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Calculating Free and Forced Vibrations of multi-story Shear Buildings by Modular method

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Abstract

The present study discretizes and calculates free and forced vibrations of a multi-story building by shear model. The present study shows that for free vibrations a dynamic system becomes a static system; as a result, static theories of linear structures can be used. Then, frequencies and modular forms are calculated. By contribution of Betti's corresponding displacement theorem, orthogonal characteristic of modules is characterized. For forced vibrations, employment of module converts multi-degrees free engaged dynamic system to several independent single-degree free dynamic systems in each of which mass, spring stiffness and forced force for new system are linear combinations of mass, spring stiffness and forced force of engaged system. For example, frequencies, modular forms and forced displacement from transverse load to story such as hurricane force and acceleration imposed on building foundation, say earthquake, are manually calculated and compared by simulating with commercial-research software package, ANSYS. Results from ANSYS for forced vibrations are more accurate that results from approximate method of SAP 2000.

Keywords: Forced Vibrations, Multi-Story building, shear model, ANSYS, SAP

Introduction

Building structural has numerous degrees of freedom due to their connection. Some analytic methods to study dynamics of structures are based on connected modelling which solves partial differential equations. Assuming linear systems and for regular geometries and properties of homogenous materials, these differential equations can be solved manually in a closed form. For irregular geometries, properties of heterogeneous materials and composition of various building materials and non-linear conditions, it is not possible to use these closed analytic solutions. The solution is to discretize and linearize connected systems and to employ numerical methods and commercial-research software packages for discretized systems.

A common method to find free and forced solution of discretized dynamic system for structures is module and finding modular normal matrix[1-4]. A simple model for discretization of multi-story buildings is shear model[1-4] in which total mass of structure is focused on floor level of each story. Stiffness of ceiling grids is assumed considerably more than stiffness of supporting columns. Effect of tensile forces within columns is supposed negligible on deformation of structure and changes in mass of columns compared to mass of ceilings. For free vibrations, module converts dynamic system to a static system[1-4]. For forced vibrations, module converts multi-degrees free engaged dynamic system to several independent single-degree free dynamic systems in each of which mass, spring stiffness and forced force for new system are linear combinations of mass, spring stiffness and forced force of engaged system[1-4].

Discretizing Multi-Story Building by Shear Model and Finding Stiffness Equations: Shear building model is achieved by following assumptions (figure 1)[5]. i. Total mass of the structure is focused on floor level of each story; ii. Stiffness of ceiling grids are considerably more than stiffness of supporting columns; iii. Effect of tensile forces within columns is negligible on deformation of structure; iv. Mass of columns is negligible compared to mass of ceilings; v. Damping effects of structures were ignored; because they are ineffective on module.

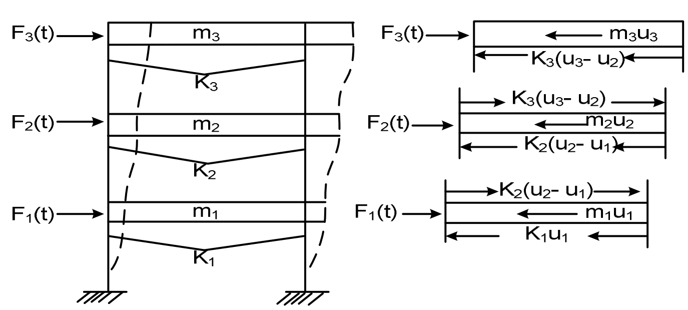

Figure-1
Shear model of multi-story building and free diagram of each story[1]

Spring constant, k_j, is a kind of force which should impose on top of the column between ceilings i and i-1 for relative displacement. For two-end-involved columns and one-end-involved and one-end-hinged columns, spring constant, k, is calculated from following equations:

$$k = \frac{12EI}{L^3}, \quad \frac{3EI}{L^3} \tag{1}$$

Where, E is elastic modulus of column; I represents cross sectional inertial moment of the column; L is height of the column.

Shear model of a three-story building is as follows:

$$\begin{cases} m_1\ddot{u}_1 + k_1u_1 - k_2(u_2 - u_1)(u_2 - u_1) = F_1(t) \\ m_2\ddot{u}_2 + k_2(u_2 - u_1) - k_3(u_3 - u_2) = F_2(t) \\ m_3\ddot{u}_3 + k_3(u_3 - u_2) = F_3(t) \end{cases} \tag{2}$$

Equation system (1) can be written in the form of following matrix:

$$[M]\{\ddot{u}\} + [K]\{u\} = \{F\} \tag{3}$$

where, mass and stiffness matrices are as follows:

$$[M] = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}, \ [K] = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} \tag{4}$$

Displacement, acceleration and force vectors are defined as follows:

$$\{u\} = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}, \ \{\ddot{u}\} = \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \end{Bmatrix}, \ \{F\} = \begin{Bmatrix} F_1(t) \\ F_2(t) \\ F_3(t) \end{Bmatrix} \tag{5}$$

Stiffness coefficient, k_{ij}, is essential force for coordinate i which displaces coordinate j one unit provided that other coordinates are constant.

