of the following form is obtained:

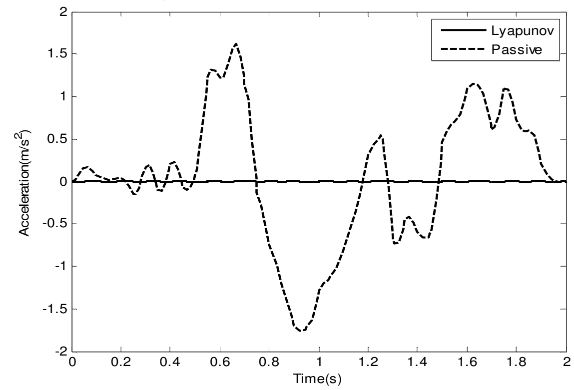
$$\dot{x}_6 = \frac{1}{\tau}(-x_6 + u) \tag{39}$$

In figure 5 the vertical accelerations and suspension travels of an active suspension system with a Lyapunov controller and a passive controller are presented and compared. In both cases, the two suspension systems are subject to two road inputs, a bump figure 4.a and a random roughness figure 4.b.

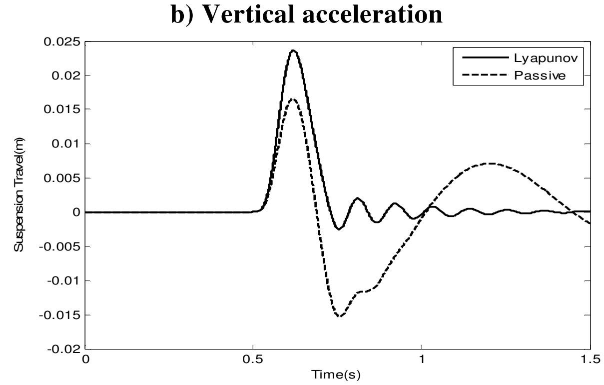
It can be seen from figure 5.a and 5.b that in the active control system, the amplitude of vertical acceleration decreases significantly which results in an increase in suspension travel, especially for the second road input. It should be noted that the increase in suspension travel must not exceed the allowed workspace limits. The allowed workspace limit in this study is in the range of ± 8 cm[10-12]. It is seen in figure 5.d that the suspension travel is close to its limits. It can be predicted that the suspension travel can exceed its limits for other road inputs. As it can be seen, with the Lyapunov controller, vertical accelerations are drastically decreased but the displacements increase to a level that is close to or exceeds the limits of the suspension system workspace. Generally, the objective of this method is only the reduction of vertical accelerations; the method doesn't take the practical constraints of the workspace into account. Hence, a trade-off between the reduction of vertical accelerations and the increase in suspension travel is not possible in this method [15].



