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Cash Management in Production Planning: An Approach for Iranian Industries

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Abstract

This paper presents a financial production planning model for Iranian industries and similar production environments. Iranian production environment entails a rather unique combination of financial aspects such as high inflation/interest rate, payments by installments and cash shortage problems. In Iranian markets, payments via installments are very common and usually buyers can choose among the available temporal patterns of payment. To incorporate this phenomenon in the production planning model, the concept of financial exchange patterns is defined. By combining these patterns with cash flows, availability of loans, time value of money and classic elements of production planning, a suitable model for Iranian industries is developed. The advantage of this model over the classic model is illustrated via extensive numerical experiments which are based on real data and reasonable scenarios.

Keywords: production planning, cash flows, loan, inflation, installments, interest.

Introduction

Production planning is an extensively reviewed and practical problem[1]. The issue that is not adequately addressed in production planning is the existence of financial exchanges and their potential effect on the production plan. Roughly expressed, a financial exchange is a process that takes place between two parties (buyer and seller). During this process, specific items (goods or services) are traded in exchange for a predefined amount of money. Financial exchanges are an inherent part of production activities. Selling products, purchasing recourses and materials, and paying the costs are examples of financial exchanges in production activities.

Financial exchanges are not always carried out in cash. In fact, in production environments like that of Iran, most of these exchanges are on credit and follow some selectable patterns of payment that results in a cash flow stream.

“When expenditures and receipts are denominated in cash, the net receipts at any time period are termed cash flow, and the series of flows over several periods is termed a cash flow stream” [2]. Assume that the cash flows are inserted in the mathematical model of production planning; if the inflation rate and bank interest rates are high, one must consider the time value of money within the model.

Along with the cash flow structure, one might have to consider the cash shortage problem. In the realm of financial engineering, this problem is recognized as the asset-liability matching, or more generally, the asset-liability management (ALM). Asset-liability matching forces the net cash flows to be positive throughout all time periods. In financial engineering, cash inflows are known as assets or more accurately current assets i.e. incomes. Similarly, cash outflows are recognized as current liabilities i.e. costs. Hence, asset-liability matching forces the coverage of liabilities by assets. If assets do not cover liabilities, the production manager may be forced to consider external financing (borrowing). In this case, a new decision variable is introduced to the production planning model (the amount of borrowing). Furthermore, if assets surpass liabilities, the manager may consider investing the surplus. Incorporating financial facets to production planning has already been addressed in a few papers. Jiaoa et al. use the option theory to incorporate a pricing model in to the flexible manufacturing systems planning[3]. Lusa et al. propose a mixed integer linear model that includes production quantities, selling price, cash management, hiring and firing and outsourcing[4]. Kirca and Koksalan put forward some examples and demonstrate that production decisions are affected by the financial state. The authors develop a linear multi-product financial production planning model[5,6]. Satir proposes a production and financial planning model using mathematical modeling and statistical methods[7].

An extension to the model of Holt, Modigliani, Muth, and Simon, namely HMMS[8], is developed by Damon and Schramm[9]. Their model combines marketing and finance decisions and cash flows. Baker and Damon develop a linear model that includes production decisions and cash flows[10]. Pizzolato makes some slight changes to Baker and Damon's model in order to make it consistent with accounting principles[11].

Yi and Reklaitis formulate a financial production problem by a special flow network structure (batch-storage network)[12]. The model is continuous and optimizes flow rates of cash flows. Integration of production planning, investment and distribution for a multinational corporation has also been considered[13,14]. Moreover, some papers have focused on the problems solving techniques[15]. Financial aspects have been addressed in the realm of supply chain management as well[16,17].

The model proposed in this paper is linear, discrete and based on cash flows. However, the distinguishing facets of the current work are as follows: i. It uses the concept of financial exchange patterns. This concept is specifically defined for Iranian markets. ii. Asset-liability matching is one of the constraints of our model. Using the concept of cash safety stock, the constraint can handle some levels of uncertainty. iii. Our model combines external financing from banks and surplus investing in banks and creates a new entity called financing pattern. If solved, the model will yield the optimal financing pattern i.e. when and how much to borrow and invest. iv. Like most of the production planning models, our model is NP-hard. We have put forward some theorems so as to provide some guiding tools for solution methods to search for the near optimal solution.

To sum up, we aim to introduce five financial elements into the production planning model: cash flow structure, time value of money, financial exchange patterns, asset-liability constraint, and financing patterns.

Material and Methods

Production environment in Iran: The production environment in Iran is unique due to having a combination of three rather special characteristics: vast employment of financial exchange patterns by markets, high interest/inflation rates and the significance of asset-liability issue. There may be other environments similar to that of Iran. However, the concept of financial exchange patterns is, to the extent of our knowledge, unprecedented. Hence, we expect the combination of the three characteristics to create a somewhat matchless environment.

To incorporate the aforementioned characteristics into the production planning model, we use the five financial elements mentioned before. First, let us describe the three characteristics of the Iranian production environment.