Natural Frequencies and Normal modular Forms for Multi-Story Shear Buildings

Free vibrations of multi-story shear buildings are as follows:

$$[M]\{\ddot{u}\} + [K]\{u\} = \{0\} \tag{6}$$

Harmonic solutions of system (6) are as follows:

$$\{u\} = \{a\}\sin(\omega t - \alpha) = \{0\} \tag{7}$$

where, $\{a\}$is amplitude vector. By substituting (7) in (6) and simplifying and arranging terms, we have:

$$[[K] - \omega^2[M]]\{a\} = \{0\} \tag{8}$$

where, determinant of coefficient matrix is assumed zero to calculate natural frequencies.

$$\|[K] - \omega^2[M]\| = 0 \tag{9}$$

Equation (9) is in the form of an-degree polynomial in terms of $\omega^2$for ann-degree free system called as system characteristic polynomial equation. Roots of a characteristic polynomial gives n value for ω^2. For each of these roots, amplitude vector, $\{a\}$ is obtained from (8) in terms of a given constant. A numeral example helps better understanding of calculations.

For a two-story shear building with steel structures and given sizes and weights, calculate natural frequencies and modular forms and write equations of motion for each story. The building has been formed from a row of frames located in 15ft from each other (figure 2 and figure 3).

Weight and concentrated mass of each story are calculated as follows:

$$W_1 = 100 * 30 + 15 * 20 + 20 * 12.5 + 15 * 2 = 52{,}500 \text{ lbf} \tag{10}$$

$$m_1 = 136 \ \frac{\text{lbf.s}^2}{\text{in}} \tag{11}$$

$$W_2 = 50 * 30 + 15 * 20 + 20 * 5 + 15 * 2 = 25{,}500 \text{ lbf} \tag{12}$$

$$m_2 = 66 \ \frac{\text{lbf.s}^2}{\text{in}} \tag{13}$$

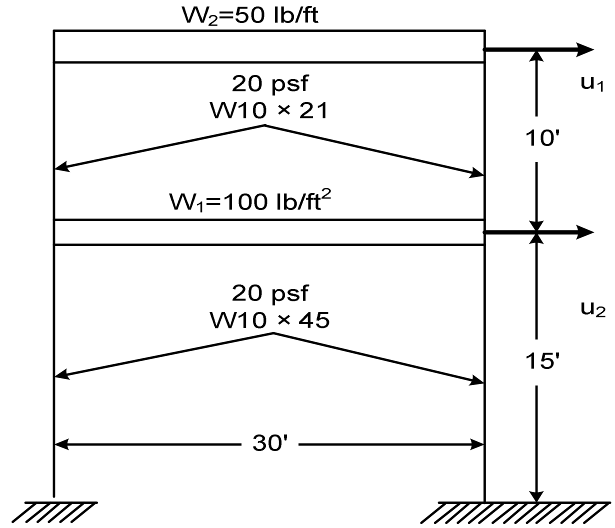

Figure-2
A two-story building for shear modelling[1]

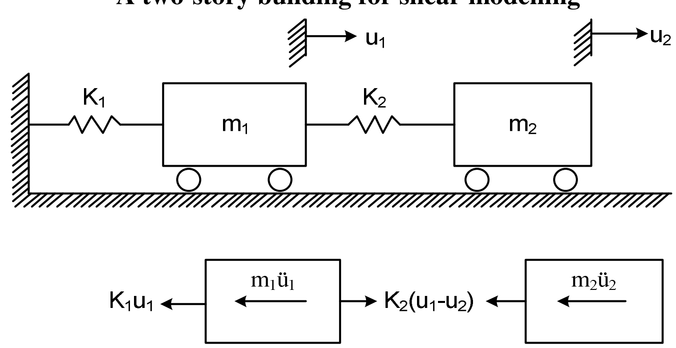

Figure-3
Discretization of a two-story building with mass and spring by shear model to calculate natural frequencies and modular forms[1]

Since ceiling grids are two-end-involved, spring constant is calculated for each story as follows:

$$k = 2\left(\frac{12EI}{L^3}\right) \tag{14}$$

$$k_1 = 2\left[\frac{12 * 30e6 * 248}{(15 * 12)^3}\right] = 30{,}700 \ \frac{\text{lbf}}{\text{in}} \tag{15}$$

$$k_2 = 2\left[\frac{12*30e6*118}{(10*12)^3}\right] = 44{,}300\ \frac{\text{lbf}}{\text{in}} \tag{16}$$

