



# Evaluating Hydrodynamic Forces of Sea Waves to Circular and Square Thin Vertical Sectioned Cylindrical Pillars with Identical Cross-section in Off-shore Structures

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## Abstract

Wave forces are considered as serious threats to off-shore structures; they can impose great, complicated and destructive forces to the structure and decrease its security and use levels. Due to random waves, it is complicated to exactly estimate hydrodynamic forces imposed on marine structures and it is impossible to easily predicate accurate situation of the structure in different moments. Meanwhile, the mathematical-experimental model of Morison used by researchers to estimate wave force on cylindrical elements of thin structures can be used for approximate estimation of wave forces to off-shore structures. Current study estimates hydrodynamic force effective on circular and square thin vertical sectioned cylindrical pillars with identical cross section using the software MATLAB and Morison's formula; it parametrically plots cylinder length and estimates total effective force and its focus as well as related moments by equivalency of plotted diagram using mathematical equations and integrating within cylinder length. Then, effectiveness of critical load and side loading capacity of pillars are examined in geometric terms. Results obtained from current study will be used to optimize designs, decrease wave forces to structures, control displacements and increase security levels of marine structures.

**Keywords:** Morison's equation, thin structures, critical load, loading capacity, hydrodynamic force.

## Introduction

Daily increasing population, industry development and need for energy, raw materials and oil production, hydrocarbon and proteins resources cause more attention to marine and marine structures. Exploitation of these resources needs design and implementation of off-shore structures. To optimal development of these structures requires exact identification of environmental forces and their influences on these structures. Building platforms for different purposes experienced considerable growth in recent century, particularly during past 50 years. Many financial resources are spent on building and establishing these platforms. Therefore, it is particularly important to establish a resistant structure during exploitation period in economic terms. To accurately identify behavior of marine structures against peripheral forces results in improvement of designs, increase in security and decrease in building costs.

According to human experiences in marine structures which is not more than one century, there are still unknowns about exact analysis of forces and structure responses and waves. Due to extensive parameters involved in it and development of human knowledge, related regulations are reviewed in shorter time periods.

An environmentally important factor influencing on design of these structures is forces resulted from waves and it is important to completely understand how they influence on marine

structures for economic and secure design of these structures and many have studied this. The current study used resources including Morison's equation<sup>1</sup>, Colgan and Carpenter's experiments and large-scale experiments in delta wave channel of Delft hydraulic laboratory, Holland, Navid Moghadam<sup>2</sup>, Sadreldini<sup>3</sup>, Afzali<sup>4</sup>, Naghipour<sup>5</sup>, Panahi<sup>6</sup>, Chegini<sup>7</sup>, Murray Rudman<sup>8</sup>, Chakrabarti<sup>9</sup>, Sarpakaya<sup>10</sup>, Fenton<sup>11</sup>, Ketabdari<sup>12</sup> and many others.

As water depth increases, design and installation of off-shore structures requires more accuracy and skill. These structures are not only influenced by dead and live loads and side loads resulted from wind or earthquake as land structures, but also exposed to different environmental loads such as loads exposed from periodic waves, marine currents, earthquake, wind and ice. During recent decades, extensive research has been conducted to estimate hydrodynamic forces of waves to off-shore structures.

However, due to importance of the subject and high costs of building and maintaining off-shore structures, it is necessary to determine more accurately wave and structure responses. The present study examined effectiveness of critical load on the structure in geometric respects and location of piles to each other within a structure. Results from current study make it possible to optimize designs, decrease wave forces to the structure, control displacements and increase security levels of marine structures.

**Objectives:** Design loads of marine structures are not as easy as shore structures; on the other hand, human experience in off-shore structures dates back to recent century. Due to extensive unknown parameters involved in it and as human knowledge develops, regulations related to this problem are reviewed in shorter time periods.

Profiles and piles in different forms are largely used to build coastal structures; while, in off-shore structures, most applications are largely assigned to cylindrical forms. The present study evaluates those forces exerted from waves to vertical piles in cylindrical, circular and square forms with similar cross-section in different arrangements. Each capacity was considered in both separate and mixed conditions. The mentioned problem was not examined in this way. Although many various theories have been developed on wave forces to cylindrical piles, the problem was not evaluated as mentioned above. The present study evaluates effectiveness of critical load in geometric aspects and location of piles to each other.