Financial exchange patterns: how markets work in Iran: Every financial exchange includes a predefined amount of money traded between the parties. Since the money is not always traded in cash, the result is a cash flow stream. A financial exchange pattern, or FEP, can roughly be defined as the pattern according to which the predefined money of a financial exchange is converted to a cash flow stream. The most obvious FEP is the cash pattern. It can be defined as “the buyer paying the price in cash to the seller”. Another hypothetical FEP can be “the buyer paying 60 percent of the price in cash and making two payments for the two following months bearing 10% of interest”. In conclusion, exercising an FEP leads to a cash flow stream.

For a production plan to be realistic, it is necessary to consider the patterns, according to which money is exchanged in the respective markets. However, FEPs are informally accepted and used by members of Iranian markets. This informality makes it difficult to investigate their characteristics. Nonetheless, the following characteristics can be expressed about these FEPs: i. Each FEP has a credit period which is the duration from the delivery of the item to the due date of the last cash flow. ii. Available FEPs tend to have less credit periods (approach the cash pattern) in periods near to national feasts like Norooz or religious months like Ramadan. iii. Each FEP has a unit net present value which is the present value of the cash flow stream resulting from trading one unit of the item under that FEP. iv. The available FEPs may vary from item to item, market to market and even from period to period. v. In a competitive market, there are several FEPs available for paying costs (payment patterns) and buyers (payers) can choose between them. vi. In a competitive market, FEPs for receiving incomes (sale patterns) depend on the behavior of customers, the time period that the trade takes place, and the market. Hence, they are not usually selectable by sellers. vii. Buyers prefer FEPs with less unit net present value as long as their asset-liability status allows. viii. Markets in Iran widely use non-cash FEPs. This can be an indication for the seriousness of asset-liability issue. ix. FEPs may include prepayments.

Loans and asset-liability matching; a challenge for Iranian industries: As a policy to empower the private sector, production centers in Iran are extensively supported by governmental facilities (such as loans). Table 1 contains some data published by Iranian Central Bank regarding the facilities extended by two famous Iranian banks to manufacturing/mining and agricultural firms from 1999 to 2008[17]. Value of supports has been increasing drastically. Hence, Iranian production centers are in a constant struggle with borrowing and paying back. Not only the struggle has a major effect on the feasibility of a production plan, but also it can be an indication for the preference of non-cash FEPs and severity of asset-liability issues.

Table-1
Facilities extended by Iranian banks to manufacturing, mining and agricultural sectors

Year*	1378	1379	1380	1381	1382	1383	1384	1385	1386	1387
Agricultural sector (billion Rials**)	4440	7039	11693	16963	22880	32201	41031	40644	44476	74494
Production and mining sector (billion Rials)	458	503	970	1825	2315	4863	6356	8525	9842	NA

*beginning of the year 1378 in Iranian calendar corresponds with 21 March 1999, **exchange rate of Iranian Rial to American Dollar reported by Iranian Central Bank is 12260[18]

High inflation and interest rates in Iran: Inflation rate is rather high in Iran. As reported by Iranian Central Bank, inflation rate for the twelve months ending on 21 December 2010 was 10.1%[19]. Inflation rate for the similar two previous periods was reported being 13.5% and 25.5%[20,21].

Interest rates of loans depend on inflation rate. Banks in Iran offer various kinds of loans with rather high interest rates. Iranian Central Bank authorizes the governmental banks' lending rates for transaction contracts to be up to 15% since 28 January 2012[22]. Private banks offer loans with interest rates as high as 20% in some cases. Depending on being short term or long term, deposit rate of Iranian banks has also been reported to be from 6% to 17% for recent years[23].

Mathematical Model: In this section, first we define the concept of financing patterns which is one of the five financial elements that we intend to introduce in our model (other elements were covered before). Then, the classic production planning is described followed by our financial production planning model. Finally the theorems are put forward.

Financing pattern: Commonly, production centers carry out their production plan until the emergence a cash-availability problem. At this point, they tend to negotiate with banks for loans. We propose a more efficient approach to solve the asset-liability unbalance.

Our approach is based on integrating financial decisions into the production planning model and proposing a financing pattern to banks based on the result of the model. The financing pattern includes two items: i. The lending cash flows from the bank to the production center, ii. The investing cash flows that the production center deposits in the bank.

The first item is the commitment of the bank and includes the loans and their delivery timetable to the production centre. The second item is the commitment of the production centre and includes the values that the production centre deposits in its account in each period. If the two parts of the financing pattern are combined, a cash flow stream consisting of transactions between the bank and the production centre during the planning horizon is created. The financing pattern is obtained from the integrated financial production planning model and, if accepted by a bank, solves the asset-liability problem of the production centre. Financing patterns have the following advantages: i. As the financing pattern includes production centre's commitments, it is easier to negotiate for more loans, less interest rates and more deposit rates. ii. Based on the financing pattern, the production centre receives its loan in accordance with a timetable that is compatible with its asset-liability needs. If the loan is delivered in one or two lumps, it can be available prior to the time it is actually needed. Consequently, an extra interest cost is inevitable. iii. As the money deposited by the production centre does not create any asset-liability problems, the financing pattern makes efficient use of the surplus cash.