Equations of free vibrations are as follows:

$$\begin{cases} m_1\ddot{u}_1 + k_1 u_1 - k_2(u_2 - u_1) = 0 \\ m_2\ddot{u}_2 + k_2(u_2 - u_1) = 0 \end{cases} \tag{17}$$

Harmonic solution of the system is as follows:

$$\begin{cases} u_1 = a_1 \sin(\omega t - \alpha) \\ u_2 = a_2 \sin(\omega t - \alpha) \end{cases} \tag{18}$$

By substituting in equations of motion and simplifying, we have:

$$\begin{bmatrix} k_1 + k_2 - m_1\omega^2 & -k_2 \\ -k_2 & k_2 - m_1\omega^2 \end{bmatrix}\begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{19}$$

Determinants of coefficients become zero to obtain characteristic polynomial.

$$\begin{vmatrix} k_1 + k_2 - m_1\omega^2 & -k_2 \\ -k_2 & k_2 - m_1\omega^2 \end{vmatrix} = 0 \tag{20}$$

$$m_1 m_2 \omega^4 - [m_1 k_2 + (k_1 - k_2) m_2]\omega^2 + k_1 k_2 = 0 \tag{21}$$

By substituting numeral sizes, following is obtained:

$$8976\ \omega^4 - 10{,}974{,}800\ \omega^2 + 1.36e6 = 0 \tag{22}$$

$$\omega_1^2 = 140,\quad \omega_2^2 = 1082 \tag{23}$$

$$\omega_1 = 11.83,\quad \omega_2 = 32.89\ \text{rad/s} \tag{24}$$

$$f_1 = \frac{\omega_1}{2\pi} = 1.88\ \text{cyc/s},\quad f_2 = \frac{\omega_2}{2\pi} = 5.24\ \text{cyc/s} \tag{25}$$

By substituting ω_1 and ω_2 in (19), modular forms are obtained. We plotted ratio of amplitudes. Real amplitudes are obtained from initial conditions (figure 4).

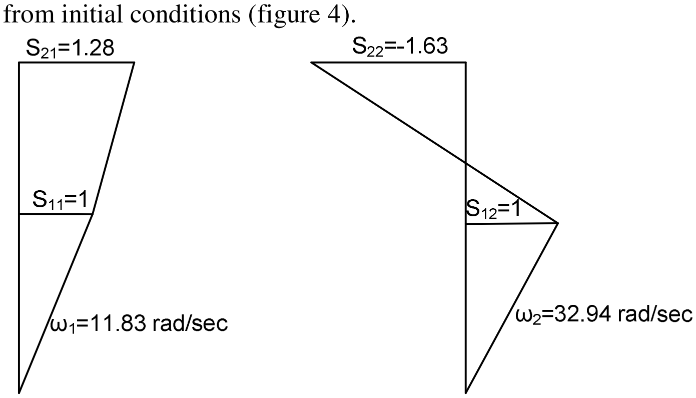

Figure-4
Modular forms for shear two-story buildings[1]

System solution is obtained from summing effects of modular forms.

$$\begin{cases} u_1 = c_1 a_{11} \sin(\omega_1 t - \alpha_1) + c_2 a_{12} \sin(\omega_2 t - \alpha_2) \\ u_2 = c_1 a_{21} \sin(\omega_1 t - \alpha_1) + c_2 a_{22} \sin(\omega_2 t - \alpha_2) \end{cases} \tag{26}$$

Four integrating constants, $c_1, c_2, \alpha_1, \alpha_2$, are obtained from four initial conditions of displacement and velocity on u_1, u_2 (figure 5).

The outputs of ANSYS software are as follows:
Natural frequencies for shear two-story building by ANSYS 12.0 is as follows
Index Of DATA SETS ON Results File

Set	Time/FREQ	Load step	SubStep	Cumulative
1	1.8827	1	1	1
2	5.2370	1	2	3

First normal modular form for shear two-story building by ANSYS 12.0 is as follows:
Load step= 1 Substep=1
FREQ=1.8827 Load Case=0
The following Degree of Freedom results are in the global coordinate system
Node UX
1 0
2 0.64369E-01
3 0.81324E-01
Maximum absolute values
Node 3
Value 0.81324E-01

The second normal modular form for shear two-story buildings by ANSYS 12.0 is as follows:
Load step= 1 Substep=2
FREQ=5.2370 Load Case=0
The following Degree of Freedom results are in the global coordinate system
Node UX
1 0
2 -0.56653E-01
3 0.92401E-01
Maximum absolute values
Node 3
Value 0.92401E-01