According to little experience of human in building off-shore structures and extensiveness of parameters involved in forces exerted on marine structures, still little is known about problems such as how arrangement of piles influences on ultimate loading capacity; or, what is the response of the structure to vortexes behind the piles, in direction of wave and current and perpendicular to it.

Combinations of various arrangements in circular and square sectioned piles with similar cross section will be clearly followed by interesting results for optimizing marine structures and many economic benefits.

The present study evaluates those forces exerted from waves to vertical piles in cylindrical, circular and square forms with similar cross-section. Each capacity was considered in both separate and mixed conditions.

Considering heavy costs of building, transportation, installation and maintenance of marine structures, to select proper sections and adequate realization of structure response against wave forces play an important role in decrease costs. According to knowledge on structure forces, concrete sections in square or rectangular forms and box-shaped profiles are largely used to build coastal structures; while, off-shore structures are mostly built by circular-sectioned cylindrical forms. The present study evaluates those forces exerted from waves to circular and square sectioned cylindrically vertical piles with similar cross-section. Each capacity was considered in both separate and mixed conditions. The mentioned problem was not examined in this way. Although many various experimental theories have been developed on wave forces to cylindrical piles, the problem was not evaluated as mentioned above. The present study evaluates effectiveness of critical load in geometric aspects and resultant of forces exerted on whole structure. The study determined critical loading capacity of piles and predicated the most

optimal condition to establish these structures. It also compared loading capacity of square and circular sectioned cylindrical piles with similar cross-section and obtained results will optimize marine structures and decrease building costs and will be followed by many economic and security benefits to build marine structures.

The following objectives were considered to conduct the study:

Effectiveness of loading capacity in geometric aspects: i. Compare side loading capacity of square and circular sectioned vertically cylindrical piles with similar cross-sections; ii. Select optimal sections to design marine structures; iii. Decrease building costs; iv. Increase structural resistance; v. Increase security level; vi. Decrease wave forces to structure.

### Estimation of Wave Force to Marine Structures

Among models suggested so far estimating wave forces, Morison's equation reasonably determines amount of wave force adjacent to cylindrical structures. Let the ratio of structural element sizes to wave length  $< 0.2$  and exclude structural vibration. Morison formula can be reasonably used to estimate hydrodynamic forces resulted from waves.

Assuming periodical waves and absence of marine currents, Morison et al.<sup>1</sup> developed a model to obtain fluid horizontal force of non-broken waves on vertical cylindrical pile from linear combination of two inertial and drag components, as follows:

$$F = f_D + f_i = 0.5 \rho C_D D |U|U + \rho C_M A \dot{U} \quad (1)$$

where,  $F$  is the force exerted on vertical cylinder length;  $U$  and  $\dot{U}$  are horizontal components of velocity and acceleration of water particles, respectively.  $C_D$  and  $C_M$  are coefficients of drag and inertia, respectively;  $\rho$  is mass density of water and  $D$  represents cylinder diameter and  $A = \frac{\pi}{4} D^2$ .

### Discussion

To use Morison's model requires assuming piles are thin; thus, dispersion or turbulence of waves can be withdrawn. Let inertia dominate the environment and structural member be small in size. Wave kinematics will not be greatly influenced by member properties. Under this condition, Freud- Kriolov theory is applicable. Through this technique, wave pressure adjacent to the member is considered to determine forces exerted on structural surface. When sizes of a structural member is considerable compared to wave length, the structure causes large changes in adjacent wave field and dispersion of waves will considerably contribute to determine wave force. Under these conditions, the structure is less influenced by loads from viscosity; but it is highly sensitive to wave dispersion adjacent to structural member. For such loading, inertia influences on the structure and dispersion theory will dominate. The current study

assumed that piles are thin. Wave forces exerted on length of circular and square sectioned cylindrical piles and resultant of these forces are calculated by integrating on submerged length of cylinder as well as moments of wave force by static relations and finally tensions are controlled for each section of cylindrical piles.