The classic production planning model: Before the mathematical formulation of proposed model is introduced, the classic production planning model is described. Additionally, for the sake of simplicity, the classic model and proposed model are both single-item and do not explicitly include facets like, backlog, labor costs and multiple resource constraints. For any particular real world case, one or some of these facets can be included in the model and this will neither introduce any fundamental change to our discussions nor considerably increase the complexity of the problem (it is already NP-hard). We are using the following notations:

c_t : unit variable cost of production in period t, $t=1,2,..,T$

d_t : demand of period t, $t=1,2,..,T$

h_t : unit inventory cost of carrying over the product from period t to $t+1$, $t=1,2,..,T-1$

b_t : maximum production capacity in period t, $t=1,2,..,T$

a_t : setup cost in period t, $t=1,2,..,T$

I_0 : initial inventory

x_t: amount of production in period t (decision variable), $t=1,2,..,T$

I_t: amount of inventory that is carried over to $t+1$ (decision variable), $t=1,2,..,T-1$

The model can be formulated as:

$$\min \sum_{t=1}^{T}\left(c_t x_t + a_t \delta(x_t)\right) + \sum_{t=1}^{T-1} h_t I_t \qquad (1)$$

$$x_t + I_{t-1} - I_t = d_t \qquad t=1,2,..,T-1 \qquad (2)$$

$$x_T + I_{T-1} = d_T \qquad (3)$$

$$x_t \le t \qquad t=1,2,..,T \qquad (4)$$

$$x_t \ge 0 \qquad t=1,2,..,T \qquad (5)$$

$$I_t \ge 0 \qquad t=1,2,..,T-1 \qquad (6)$$

where $\delta(A)=1$ if $A \ge 0$ and $\delta(A)=0$ if $A<0$. The objective function consists of production and inventory costs. Expressions 2 and 3 are the inventory balance constraints and Expression 4 mirrors the capacity constraint. The problem is NP-hard[24].

The integrated financial production planning model: Following indices, sets and parameters are used for the financial production planning model:

$t=1,2,..,T$: periods

$M(t)$: set of available sale patterns in period t

sp_{mtk} : cash flow induced to period k if one unit of product is sold in period t under the pattern m, $m \in (t), k$ and $t=1,2,..,T$

α_{mt} : proportion of the demand of period t that, according to surveys, is satisfied by sale pattern m, $m \in M(t), t = 1,2,..,T$, $0 \le \alpha_{mt} \le 1$, $\sum_{j \in M(t)} \alpha_{jt} = 1$

$N(t)$: set of available patterns in period t for variable production costs

vp_{mtk} : cash flow induced to period k if one unit of product is produced in period t and its variable costs is paid under pattern n, $n \in N(t), k \text{ and } t = 1,2,..,T$

$O(t)$: set of available patterns in period t for fixed production costs (setup costs)

fp_{mtk} : cash flow induced to period k if the production center is set up for production in period t and the corresponding fixed costs are paid under pattern o, $o \in O(t), k \text{ and } t = 1,2,..,T$

icf_t : initial cash flow of period t, $t = 1,2,..,T$

i_1 : lending rate of the bank i.e. interest rate of loans

i_2 : deposit rate of the bank

i_3 : inflation rate

css : cash safety stock (reserved for unpredictable situations)

l_{max} : maximum amount of money that the bank lends to a single entity (not time valued)

d_t, h_t, I_0 and b_t are similar to the classic model. Decision variables are:

x_t and I_t similar to the classic model

e_{nt} : binary variable indicating whether pattern n is adopted in period t, $n \in N(t), t = 1,2,..,T$

f_{ot} : binary variable indicating whether pattern o is adopted in period t, $o \in O(t), t = 1,2,..,T$

y_{tk} : amount of money borrowed from the bank in period t and paid back with interest in period k, $t = 1,2,..,T, k = t+1, t+2,..,T_{max}$

cf_t : net cash flow in period t, $t = 1,2,..,T$

cfc_t : amount of cash carried over from period t to $t+1$, $t = 1,2,..,T-1$

al_t : net asset-liability balance in period t before borrowing and cash carry-over, $t = 1,2,..,T_{max}$

Before the model is proposed, let us outline some issues using the above notations: i. T_{max} is the latest theoretical period that a cash flow can be induced to, ii. One can define $Y_t = \sum_{k=t+1}^{T_{max}} y_{tk}$ as the amount of total external financing from the bank in period t. Thus, Y_t s are the bank's commitment in the financing pattern. iii. cf_t s are the commitment of the production centre in the financing pattern. iv. icf_t s include exogenous liabilities or incomes that were initiated in the previous planning horizons (such as the due date of long-term debts) v. The assumption of no arbitrage requires $i_1 > i_2$. Also it is logical to assume $i_2 > i_3$.