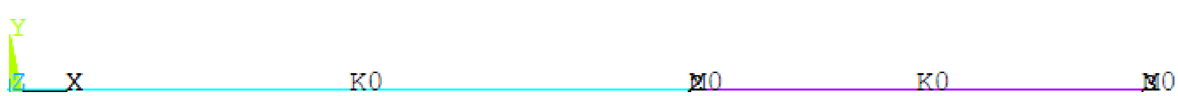

Figure-5
Simulating a shear two-story building by elements Combin14 and Mass21 using ANSYS 12.0

Orthogonal Characteristic of Modular Forms

Summing effects by combining modules is based on orthogonal characteristic of modular forms. Equations of free vibrations in an n-story shear building can be obtained as follows:

$$[[K] - \omega^2[M]]\{a\} = \{0\} \quad (27)$$

For a given two-story buildings, the equations are as follows:

$$\begin{cases} (k_1 + k_2)a_1 - k_2 a_2 = \omega^2 m_1 a_1 \\ -k_2 a_1 + k_2 a_2 = \omega^2 m_2 a_2 \end{cases} \quad (28)$$

Obviously, dynamic system has converted to a static system in which external forces, $\omega^2 m_1 a_1$ and $\omega^2 m_2 a_2$ have influenced on masses $m_1$and m_2. Modular forms are static deformation for a degree of freedom influenced by a static force. Since a dynamic system has changed to a static system, static theories of linear structures can be used by Betti's corresponding displacement theorem. Consider two loading systems, 1 and 2, and corresponding displacements, 1 and 2, on a structure. According to Betti's theorem, the work done by loading system 1 through displacements 2 is equal to the work done by loading system 2 through displacements 1.

Load System 1, Forces: $\omega_1^2 m_1 a_{11}$, $\omega_1^2 m_2 a_{21}$, Displacements: a_{11}, a_{12} (29)
Load System 2, Forces: $\omega_2^2 m_1 a_{12}$, $\omega_2^2 m_2 a_{22}$, Displacements: a_{12}, a_{22} (30)

Betti's theorem is used as follows for these two systems:

$$(\omega_1^2 m_1 a_{11})(a_{12}) + (\omega_1^2 m_2 a_{21})(a_{22}) = (\omega_2^2 m_1 a_{12})(a_{11}) + (\omega_2^2 m_2 a_{22})(a_{21}) \quad (31)$$

By simplifying, we have:

$$(\omega_1^2 - \omega_2^2)(m_1 a_{11} a_{12} + m_2 a_{21} a_{22}) = 0 \quad (32)$$

Since $\omega_1^2 - \omega_2^2 \neq 0$, we have:

$$m_1 a_{11} a_{12} + m_2 a_{21} a_{22} = 0 \quad (33)$$

Equation (33) indicates orthogonal characteristic of modular forms expressed in a matrix form:

$$\langle a_{11} \quad a_{21} \rangle \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} a_{12} \\ a_{22} \end{Bmatrix} = 0 \quad (34)$$

For an n-degree free system, orthogonal characteristic of modular forms can be expressed between two degrees of freedom, iand j, as follows:

$$\langle a \rangle_i [M] \{a\}_j = 0, \; i \neq j \quad (35)$$

Modular forms can be expressed in a normal form:

$$\phi_{ij} = \frac{a_{ij}}{\sqrt{\langle a \rangle_i [M] \{a\}_j}} \quad (36)$$

where, ϕ_{ij} is normalized element of i from modular vector, j. If mass matrix is diagonal, orthogonal characteristic of modular forms and normalization of eigenvectors will be as follows:

$$\sum_{k=1}^{n} m_k a_{ki} a_{kj} = 0, \; i \neq j \quad (37)$$

$$\phi_{ij} = \frac{a_{ij}}{\sqrt{\sum_{k=1}^{n} m_k a_{kj}}} \quad (38)$$

For normal eigenvectors, orthogonal characteristic will be as follows:

$$\langle \phi \rangle_i [M] \{\phi\}_j = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases} \quad (39)$$

Normal modular matrix is obtained by arranging normal modular vectors in a column.

$$[\Phi] = \begin{bmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1n} \\ \phi_{21} & \phi_{22} & \cdots & \phi_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{n1} & \phi_{n2} & \cdots & \phi_{nn} \end{bmatrix} \quad (40)$$

Orthogonal characteristic in terms of modular matrix is as follows:

$$[\Phi]^T [M] [\Phi] = [I] \quad (41)$$

Various numeral methods including Jacobi, Rayleigh Quotient and Langzoshave been proposed to calculate eigenvalues and eigenvectors of free vibrations suitable for multi-story buildings by shear model. Some methods have been incorporated in commercial-research software packages including ANSYS and SAP2000 (figures 6-13).