The current study evaluates results obtained for all water depths assuming that water is deep. The results were supported by software MATLAB. Here, only one example is covered, as follows:

**Hypotheses:** Depth of the water where wave flows is 100m, wave height is 8m, wave length is 99.84m and frequency time is 8s. Characteristics of circular-sectioned cylinder (external diameter: 120cm, internal diameter: 108cm, wall thickness: 6cm) and square-sectioned cylinder (external sides: 96×96cm, internal sides: 84×84, wall thickness: 6cm) and drag and inertia coefficients were selected according to Table 1 (Carl-A-Torsen) which is highly consistent with Persian Gulf condition.

The program by which software MATLAB examined the problem assumed water depth ( $d$ ) and wave period ( $T$ ) constant in each run; wave movement was considered two dimensional; waves are propagated in a constant form; the liquid is assumed incompressible; effects of viscosity, turbulence and surface tension are excluded and wave height is considered less than wave length; on the other word,  $H/L \ll 1$ .

The current study simulated changes in force to water height for different depths, changes in force to different phases and tensions in both sections by software MATLAB; related computations are given only for a certain depth ( $d=100$ ); but shapes and variables are parametric.

**Table -1**  
**Drag and inertia coefficients for cylindrical piles with circular and square cross section**

Pile shape	Drag coefficient $C_D$	Inertia coefficient $C_M$
Cylinder	1	2
Cube with a square cross-section	2	2.19

Those waves with period  $< 30$ s are short period waves. Let wave propagate in the water in uniform depth  $d$  so that axis  $x$  be in propagation direction,  $y$  be vertical to propagation direction and  $z$  be upward. Viscosity of the liquid is often discarded due to its low effect on wave movement.  $\phi$  is velocity potential of wave movement and  $U$  and  $w$  represent velocity components in  $x$  and  $z$  axis, respectively.

$$\partial^2 \phi / \partial x^2 + \partial^2 \phi / \partial y^2 = 0 \quad (2)$$

The equation is called as Laplace's equation.

The equations are related to boundary conditions in non-linear free surface; thus, analysis of wave problem as stated above results in a non-linear problem. The present study will use hypotheses of the problem and wave height which is considered very small compared to its length to make it linear<sup>7</sup>.

$$\partial^2 \phi / \partial x^2 + \partial^2 \phi / \partial z^2 = 0 \quad (3)$$

Given that  $a$  is acceleration;  $H$  is wave amplitude;  $k = \frac{2\pi}{L}$  is wave number;  $\sigma = \frac{2\pi}{T}$  represents angular frequency;  $T$  is wave period and  $\frac{\sigma}{k}$  wave velocity; we have:

For circular-sectioned cylindrical pile:

$$U = \frac{\partial \phi}{\partial x} = \frac{\pi H}{T} \frac{\cosh k(d+z)}{\sinh kd} \sin \left( kx - \frac{2\pi}{T} t \right) \quad (4)$$

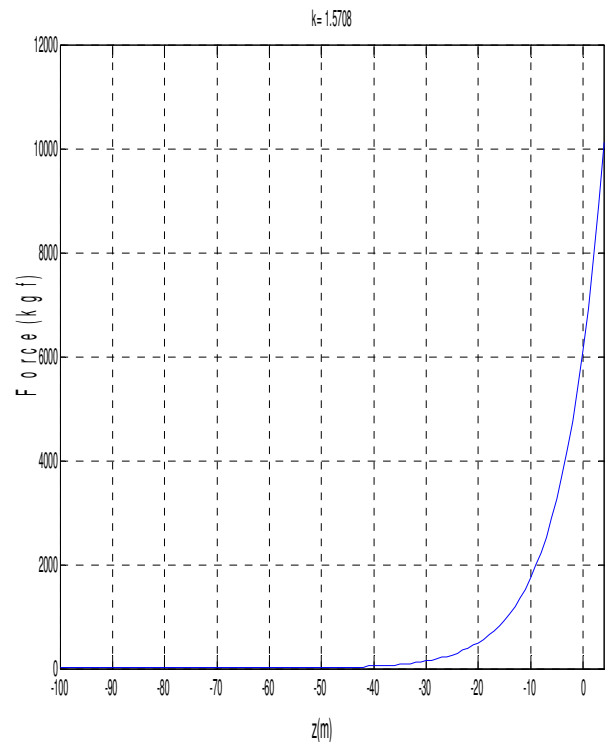
$$W = \frac{\partial \phi}{\partial z} = \frac{\pi H}{T} \frac{\cosh k(d+z)}{\sinh kd} \cos \left( kx - \frac{2\pi}{T} t \right) \quad (5)$$