The mathematical model can be formulated as:

$$Max\ z = FV_{t=1}^{T}(cf_t, i_2, T) + FV_{t=T+1}^{T_{max}}(al_t, i_3, T) \quad (7)$$

Subject to:

$$al_1 = \sum_{k=1}^{T} \sum_{m \in M(k)} \alpha_{mk} d_k sp_{mk1} - \sum_{k=1}^{T} \sum_{n \in N(k)} vp_{nk1} x_k e_{nk} - \sum_{k=1}^{T} \sum_{o \in O(k)} fp_{ok1} \delta(x_k) f_{ok} - h_1 I_1 + icf_1 \quad (8)$$

$$al_t = -\sum_{k=1}^{t-1} y_{kt}(1+i_1)^{t-k} + \sum_{k=1}^{T} \sum_{m \in M(k)} \alpha_{mk} d_k sp_{mkt} - \sum_{k=1}^{T} \sum_{n \in N(k)} vp_{nkt} x_k e_{nk} - \sum_{k=1}^{T} \sum_{o \in O(k)} fp_{okt} \delta(x_k) f_{ok} - h_t I_t + icf_t \qquad t = 2,3,...,T \quad (9)$$

$$al_t = -\sum_{k=1}^{T} y_{kt}(1+i_1)^{t-k} + \sum_{k=1}^{T} \sum_{m \in M(k)} \alpha_{mk} d_k sp_{mkt} - \sum_{k=1}^{T} \sum_{n \in N(k)} vp_{nkt} x_k e_{nk} - \sum_{k=1}^{T} \sum_{o \in O(k)} fp_{okt} \delta(x_k) f_{ok}$$
$$t = T+1, T+2,..,T_{max} \quad (10)$$

$$cf_1 = \sum_{k=2}^{T} y_{1k} + al_1 - cfc_1 \quad (11)$$

$$cf_t = \sum_{k=t+1}^{T_{max}} y_{tk} + al_t + cfc_{t-1} - cfc_t \quad t = 2,3,...,T$$

$$cf_t \ge css \qquad t = 1,2,3,...,T \quad (12)$$

$$\sum_{n \in N(t)} e_{nt} = 1 \qquad t = 1,2,...,T \quad (14)$$

$$\sum_{o \in O(t)} f_{ot} = 1 \qquad t = 1,2,...,T \quad (15)$$

$$\sum_{t=1}^{T} \sum_{k=t+1}^{T_{max}} y_{tk} \le l_{max} \quad (16)$$

$$x_t + I_{t-1} - I_t = d_t \qquad t = 1,2,..,T-1$$

$$x_T + I_{T-1} = d_T \quad (18)$$

$$x_t \le b_t \qquad t = 1,2,..,T \quad (19)$$

$$e_{nt} \in \{0,1\} \quad n \in N(t) \quad t = 1,2,..,T \tag{20}$$
$$f_{ot} \in \{0,1\} \quad o \in O(t) \quad t = 1,2,..,T \tag{21}$$
$$y_{kt} \geq 0 \quad t = 1,2,..,T \quad k = t+1, t+2,..,T_{max} \tag{22}$$
$$cfc_t \geq 0 \quad t = 1,2,..,T-1 \tag{23}$$
$$I_t \ \& \ x_t \geq 0 \quad t = 1,2,..,T \tag{24}$$

$FV_{t=t_1}^{t_2}(CF_t, i, t')$ is the future value of cash flow stream $CF_{t_1}, CF_{t_1+1}, CF_{t_1+2} ..., CF_{t_2}$ under discount rate i at the time t'. The first part of expression 7 is the future value of all the cash flows which can be deposited in the bank. The second part includes the future value of the cash flow stream beyond the planning horizon which is compounded by inflation rate (because no investment decision is made within the scope of this model for this stream). Expressions 8 to 10 are the definition of al_t. Expressions 11 and 12 are the definition of cf_t. Expression 13, the asset-liability matching constraints, states that there should always be a minimum amount of cash available in order to deal with unpredictable events. Expressions 14 to 15 state that only one pattern can be adopted for paying the costs in each period. Expression 16 is the regulation based loan limit. Expressions 17 to 19 are similar to the classic model. The rest of the expressions are domain definitions.

Characteristics of the model: By relaxing some assumptions, the aforementioned model can be converted to the classic production planning which is NP-hard. Therefore, heuristic techniques are required to solve the model. Here, we put forward some theorems that can be the basis of those heuristic methods. For not prolonging the matter, the proofs are not given in a detailed mathematical language. However, concise outline of the proofs are given and the detailed proofs are available to interested readers.

Theorem 1. For any optimal solution and $\forall t = 1,2,..,T-1$, $Y_t \times cfc_t = 0$.