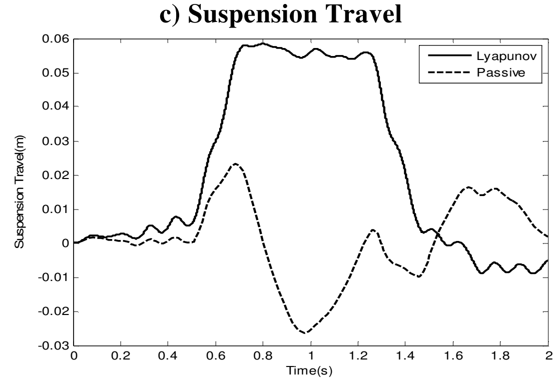
a) Vertical acceleration



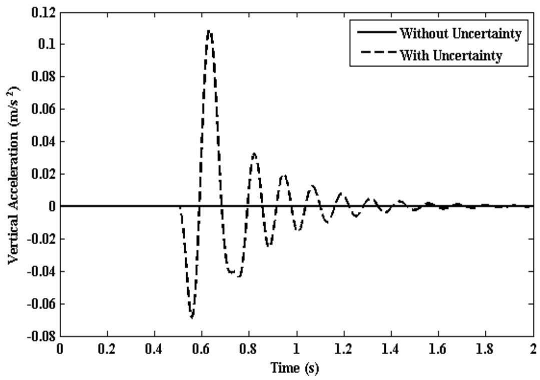
b) Vertical acceleration



c) Suspension Travel



d) Suspension Travel



e) Robustness analyze
Figure-5
Lyapunov Controller for suspension system

On the other hand, in the controller design, nominal parameter values were assumed. Such an assumption cannot be made in practice and deviations from nominal values are inevitable. For example, the vehicle mass varies based on factors such as its load, number of passengers, and the amount of fuel. Also, the elasticity of vehicle tires changes based on vehicle speed, temperature increase, and road quality. Also robustness analyze perform for this controller and that presented in figure 5.e. It can be observed that the Lyapunov controller is very sensitive to changes in system parameter values and the amplitude of vertical acceleration with the new parameter values is a multiple of that with nominal parameter values.

On the whole, the results of studying the Lyapunov controller indicate that although this controller acts successfully to improve ride comfort by reducing vertical accelerations, it is not robust against model uncertainties. In addition, for various road inputs, suspension travel is increased and cannot be controlled. These results show the necessity of designing a nonlinear controller with which a trade-off is possible between vertical acceleration and suspension travel for various road inputs.

To solve the problems of the designed Lyapunov controller with respect to the practical aspects of the suspension system, the sliding mode method, being a non-linear and robust one, was used for designing the controller.

Sliding Mode Controller Design

By designing such a controller, achieving both ride comfort and suitable handling would be possible. Therefore, the sliding mode control method with the nonlinear and robust nature taken into account was used and its sliding surfaces were obtained according to the sky-hook model which was selected as a reference. However, to have a tradeoff between vertical acceleration and suspension displacements that was the requirement for optimization, an optimized linear controller was first designed for the sky-hook reference model in which by choosing suitable weight coefficients, a compromise could be created between acceleration reduction and increased suspension displacement, something that has been done in the previous section related to PID controller. The sliding mode controller tracked the behavior of the adjusted reference model. This controller will be introduced in the following sections of the article with its relevant simulation results.

Comparing equation (12) and the fourth equation in (4) which represents the acceleration of the sprung mass in the nonlinear model and in the sky-hook model, respectively, one can write:

$$Ax_5 = u - C x_2 \quad (40)$$

Using the feedback law,

$$u = -k x \quad (41)$$

the actuator force can be obtained from:

$$x_{5d} = -\frac{k x + C x_2}{A} \quad (42)$$

Here the error vector is defined as $e_1 = x_5 - x_{5d}$ where x_5 is the hydraulic pressure drop across the piston and x_{5d} is the desired pressure drop. Since equation (13) is of first order, the first sliding surface is the error vector and is defined as below:

$$s_1 = x_5 - x_{5d} \quad (43)$$

Selecting the sliding condition as the following first-order differential equation,

$$\dot{s}_1 = -k_1 s_1 \quad (44)$$

The term s_1 will tend to zero as time passes by. A value of zero for s_1 means that the system has reached the desired state. Satisfying this condition results in x_{6d}, the spool valve displacement, is obtained as:

$$x_{6d} = \frac{\alpha A(x_4 - x_2) + (\beta - k_1)x_5 - k_1\left(\frac{k x + C_s x_2}{A}\right) - \left(\frac{k \dot{x} + C_s \dot{x}_2}{A}\right)}{\gamma\sqrt{P_s - x_5 sign(x_6)}} \quad (45)$$

To track the desired behavior of the controlled linear reference model, two sliding surfaces are needed to be designed. The first sliding surface is defined with the purpose of tracking the force of the linear actuator and the second is defined for tracking the desired spool valve displacement and obtaining the system input actuator voltage. The actuator voltage is defined in a way that the spool valve displacement coincides with its desired value, which is obtained from satisfying the sliding condition. The second error vector is defined in equation (46). This equation, similar to equation (14), is of first order. The sliding surface is defined in equation (47):

$$e_2 = x_6 - x_{6d} \quad (46)$$

$$s_2 = x_6 - x_{6d} \quad (47)$$

$$\dot{s}_2 = -k_2 s_2 \quad (48)$$

The actuator voltage is obtained from the following equation:

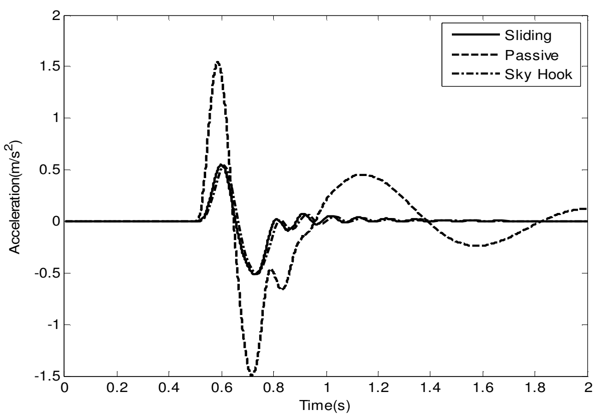
$$u = (1-\tau k_2)x_6 + \tau k_2 x_{6d} + \tau \dot{x}_{6d} \quad (49)$$

Where $k_1 = k_2 = 90$ and $C = 1000$ have been chosen [1].

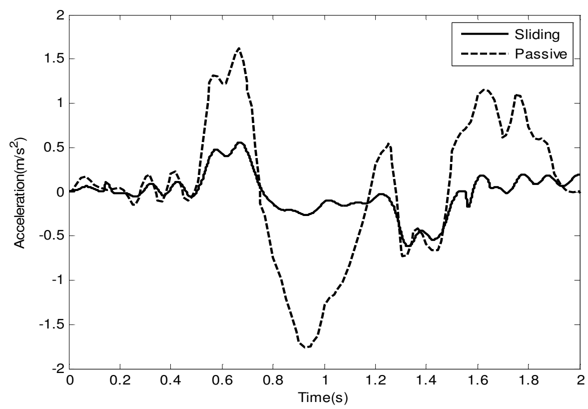
To simulate the system behavior, in Figure 6, the vertical acceleration and suspension travel have been shown for the actively controlled system with the sliding mode controller and the passively controlled system have been presented for the road inputs in figures 3.a and 3.b.

Figure 6 shows that the nonlinear model tracks the behavior of the controlled linear model well and that the vertical acceleration of the active suspension system is smaller than that of the passive system and suspension travel is within bounds for both road inputs. In comparison to the Lyapunov method, the sliding mode controller, not only compensates for the nonlinearities in the model, but it also uses optimization in making a trade-off between ride comfort and practical constraints. This is because the sliding mode controller tracks the behavior of the controlled sky-hook model. The behavior of the controlled sky-hook model can be modified using the weighting matrices Q and R based on design specifications. Figure 7 presents the robustness analysis of the sliding mode controller. It can be seen that the sliding mode controller is more robust against parameter variations. Figure 8 shows the actuator forces of the sliding mode and Lyapunov controllers and table 2 gives their respective control energy obtained from the following equation:

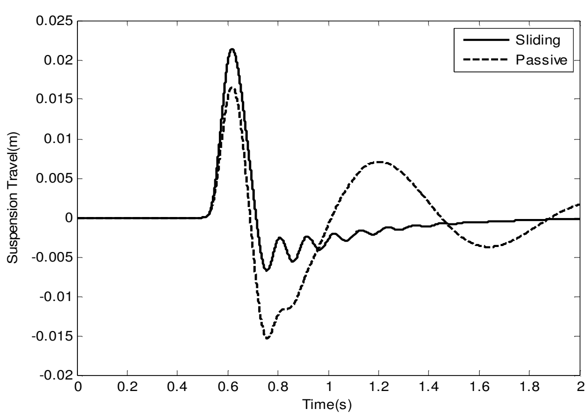
$$E = \int_0^t u^2 dt \quad (50)$$



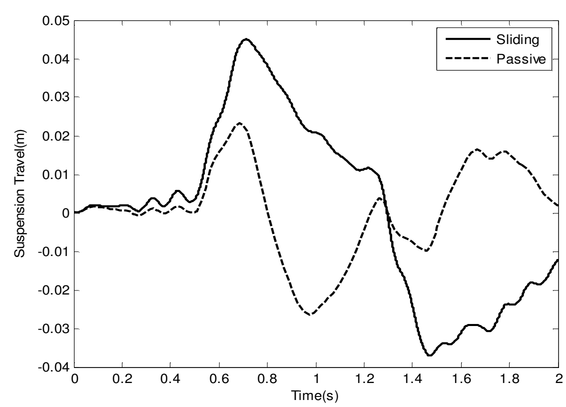
a) Vertical acceleration of the active and passive systems and the reference model



b) Vertical acceleration of the active and passive systems and the reference model



c) Suspension travel of the active and passive systems



d) Suspension travel of the active and passive systems

Figure-6
Sliding mode controller for suspension system

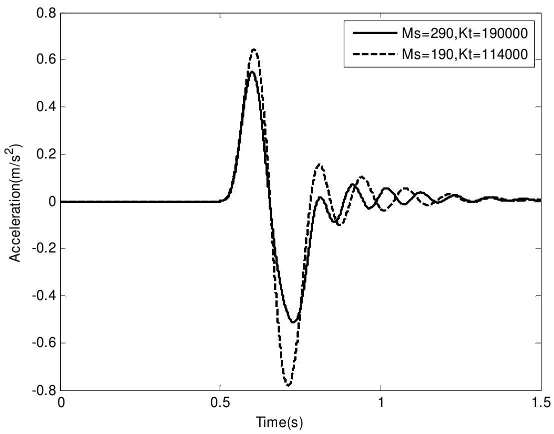


Figure-7
Robustness analysis for the sliding mode controller

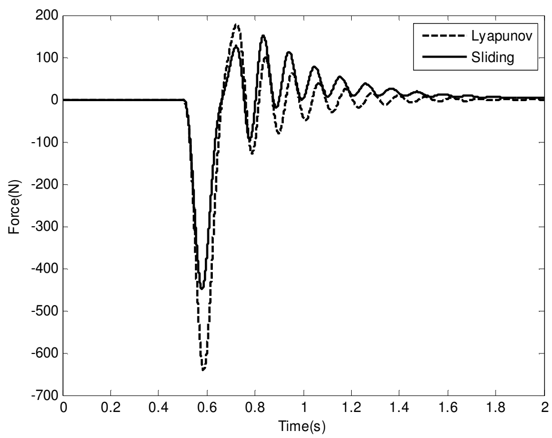


Figure-8
Actuator forces for sliding mode and Lyapunov controllers

Table-2
Control energy

Lyapunov Controller	Sliding Mode Controller
26590 J	13740 J

Conclusion

In this article, a PID linear controller and two nonlinear controllers were designed using the sliding mode and Lyapunov methods for a vehicle active suspension system. Due to the linear nature of the PID controller, it was not successful in controlling the system. Though it had a relatively good tracking behavior, its control signal could not be applied and the system was highly dependent on parameters and did not prove to be robust at all. In Lyapunov method, by choosing the vertical acceleration square as the candidate Lyapunov function, the controller was designed in a way that the chosen function would be a real Lyapunov function. This controller did not control the effective factors of a vehicle's handling and stability and excessive reduction of acceleration led to increased suspension displacement close to the set boundaries. Also, Lyapunov controller did not have a good robustness against parameter variation. However, by choosing a suitable sliding surface in the sliding mode method, the nonlinear model followed the improved desired linear model's behavior. Since the LQR optimal controlling method was chosen for controlling the sky-hook reference model, system behavior could be desirably changed by adjusting R and Q weight matrices. Moreover, robustness analysis results indicated that the sliding mode controller showed a good robustness against parameter changes.

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