$$a = \frac{dw}{dt} = -\frac{2\pi^2 H}{T^2} \frac{\cosh k(d+z)}{\sinh kd} \cos \left( kx - \frac{2\pi}{T} t \right) \quad (6)$$

$$C_M = 2, C_D = 1, D = 1.2 \quad (7)$$

$$K = \frac{2\pi}{T} x - \frac{2\pi}{T} t \quad (8)$$

$$F = 618 U |U| + 2329.86 \times a \quad (9)$$



**Figure-1**  
**Force changes in length of circular-sectioned cylindrical pile for phase  $k=\pi/2$**

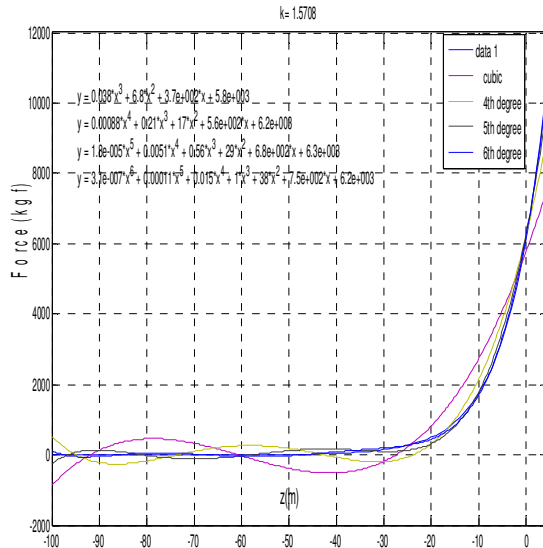


Figure-2

Corresponds to the force of the waves Cylinder circular cross section with a mathematical equation

With Equivalent of the (figure 1) with (figure 2) through one of Integration in active form (2) and equation by equation (10) on the submerged length of the column, the total force is Vardbrpayh.

$$F_t = \int_{-100}^4 F \cdot dZ \quad (10)$$

Let  $\bar{z}$ , effect center of fore resultant to pile, be assumed from water level. To calculate  $\bar{z}$  we use:

$$\bar{z} = \frac{\int Z \cdot F \cdot dZ}{\int F \cdot dZ} \quad (11)$$

$$M_A = \frac{P \cdot a \cdot b^2}{I \cdot a^2} \quad (12)$$

$$M_B = \frac{P \cdot b \cdot a^2}{I \cdot b^2} \quad (13)$$

$$I = t \cdot \pi r^3 \quad (14)$$

$$f = \frac{M \cdot y}{I} \quad (15)$$

Table -2

Place of forces, stresses and moments generated circular cross section of the wave of the phase  $k = \pi/2$

Tension- (t/m <sup>2</sup> )	(t - m)M <sub>B</sub>	(t - m)M <sub>A</sub>	Z Deepsea- (m)	(m)	K
2706.26	133.96	24.09	100	11.85	$\pi/2$