Proof. Theorem 1 states that the money borrowed in a period cannot be carried over and must be spent during that period. It can be proved by contradiction. Assume for an optimal solution there exists at least one period, say z, so that $Y_z \times cfc_z \neq 0$. Let us define A as the amount of cash borrowed in period z and carried over to period $z+1$. If $Y_z \geq cfc_z$ then $A = cfc_z$ and if $Y_z < cfc_z$ then $A = Y_z$. Now, it is easy to verify that if Y_z is modified to $Y_z - A$, cfc_z changes to $cfc_z - A$ and Y_{z+1} is modified to $Y_{z+1} + A$, a new solution is obtained for which the objective function improves and none of the constraints are violated. Hence, the original solution is not optimal. Clearly, Theorem 1 holds for the new solution.

Theorem 2. For every optimal solution and $\forall t = 1,2,..,T$, if $al_t \leq css$ then $cf_t = css$.

Proof. Theorem 2 can also be proved by contradiction. Assume for an optimal solution there exists at least one period, say z, so that $al_z \leq css$ and $cf_z > css$. Let us define $B = cf_z - css > 0$. B can be interpreted as the optimal value of money deposited in period z in addition to css. As $al_z \leq css$, Expression 12 states that B can either be funded via borrowing in period z or carrying over cash from period $z-1$. However, neither of these two can happen. No cash can be borrowed in one period and deposited in the same period since the assumption of no arbitrage entails this act to be non-optimal. Also, no cash can be carried over to and deposited in a period since depositing in the previous period is more profitable. In this way, $B = 0$ and consequently cf_z cannot be greater than css.

Theorem 3. For every optimal solution and $\forall t = 1,2,..,T$, if $al_t \geq css$ then $Y_t = 0$.

Proof. If $cfc_t > 0$ then $Y_t = 0$ (Theorem 1). If $cfc_t = 0$, since $al_t \geq css$, if money is borrowed in period t it should be deposited in period t as well. However, as mentioned before, no cash can be borrowed in one period and deposited in the same period because of the assumption of no arbitrage.

Results and Discussion

This section is divided to two parts. Firstly, test problems are derived from the production environment of Iran in order to evaluate our model. Part 2 is devoted to interpreting the results of test problems.

Details of test problems: In order to have valid test problems for the evaluation of the proposed model, we use two approaches. Firstly, we try to use real data as much as possible to picture the real production environment of Iran. Secondly, since real data may not be available due to innovative elements of the model, we use reasonable scenarios. Below are the parameter settings of our test problems according to the aforementioned approaches. Unless mentioned otherwise, all prices are reported in 10 million Iranian Rials.

Rate scenarios: Three rate scenarios are assumed. In scenario 1 $i_1 = 0.15$, $i_2 = 0.12$, $i_3 = 0.1$. In scenario 2 $i_1 = 0.2$, $i_2 = 0.15$, $i_3 = 0.12$. Scenario 3 is the extreme situation in which $i_1 = 0.22$, $i_2 = 0.18$, $i_3 = 0.16$.

Data Regarding the Production Centre: Table 2 contains some real data from an automotive parts manufacturer which was reported for the year beginning on 21 March 2009. A full year comprising of 4 planning periods (seasons) is considered which complies with the fiscal year of Iranian institutions and the seasonal behavior of Iranian automotive parts manufacturers.

Table-2
General test problem parameters

Period	1	2	3	4
d	325	837.5	562.5	1100
h	1	0.5	1	0.75
b	1800	1800	1800	2700
icf	3000	-3000	-4200	-7500

Scenarios regarding FEPs: As mentioned before, there has not been any research on FEPs of Iranian markets. Tables 3 and 4 contain our best estimate on FEPs in automotive parts market. These data are based on some interviews with marketing experts of automotive parts. The detailed data of the interviews and statistical analysis are not mentioned but available to interested readers. i. Provided that one unit of product is sold (produced) in the arbitrary period *t*, Table 3 (4) contains the data of the sale (payment) patterns. It is obvious that $T_{max}=6$. ii. In order to give the FEPs more generality, some scenarios have been considered. Sale patterns either exist in the market (1) or not (0). If they do not exist, sales are conducted in cash. If they exist, Table 3 is considered.

Three scenarios are considered for payment patterns. In scenario 1, patterns are not available and all the expenses are paid in cash. In scenario 2, however, there is limited availability of FEPs and the first three patterns of Table 4 are considered for variable and fixed costs. Yet, the third scenario entails full availability of patterns.

Table-3
Sale patterns data assuming that one unit of product is sold in period t

Pattern (*m*)	$(sp_{mtt}, sp_{m,t,t+1}, sp_{m,t,t+2})$	$(\alpha_{m1}, \alpha_{m2}, \alpha_{m3}, \alpha_{m4})$
1	(100,0,0)	(10,15,20,45)
2	(60,43.2,0)	(40,40,40,30)
3	(30,35,35)	(50,45,40,25)

Scenarios regarding other parameters: Apart from the previously described parameter settings, we have considered the following scenarios: i. The cash flow structure and time value of money are either included in the model or not. If not, the borrowing possibility and asset-liability matching should also be excluded which results in the classic production planning. ii. *css* can be -∞, 0, or 1500. If *css*=-∞, Expression 13 states that the asset-liabilty matching is excluded. Moreover, there will be no sense in borrowing (l_{max} is irrelevant). *css*=-∞ can be an indication that the production centre does not have any cash availability problems. Higher values of *css* indicate higher uncertainty level in the environment (which forces the production centre to use greater cash buffer) iii. l_{max} can be 0,15000, ∞.