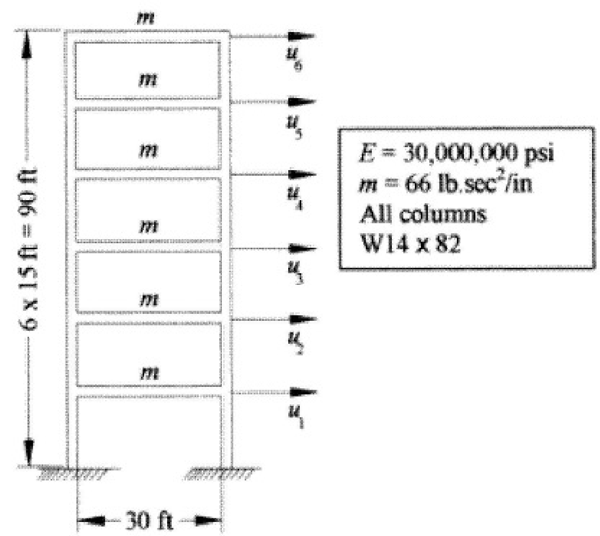

Figure-6
A six-story building for shear modelling[1]

Natural frequencies for a six-story shear building by SAP 2000 is shown in table 1.

Normalized modular matrix for a six-story shear building by SAP 2000 is as follows:

$$[\Phi] = \begin{bmatrix} 0.016 & 0.045 & 0.064 & 0.068 & 0.056 & 0.032 \\ 0.032 & 0.068 & 0.045 & -0.016 & -0.64 & -0.056 \\ 0.045 & 0.056 & -0.032 & -0.064 & 0.016 & 0.068 \\ 0.056 & 0.016 & 0.068 & 0.032 & 0.045 & -0.064 \\ 0.064 & -0.032 & -0.016 & 0.056 & -0.068 & 0.045 \\ 0.068 & -0.064 & 0.056 & 0.056 & 0.032 & -0.016 \end{bmatrix}$$

Table-1
Modal periods and natural frequencies

Mode	Period (Sec)	Frequency (cps)	Frequency (rad/sec)	Eigenvalue $(rad/sec)^{**2}$
1	0.684	1.463	9.191	84.474
2	0.245	4.085	25.667	658.802
3	0.162	6.158	38.693	1497.140
4	0.129	7.752	48.707	2372.376
5	0.112	8.911	55.988	3134.665
6	0.104	9.622	60.459	3655.307

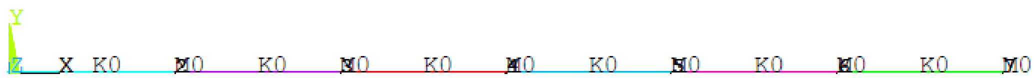

Figure-7
Simulating a two-story shear building with elements Combin14 and Mass21 by ANSYS 12.0

```
 LOAD STEP=     1  SUBSTEP=     1
  FREQ=    1.4633      LOAD CASE=   0

 THE FOLLOWING DEGREE OF FREEDOM RESULTS ARE IN THE GLOBAL COORDINATE SYSTEM

   NODE       UX
      1   0.0000
      2  0.16340E-01
      3  0.31731E-01
      4  0.45277E-01
      5  0.56192E-01
      6  0.63842E-01
      7  0.67781E-01

 MAXIMUM ABSOLUTE VALUES
 NODE          7
 VALUE   0.67781E-01
```

Figure-8
First normal modular form for a six-story shear building by ANSYS 12.0

```
 LOAD STEP=     1  SUBSTEP=     2
  FREQ=    4.3049      LOAD CASE=   0

 THE FOLLOWING DEGREE OF FREEDOM RESULTS ARE IN THE GLOBAL COORDINATE SYSTEM

   NODE       UX
      1   0.0000
      2  0.45277E-01
      3  0.67781E-01
      4  0.56192E-01
      5  0.16340E-01
      6 -0.31731E-01
      7 -0.63842E-01

 MAXIMUM ABSOLUTE VALUES
 NODE          3
 VALUE   0.67781E-01
```

Figure-9
Second normal modular form for a six-story shear building by ANSYS 12.0

```
 LOAD STEP=     1  SUBSTEP=     3
  FREQ=    6.8962      LOAD CASE=   0

 THE FOLLOWING DEGREE OF FREEDOM RESULTS ARE IN THE GLOBAL COORDINATE SYSTEM

   NODE       UX
      1   0.0000
      2 -0.63842E-01
      3 -0.45277E-01
      4  0.31731E-01
      5  0.67781E-01
      6  0.16340E-01
      7 -0.56192E-01

 MAXIMUM ABSOLUTE VALUES
 NODE          5
 VALUE   0.67781E-01
```