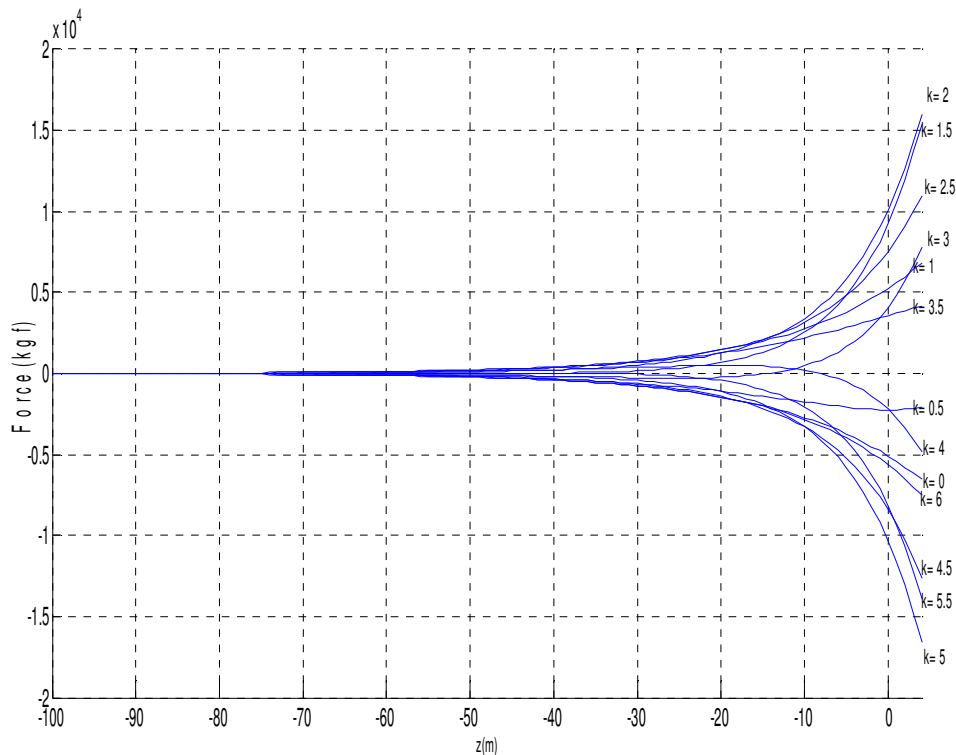


Figure-3

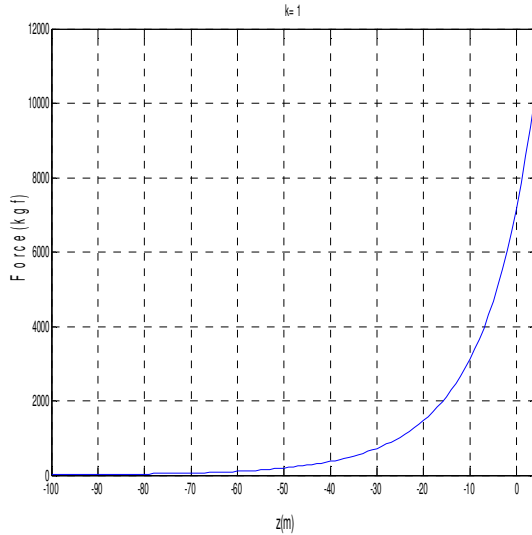
Force changes in length of circular-sectioned cylinder for a certain depth in different phases

**Square-sectioned cylindrical pile:**

$$F = f_D + f_i = 0.5 \rho c_D D |U|U + \rho c_M \frac{\pi D^2}{4} \dot{U} \quad (16)$$

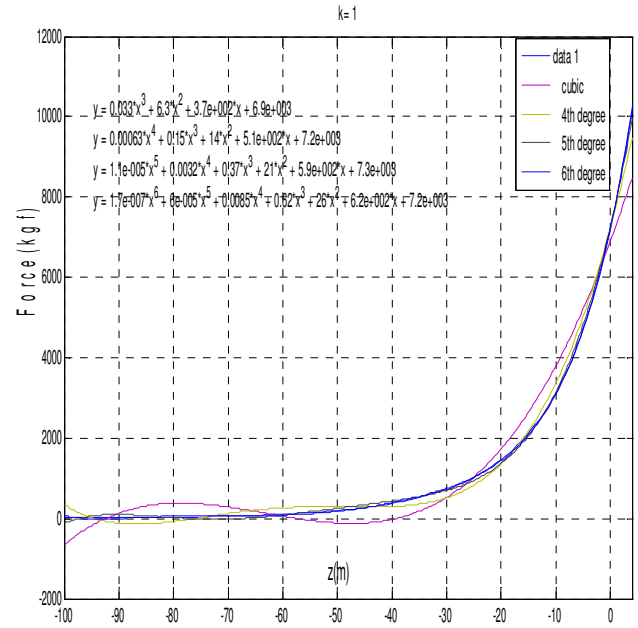
$$c_{D=2}, c_M = 2.19 \cdot D = 0.96m \quad (17)$$