Table-4
Payment patterns assuming that one unit of product is produced in period t

Pattern(*n*)	$(vp_{ntt}, vp_{n,t,t+1}, vp_{n,t,t+2})$	$(fp_{ntt}, fp_{n,t,t+1}, fp_{n,t,t+2})$
1	(90,0,0)	(20000,0,0)
2	(63, 28.35,0)	(14000, 6420,0)
3	(45, 24.75, 27.225)	(10000, 5600, 6272)
4	(31.5, 36.225, 35.708)	(7000, 8190, 8213.4)
5	(18, 43.2, 51.84)	(4000, 9760, 11907.2)

Test problems: Combining the aforementioned scenarios and data, 112 test problems are created. The result is depicted in Tables 5 and 6. Table 5 contains the information regarding the test problems that are, in some manner, related to classic production planning. In particular, in Test Problem 1 cash flow structure and time value of money are excluded. Hence, other financial elements cannot be applied as well. Thus, the resultant model is the classic production planning. Noticeably, while values of icf_t s are included in Test Problem 1, they have no effect on the optimal production plan since they are fixed values added to the objective function. In Test problems 2, 3 and 4 time value of money is included but other financial elements are excluded. The only difference between these test problems lies in the rate scenario. Table 6 contains the rest of the test problems.

Table-5
Detailed description of test problems 1 through 4

Cash flow structure and time value of money	*css*	l_{max}	FEP scenario	rate scenario	TP*
Not included	not applicable	not applicable	not applied	not applicable	1
Included	-∞ (no ALM)			1	2
				2	3
				3	4

* Test Problem Number

Table-6
Detailed information of test problems 5 through 112

rate scenario 1					rate scenario 2					rate scenario 3				
css	l_{max}	Ssc*	Psc**	TP***	*css*	l_{max}	Ssc	Psc	TP	*css*	l_{max}	Ssc	Psc	TP
0	0	0	1	5	0	0	1	1	41	0	0	0	1	77
			2	6				2	42				2	78
			3	7				3	43				3	79
		1	1	8			2	1	44			1	1	80
			2	9				2	45				2	81
			3	10				3	46				3	82
	15000	0	1	11		15000	0	1	47		15000	0	1	83
			2	12				2	48				2	84
			3	13				3	49				3	85
		1	1	14			1	1	50			1	1	86
			2	15				2	51				2	87
			3	16				3	52				3	88
	∞	0	1	17		∞	0	1	53		∞	0	1	89
			2	18				2	54				2	90
			3	19				3	55				3	91
		1	1	20			1	1	56			1	1	92
			2	21				2	57				2	93
			3	22				3	58				3	94
1500	0	0	1	23	1500	0	0	1	59	1500	0	0	1	95
			2	24				2	60				2	96
			3	25				3	61				3	97
		1	1	26			1	1	62			1	1	98
			2	27				2	63				2	99
			3	28				3	64				3	100
	15000	0	1	29		15000	0	1	65		15000	0	1	101
			2	30				2	66				2	102
			3	31				3	67				3	103
		1	1	32			1	1	68			1	1	104
			2	33				2	69				2	105
			3	34				3	70				3	106
	∞	0	1	35		∞	0	1	71		∞	0	1	107
			2	36				2	72				2	108
			3	37				3	73				3	109
		1	1	38			1	1	74			1	1	110
			2	39				2	75				2	111
			3	40				3	76				3	112

*sale pattern scenario, ** payment pattern scenario, *** test problem

Results of test problems, comparisons and interpretations: We have solved the test problems with Lingo on an Intel core™2 computer that operates at 1.83 GHz. Detailed optimal values of production quantities (x_t's) and total external financings (Y_t's) are reported in table 7. Existence of unfeasible test problems reveals that failing to see the real (financial) facets of the production environment may result in unexpected unfeasibility. Following is a more detailed analysis of table 7.