Figure-10
Third normal modular form for six-story shear building by ANSYS 12.0

```
 ***** POST1 NODAL DEGREE OF FREEDOM LISTING *****

 LOAD STEP=     1  SUBSTEP=     4
  FREQ=    9.0868      LOAD CASE=   0

 THE FOLLOWING DEGREE OF FREEDOM RESULTS ARE IN THE GLOBAL COORDINATE SYSTEM

   NODE       UX
      1   0.0000
      2  0.67781E-01
      3 -0.16340E-01
      4 -0.63842E-01
      5  0.31731E-01
      6  0.56192E-01
      7 -0.45277E-01

 MAXIMUM ABSOLUTE VALUES
 NODE          2
 VALUE   0.67781E-01
```

Figure-11
Fourth normal modular form for a six-story shear building by ANSYS 12.0

```
LOAD STEP=     1  SUBSTEP=     5
 FREQ=    10.749      LOAD CASE=   0

THE FOLLOWING DEGREE OF FREEDOM RESULTS ARE IN THE GLOBAL COORDINATE SYSTEM

  NODE      UX
     1   0.0000
     2 -0.56192E-01
     3  0.63842E-01
     4 -0.16340E-01
     5 -0.45277E-01
     6  0.67781E-01
     7 -0.31731E-01

MAXIMUM ABSOLUTE VALUES
NODE          6
VALUE   0.67781E-01
```

Figure-12
Fifth normal modular form for a six-story shear building by ANSYS 12.0

```
LOAD STEP=     1  SUBSTEP=     6
 FREQ=    11.787      LOAD CASE=   0

THE FOLLOWING DEGREE OF FREEDOM RESULTS ARE IN THE GLOBAL COORDINATE SYSTEM

  NODE      UX
     1   0.0000
     2  0.31731E-01
     3 -0.56192E-01
     4  0.67781E-01
     5 -0.63842E-01
     6  0.45277E-01
     7 -0.16340E-01

MAXIMUM ABSOLUTE VALUES
NODE          4
VALUE   0.67781E-01
```

Figure-13
Sixth normal modular form for a six-story shear building by ANSYS 12.0

Forced Vibrations of Multi-Story Shear Building by Combination of modules: Free vibrations of two-story shear buildings are as follows:

$$\begin{cases} m_1\ddot{u}_1 + k_1 u_1 - k_2(u_2 - u_1) = F_1(t) \\ m_2\ddot{u}_2 + k_2(u_2 - u_1) = F_2(t) \end{cases} \quad (42)$$

We search for a conversion which converts involved system of equations to an independent system of equations. For free vibrations, solution was considered as a combination of sinusoidal functions and normal modules. For forced vibrations, general arbitrary functions are considered instead of Sinusoidal functions.

$$\begin{cases} u_1(t) = a_{11} z_1(t) + a_{12} z_2(t) \\ u_2(t) = a_{21} z_1(t) + a_{22} z_2(t) \end{cases} \quad (43)$$

By substituting (43) in (42), we have:

$$\begin{cases} m_1 a_{11}\ddot{z}_1 + (k_1 + k_2)a_{11} z_1 - k_2 a_{21} z_1 + m_1 a_{12}\ddot{z}_2 + (k_1 + k_2)a_{12} z_2 - k_2 a_{22} z_2 = F_1(t) \\ m_2 a_{21}\ddot{z}_1 - k_2 a_{11} z_1 + k_2 a_{21} z_1 + m_2 a_{22}\ddot{z}_2 - k_2 a_{12} z_2 + k_2 a_{22} z_2 = F_2(t) \end{cases} \quad (44)$$

To reach suitable functions, $z_1(t)$ and $z_2(t)$, which convert (44) to independent equations, orthogonal characteristics of modules are used. Once, first equation is multiplied by a_{11}, and the second equation is multiplied by a_{21}; then, they are summed. Once again, the first equation is multiplied by a_{12}, and the second equation is multiplied by a_{22}; then, they are summed. By simplifying, following equations are obtained:

$$\begin{cases} (m_1 a_{11}^2 + m_2 a_{21}^2)\ddot{z}_1 + \omega_1^2(m_1 a_{11}^2 + m_2 a_{21}^2)z_1 = a_{11}F_1(t) + a_{21}F_2(t) \\ (m_1 a_{12}^2 + m_2 a_{22}^2)\ddot{z}_2 + \omega_2^2(m_1 a_{12}^2 + m_2 a_{22}^2)z_2 = a_{12}F_1(t) + a_{22}F_2(t) \end{cases} \quad (45)$$

where, degrees of freedom have been separated. These equations indicate a physical characteristic that effective force in stimulating a module is equal to work done by external forces through displacements of its module. Obviously, mass, spring stiffness and external force has been obtained as a combination of mass, spring stiffness and external force for involved system and normal modular forms. Mass, spring stiffness and modular external forces are defined as follows:

$$\begin{cases} M_1 \equiv m_1 a_{11}^2 + m_2 a_{21}^2 \\ M_2 \equiv m_{12} a_{12}^2 + m_2 a_{22}^2 \\ K_1 \equiv \omega_1^2 M_1 \\ K_2 \equiv \omega_2^2 M_2 \\ P_1(t) = a_{11}F_1(t) + a_{21}F_2(t) \\ P_2(t) = a_{12}F_1(t) + a_{22}F_2(t) \end{cases} \quad (46)$$