$$F = 988.8U \cdot |U| + 2078.8531 \cdot a \quad (18)$$



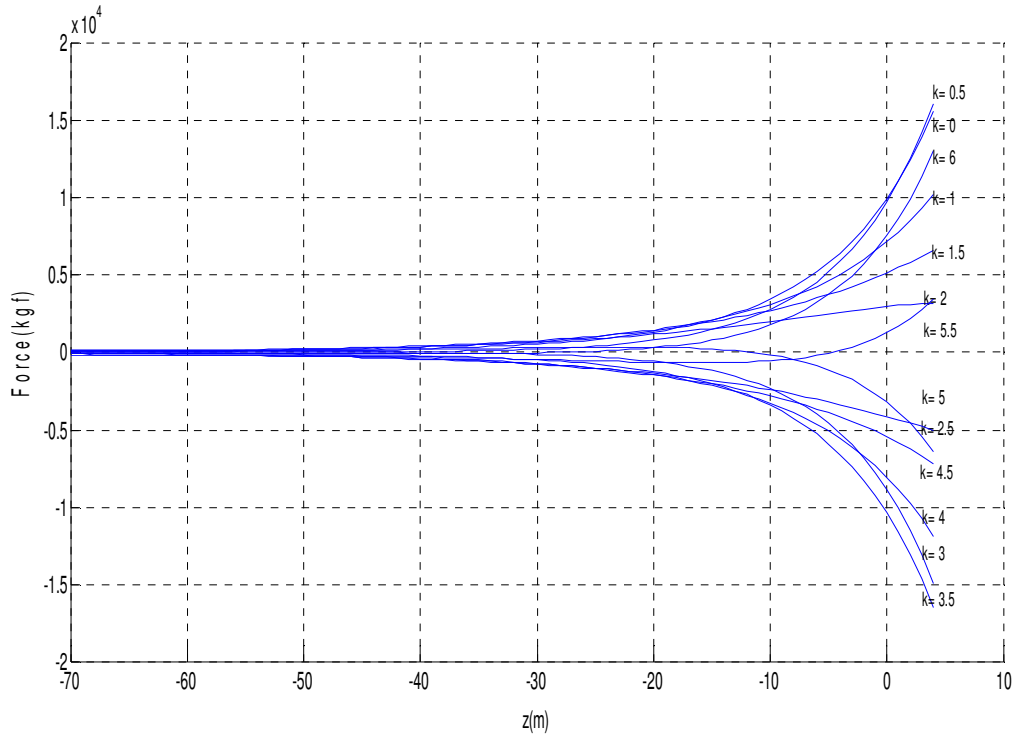
**Figure-4**

**Force changes in length of square-sectioned cylindrical pile for phase k=1**



**Figure-5**

**force changes in length of square-sectioned cylindrical pile for phase k=1.5708**



**Figure-6**

**Force changes in length of square-sectioned cylinder for a certain depth in different phases**

With Equivalent of the figure 4 with figure 5 through one of Integration in active form (5) and equation by equation (19) on the submerged length of the column, the total force is Vardbrpayh.

$$F_t = \int_{-100}^4 F \cdot dZ \quad (19)$$

$$\bar{z} = \frac{\int Z \cdot F \cdot dZ}{\int F \cdot dZ} \quad (20)$$

$$M_A = \frac{F \cdot a \cdot b^2}{I^2} \quad (21)$$

$$M_B = \frac{F \cdot a \cdot b^2}{I^2} \quad (22)$$

$$f = \frac{M_y}{I} \quad (23)$$

**Table -3**

**Place of forces, stresses and moments generated by the force of the waves square section**

Tensio (n-t/m2)	(t - m)M <sub>B</sub>	(t - m)M <sub>A</sub>	(Deepsea-)Z (m)	(m)	K
1621.20	119.53	49.21	100	11.85	$\pi/2$

As table (2) and table (3) show, tension of cylinder is more than that of cube.

### Conclusion

Wave radiation may be the most critical phase ( $k = \pi / 2$ ), which is designed to control the amount of tension generated at Cylinder with Section square tables (2) and (3), 67% Less From the same circle. Maximum and minimum values of wave forces on cylindrical pillars with circular and square cross sections are not identical, so that happen at Cylinder the maximal sometimes circular, sometimes square Cylinder. With regard to the second point, using a combination of cylindrical pillars with circular and square cross sections and reduce the resultant forces on offshore structures appropriate to the whole structure is waves. Using cylindrical columns with square cross the resultant less force due to waves, reducing the structural deviation from the vertical is placement is one of the controlling factors in the designs of offshore structures. Results 1 to 4 of the rows to

optimized signs, reduce wave forces on structures, shifts and increase safety levels are better controlled, the pillars of marine structures composed of cylindrical pillars with circular and square selection.

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