Table-7
Detailed results of test problems

TP	$x_1/x_2/x_3/x_4/Y/z$	TP	$x_1/x_2/x_3/x_4/Y/z$	TP	$x_1/x_2/x_3/x_4/Y/z$
1	1725/0/0/1100/-/31369	39	1163/0/1663/0/109/9490	76	325/1400/0/1100/32.1/7606
2	1163/0/1663/0/-/12514	40	1163/0/1663/0/93.2/11585	77	Not feasible
3	1163/0/1663/0/-/1021	41	1163/0/1663/0/195.6/2442	78	325/838/563/1100/0/-2498
4	325/1400/0/1100/-/-7951	42	325/838/563/1100/0/-4480	79	325/1400/0/1100/0/18028
5	Not feasible	43	325/1400/0/1100/0/12851	80	Not feasible
6	325/838/563/1100/0/-5873	44	Not feasible	81	Not feasible
7	325/1400/0/1100/0/10120	45	Not feasible	82	325/838/563/1100/0/-4732
8	Not feasible	46	325/838/563/1100/0/-5924	83	325/838/563/1100/15/-14134
9	Not feasible	47	325/838/563/1100/15/-13669	84	325/1400/0/1100/14.2/11131
10	325/838/562/1100/0/-6093	48	325/1400/0/1100/14.2/10123	85	325/1400/0/1100/0/18028
11	325/838/563/1100/15/-11646	49	325/1400/0/1100/0/12851	86	Not feasible
12	325/1400/0/1100/14.2/10126	50	Not feasible	87	Not feasible
13	325/1400/0/1100/14.2/10126	51	Not feasible	88	325/1400/0/1100/10/4617
14	Not feasible	52	325/1400/0/1100/7.5/4231	89	325/1400/0/1100/56/-3202
15	Not feasible	53	325/1400/0/1100/69.3/-1407	90	325/1400/0/1100/22.7/11434
16	325/1400/0/1100/13.8/5555	54	325/1400/0/1100/14.2/10123	91	325/1400/0/1100/0/18028
17	1163/0/1663/0/190.9/2866	55	325/1400/0/1100/0/12851	92	325/1400/0/1100/121.6/-20733
18	1163/0/1663/0/102.6/12374	56	325/1400/0/1100/119.7/-14805	93	325/1400/0/1100/87.9/-2461
19	1163/0/1663/0/102.6/12374	57	325/1400/0/1100/67.4/1396	94	325/1400/0/1100/26.4/7961
20	1163/0/1663/0/214.5/-46	58	325/1400/0/1100/26.4/8309	95	Not feasible
21	1163/0/1663/0/106/10152	59	Not feasible	96	325/838/563/1100/0/-3540
22	1163/0/1663/0/86.8/12062	60	325/838/563/1100/0/-5219	97	325/1400/0/1100/0/18028
23	Not feasible	61	325/1400/0/1100/0/12851	98	Not feasible
24	325/838/563/1100/0/-6334	62	Not feasible	99	Not feasible
25	325/1400/0/1100/0/10120	63	Not feasible	100	325/838/563/1100/0/-7282
26	Not feasible	64	325/838/563/1100/0/-9087	101	Not feasible
27	Not feasible	65	Not feasible	102	325/838/563/1100/11.4/-659
28	325/838/562/1100/0/-10201	66	325/838/563/1100/11.4/-3190	103	325/1400/0/1100/0/18028
29	Not feasible	67	325/1400/0/1100/0/12851	104	Not feasible
30	325/838/563/1100/11.4/-4769	68	Not feasible	105	Not feasible
31	325/1400/0/1100/0/10120	69	Not feasible	106	325/1400/0/1100/10.5/4153
32	Not feasible	70	325/1400/0/1100/11.2/3669	107	325/1400/0/1100/73/-4094
33	Not feasible	71	325/1400/0/1100/78.9/-2544	108	325/1400/0/1100/22.1/11230
34	325/1400/0/1100/14.5/4638	72	325/1400/0/1100/15.7/9879	109	325/1400/0/1100/0/18028
35	1163/0/1663/0/195.6/2442	73	325/1400/0/1100/0/12851	110	325/1400/0/1100/129.1/-21625
36	1163/0/1663/0/105.6/12272	74	325/1400/0/1100/127.3/-15942	111	325/1400/0/1100/97.8/-3353
37	1163/0/1663/0/105.6/12272	75	325/1400/0/1100/83.6/259	112	325/1400/0/1100/29.4/7497
38	1163/0/1663/0/197.2/-708		nil		nil

*all the values are rounded and values of Y are reported in 10^{10} Iranian Rials

Comparisons between the classic and financial production planning: The production centre, on which we based our test problems (Table 2), uses the classic production planning approach. Thus, the actual production plan adopted in the year 2009 is expectedly the same as Test Problem 1: $x_1=1725$, $x_2=x_3=0$ *and* $x_4= 1100$.

Before comparing classic and financial production planning, let us address an important question; does considering time value of money affect the optimal solution? According to the result of the first four test problems, the answer is positive. If rate scenario 1 is considered, the optimal x_t's change from 1725/0/0/1100 to 1163/0/1663/0. In fact, time value of money can have drastic effects as one can observe that altering between rate scenarios 1 and 2 changes the objective function from a 12514 profit to a 7951 loss.