According to definitions (46), equations (45) become as follows:

$$\begin{cases} M_1\ddot{z}_1 + K_1 z_1 = P_1(t) \\ M_2\ddot{z}_2 + K_2 z_2 = P_2(t) \end{cases} \quad (47)$$

Equations (47) are normalized as follows:

$$\begin{cases} \ddot{z}_1 + \omega_1^2 z_1 = p_1(t) \\ \ddot{z}_2 + \omega_2^2 z_2 = p_2(t) \end{cases} \quad (48)$$

Modular forces of (48) are in terms of normal modules:

$$\begin{cases} p_1(t) = \phi_{11}F_1(t) + \phi_{21}F_2(t) \\ p_2(t) = \phi_{12}F_1(t) + \phi_{22}F_2(t) \end{cases} \quad (49)$$

The above technique for independent forced vibrations for an arbitrary-story building can be extended[1] and shown in figures 14-17.

$$F_1(t) = 10{,}000\left(1 - \frac{t}{t_d}\right) lb$$

$$F_2(t) = 20{,}000\left(1 - \frac{t}{t_d}\right) lb\ ; \qquad \text{for } t \le 0.1 \text{ sec}$$

$$u_{1max} = \sqrt{(0.06437 * 9.62)^2 + (0.0567 * 1.44)^2} = 0.62 \text{ in}$$

$$u_{2max} = \sqrt{(0.08130 * 9.62)^2 + (-0.0924 * 1.44)^2} = 0.79 \text{ in}$$

$$m_1\ddot{u}_{r1} + (k_1 + k_2)u_{r1} - k_2 u_{r2} = -m_1\ddot{u}_s$$
$$m_2\ddot{u}_{r2} - k_2 u_{r1} + k_2 u_{r2} = -m_2\ddot{u}_s$$
$$\ddot{u}_s = 0.28 * 0.386 = 108.47 \text{ in/sec}^2$$

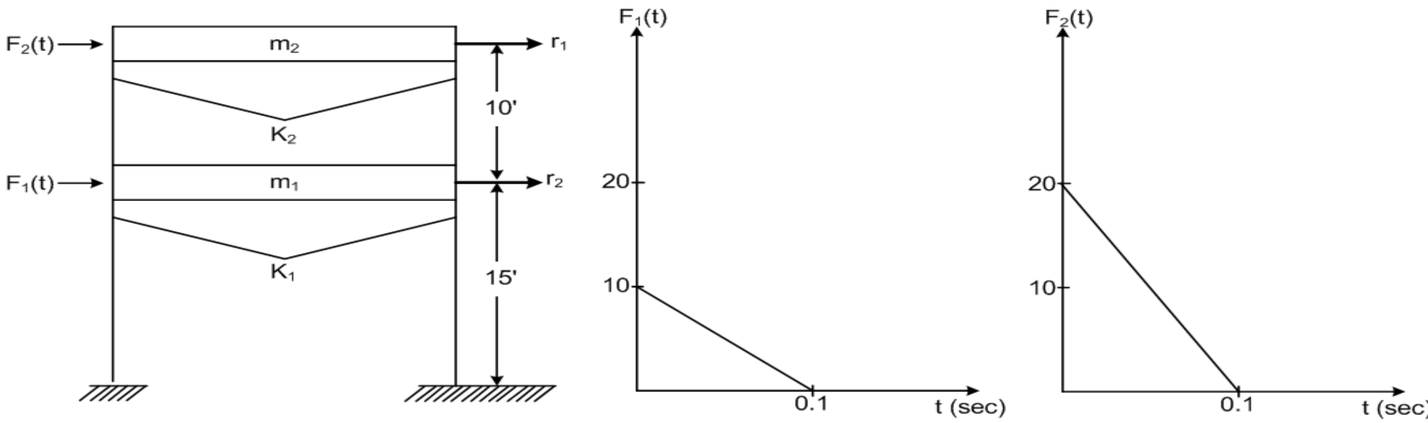

Figure-14
Forced vibrations for two-story shear building by non-impact transverse forces to a story[1]