To compare classic and financial production planning, we use an example and then generalize the result. Assume the real situation of production environment entails rate scenario 2, full availability of FEPs, unlimited borrowing and no uncertainty (css=0). This situation depicts Test Problem 58. If the production manager fails to see these characteristics and uses the classic model (Test Problem 1), he will compute optimal values of production plan as 1725/0/0/1100 and estimate to make a 31369 profit (table 7). In reality, however, he is opposed to asset-liability mismatch and FEPs; he is forced to borrow; and time value of money forcedly affects him. If he uses his production plan, one can compute that he will borrow 141453 and the actual objective function will be - 8763. If he had considered the financial aspects of his production environment, he would choose a different production plan and his objective function would be 8309 (Test problem 58). This shows a 205% loss. If the production manager had at least considered time value of money (Test Problem 3), the same computations show that he would lose only 169%.

Similar computations for the feasible test problems of table 6 show that, if no financial aspects are considered, 65% of the test problems become infeasible (i.e. the results of classic model cannot be used in reality). For the remaining test problems, an average 144% of loss is observed. If at least time value of money is considered (rate scenario 1), 51% of the test problems become infeasible and an average 92% of loss is observed for the remaining test problems.

The effect of asset-liability matching on the results: If $css=-\infty$, we are in the realm of classic production planning (table 5). Table 7 shows that, other things equal, changing the value of *css* from 0 to 1500 declines the average value of the objective function by 66.1%. This can be interpreted as the cost of uncertainty or instability in the production environment. Test problems 11, 41, 47 and 83 become infeasible if *css* is changed from 0 to 1500.

The effect of borrowing on the results: From the view point of maximum external support (l_{max}), test problems of Table 6 can be categorized to four groups. Since each test problem reflects a production environment, we use the word "environment" for this categorization: i. Unsustainable environment: regardless of the limit of external support, a production centre cannot survive (cannot generate a feasible production plan) in this environment. ii. Little sustainable environment: only if a production centre is unlimitedly supported ($l_{max}=\infty$), it may survive in this environment. iii. Sustainable environment: for a production centre to survive in this environment, a limited amount of external support ($l_{max}=15000$) is necessary. iv. Profitable environment: regardless of the limit of external support, a production centre survives in this environment.

Considering the above description, table 6 contains 36 production environments: 18 cases of profitable environment, 3 cases of sustainable environment, 15 cases of little sustainability and no case for unsustainable environment.

Existence of no case for unsustainable environment expresses that if a production plan is accomplishable from the technical (not financial) view point, it is feasible (not necessarily profitable) provided that enough financial support is present. Consider Test Problem 105 as an example. With a 15000 external support, this test problem is infeasible. However, if the external support is raised to 97800, it becomes feasible (Test Problem 111). The 18 cases of little sustainable and sustainable environment confirm the fact that availability of more external support can alter the infeasibility status.

Now, let us consider profitable environment. According to table 7, if l_{max} changes from 0 to 15000, the average objective function for this environment rises from 596.4 to 7312.6 (1126%). If unlimited borrowing is allowed, the average objective function rises to 11596.5 (59%). Thus, the effect of external borrowing on the profitability is enormous.

The effect of rate scenario on the results: According to table 7, changing the rate scenario never alters feasibility status. No strong dependency between profitability and rate scenario is also observed.

The effect of FEPs on the results: Table 7 shows that, other things equal, payment pattern scenario 2 never yields less profit than scenario 1 and payment pattern scenario 3 never yields less profit than scenario 2. Consider Test Problem 53 as an example. If the payment pattern scenario changes from 1 to 2, the optimal profit rises from -1408.6 to 10122.8 (Test Problem 54). If the payment pattern scenario changes to 3, the optimal profit rises to 12851 (Test Problem 55). It can roughly be said that payment pattern scenarios with more non-cash flexibility yield more profit.

Table 7 also shows that, other things equal, sale pattern scenario 1 never yields more profit that scenario 0. Thus, one can roughly say that sale pattern scenarios which entail more non-cash exchanges yield less profit.

Conclusion

This paper proposed a financial production planning model for discrete production activities. The financial facets of the model were derived from the real production environment of Iran. However, the model is applicable in any environment which entails high inflation/interest/deposit rates, significance of asset-liability problems and non-cash financial exchanges. Reasonable test problems were designed that justified the need for this model. Under the assumptions considered for Iranian production environment and for the test problems designed, the following conclusions can be drawn: i. Unrealistic assumptions of classic production planning can lead to either production plans that cannot be adopted in reality or considerable losses gained. ii. If uncertainty or instability increases in a production environment, more cash must be kept in reserve which can lead to either infeasibility of production plan or loss. iii. Considering time value of money can introduce changes to optimal production plan. iv. Changes in the inflation/interest/deposit rate do not alter feasibility status. They also do not necessarily cause profit or loss. v. If a production planning model is feasible, raising the maximum limit of external support makes it much more profitable. If it is not feasible, large enough external support makes it feasible. vi. Roughly expressed, sale pattern scenarios entailing more non-cash exchanges yield less profit and payment pattern scenarios with more non-cash flexibility yield more profit. vii. In the proposed model, uncertainty of financial behavior of the production environment was handled by means of cash safety stock. However, future studies can be focused on more precise approaches based on the duration and convexity of cash flow streams[2]. Moreover, asset-liability management for continuous production planning and scheduling can be of value.

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