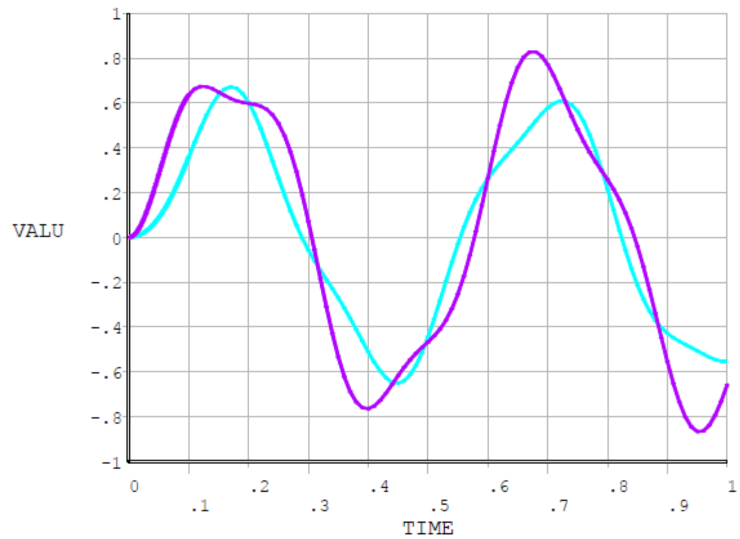

Figure-15
Displacements of a story for two-story shear building by non-impact transverse forces to a story by ANSYS 12.0

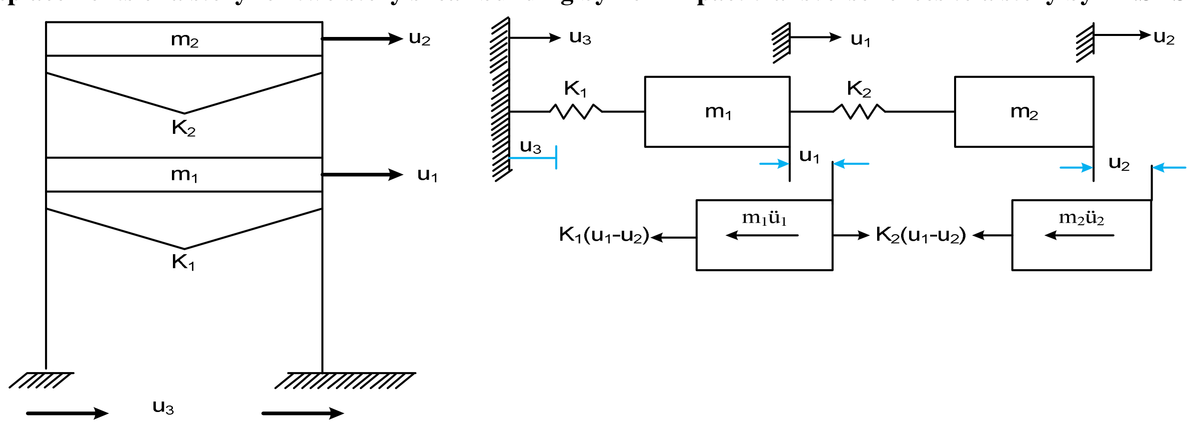

Figure- 16
vibrations of a two-story shear building by transverse earthquake acceleration to the building equal to 0.28 of gravity acceleration[1]

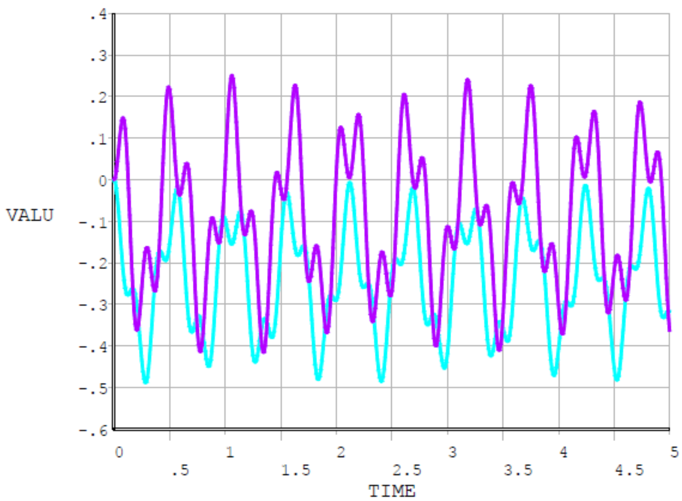

Figure-17
Vibrations of a two-story shear building by transverse earthquake acceleration to the building equal to 0.28 of gravity acceleration by ANSYS 12.0

Conclusion

Although there are particular software packages, including SAP 2000 and ETABS, for dynamics of constructional structures, we showed that multipurpose commercial-research software package, ANSYS, can simulate and study static and dynamics of building structures including multi-story buildings. Here, simple theory of discretizing multi-story building by shear model was used to obtain solutions of transverse free and forced vibrations and acceleration load of earthquake to building base by module. Considering capabilities of ANSYS, it can conduct connected, non-linear andtensile modellings for building structures. Results from forced vibrations by ANSYS are more accurate than SAP 2000.

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